SHEAR FACTOR : A New Way To Characterize Fiber Suspension Shear

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ABSTRACT

A new parameter called shear factor λ is proposed to describe the shear behavior of wood pulp fiber suspensions in specific cases. It is based on a rheological analogy, a flow model, and empirical equations for fiber suspension flow. A plot of friction factor f versus the Metzner-Reed generalized Reynolds number *Re'* on logarithmic coordinates of resulted in a straight line for all friction loss data obtained for nine very heavily beaten pulps. The correlation is identical to the correlation for homogeneous, non Newtonian fluids even though laminar flow does not exist in either case. It is shown that shear factor λ reverts to a non Newtonian apparent viscosity μ_a in these two limiting cases : quasi-homogeneous (continuum-like) flow, and also flow in very small diameter pipes.

Shear factor λ is applied successfully to the limiting case for highly beaten pulps (> 87 °*SR, <28 CSF*) and also flow in small shear gaps and pipes. Shear factor λ can be used to incorporate fiber-fiber and fiber-solid surface interactions, acceleration and deceleration effects, in addition to liquid shear. Shear factor λ has been used to predict disintegrator performance and can also be applied to refiner operations and their characterization.

INTRODUCTION

The rotor or moving element in a beater, refiner or disintegrator causes intense fiber-fiber and fiber-solid surface interactions. The shear gap is usually small in these processes and the fiber concentration often high. The highly localized shear flow is non-uniform and is further complicated by acceleration and deceleration effects across the bars or elements as well as direct impact of fibers on their leading edges. Modeling several shear mechanisms simultaneously is difficult, but it is possible to introduce a single term that effectively incorporates many of these effects. It is also possible to view the narrow-gap shear field as the limiting case of a fiber suspension flowing in a small diameter tube where all the fibers may be active in the flow field. From the point of view of suspension uniformity, more fluid-like behavior tends to occur at low fiber concentrations, in small shear gaps, with very heavily-beaten (low freeness, high Schopper Riegler) pulps, or with a combination of these. Flow in small diameter pipes and heavily beaten glassine-like pulps are examined in this investigation to gain further insight into the behavior of fiber suspensions in narrow shear gaps.

Plug flow exists in pipelines over a wide range of flow rates and fiber concentrations. Consequently laminar flow never exists throughout the entire pipe at concentrations above the sedimentation concentration. This brings into question the meaning of viscosity when used in conjunction with fiber suspension flows. The pipe diameters used in industrial systems are much larger than the mean floc size and the plug network has integrity resulting from interfloccular strength as well as intrafloccular fiber coherence radially across the pipe. In the plug flow regime only the peripheral annular layer is in shear with over 95 percent of the fiber plug being undisrupted. As the pipe diameter is reduced the volume-percentage of the total suspension actively engaged in shear increases exponentially below a diameter of about 10 mm. Radial network coherence decreases as the plug diameter approaches one or two floc diameters in size because the radial interfloccular coherence is weakened. In the limit in small tube diameters, either single distorted flocs exist as a loosely connected longitudinal chain in the pipe, or the flocs are disrupted and dispersed, and the fibers exist as a long cylindrical, continuum-like network. It is the limiting state which has aroused fresh interest as it approximates to the narrow shear gap situation between a rotor and stator in a disintegrator or blender. In the case of a refiner with even smaller shear gaps it is likely that water is squeezed out into the grooves resulting in more solid-solid like shear between the bars.

In tubes where the diameter dimension approaches typical gap sizes the velocity profile would still be uniform and developed. However, in disintegrators and refiners the shear field is asymmetric and undeveloped, and the path length in the shear zone much shorter. Most fibers would experience intense and irregular flow disturbances, mechanical shear, and possibly slippage between each other and the shear surface, as often one surface is stationary and one is moving. This is different from pipe flow where the fibers travel together in bulk flow and the surfaces are stationary. The effective resistance to shear would be expected in the limit to be similar and strongly dependent on fiber population.

A new parameter called shear factor λ is introduced based on a rheological analogy, a flow model, and empirical equations for fiber suspension flow. It is applied successfully to a limiting case for quasi-homogeneous, highly beaten pulps (> 87 °*SR, <28 CSF*) and it is shown that it can be applied to small shear gaps. Shear factor λ has been used to predict disintegrator performance *[1]* and flow in small diameter tubes *[2]*.

CORRELATING PIPE FRICTION FLOW DATA

In experimental fluid mechanics relationships are usually obtained between the wall shear stress τ_w and the wall shear rate $\dot{\gamma}_w$. In pipeline flow the hydraulic gradient is incorporated in the wall shear stress τ_w and for laminar flow the mean shear rate at the wall is *[8U/D]* where *U* is the bulk velocity and *D* the internal pipe diameter (see Appendix). Experimental data are presented graphically as τ_w versus *[8U/D]*, called a rheogram. The coefficient of proportionality of the equation, or the slope in the rheogram, is called the viscosity μ. When the coefficient varies with τ_w (varying slope) an apparent viscosity μ_a is often used. However, in concert with the viscosity parameter comes the mechanistic concept of sliding layers of fluid and thus the combined mathematical and physical meaning of viscosity has traditionally become inseparable.

A dilemma arises for researchers and practitioners working with flowing wood pulp fiber suspensions. As explained previously, because laminar shear does not occur throughout the fiber suspension, the application of the concept of viscosity with lamella-type shear is misleading, and out of character with classical fluid mechanics. Hence, it is proposed for fiber suspension flows that the variable coefficient termed shear factor λ be used as there is no connotation or linkage with sliding shear layers or lamellae.

$$\tau_w = \lambda\left[\frac{8U}{D}\right] \qquad (1)$$

The proportionality coefficient or shear factor λ can incorporate turbulence, and acceleration/deceleration effects as there is no implied mechanistic constraint. In the limits of very low fiber concentration, continuum-like behavior and/or small shear gaps, λ moves towards an expression for viscosity μ.

One of the problems that has arisen from the direct transfer of classical rheological models to structured fiber suspension flows has been the proliferation of models and conflicting concepts that are in essence not applicable to fiber suspensions and convey ideas that are misleading.

(a) Non Newtonian laminar flow equations applied to non laminar fiber suspension flow

Metzner and Reed *[3]* were the first to propose a method for correlating the laminar flow of non Newtonian fluids based on the replacement of the conventional viscosity μ with an apparent viscosity μ_a (see Appendix equation (26)), which was used in the Metzner-Reed generalized Reynolds number *Re'* (Appendix equation (15)). Guthrie *[4]* attempted a few years later to apply this method to flowing hardwood pulp fiber suspensions (0.5-2.0%) in small diameter pipes. His Reynolds number range was 300 to 70,000. Assuming laminar flow he used a power law model (Appendix equation (25)) and found that the flow parameters *K' and n'* were a strong functions of fiber concentration. However, at fiber concentrations greater than about 0.6 to 0.8 percent plug flow would have existed and the flow would have been non-laminar. Guthrie was primarily interested in correlating data for the design of flow systems in paper mills and did not investigate the mechanisms of flow. It is likely that later researchers have followed suit and simply used the *f-Re'* method without proving that the flow is laminar in the first place.

Luthi *[5]* attempted to apply the basic rheological equations to medium consistency MC fiber suspension flow. He assumed that MC fiber suspensions were pseudoplastic and derived an expression for laminar flow in the region before the maximum in the friction loss curve where plug flow exists and the predominant shear mechanism is fiber and floc drag on the pipe wall. This led to calculated velocity profiles and consistency gradients that were not in agreement with experimental findings obtained previously by Mih and Parker*[6]* and Lee and Duffy *[7]*. It had already been reported by Duffy et al. *[8]* that it was possible to rework the empirical equation for the regime before the maximum in the friction loss curve and develop an equation similar to the power law model (Appendix equation (17)). Mathematically the data equation may be similar to the power law model but that is where the similarity ends.

Myreen *[9,10]* for example, assumed that pulp suspensions could be treated as pseudoplastic fluids and obtained velocity profiles which differed greatly from experimentally obtained profiles *[6,7]*. He also assumed that both the sheared layer and the central plug (> 95% behaving as a solid-like network) were in laminar flow. In fact strong fiber/wall interactions occur in plug flow and when hydrodynamic shear does occur the annular film is a thin non uniform laminar layer which at higher flow rates becomes turbulent. Again neglecting the uniqueness of fiber suspension flows and making hypothetical assumptions led to equations and velocity profiles that differed greatly from experimental results.

It is important to realize that wood pulp fiber suspensions are inhomogeneous in most industrial operations. The coincidental mathematical fitting of a simple power law model to fiber suspension flows does not mean that fiber suspensions behave as a pseudoplastic, or even a Bingham plastic fluid. For example, the yield stress (to flow) in a Bingham plastic fluid is due to the fluid, but in a fiber suspension, it is the minor component of elastic fibers trapped in strained configurations dragging on the pipe wall that cause the high initial yield stress, and not the fluid system itself. The direct application of classical rheological models to structured fiber suspension flow is therefore suspect and can lead to erroneous results.

(b) A theoretical model and an empirical equation for plug flow

(i) Empirical model

It is well established that wood pulp fibers in water suspension interact strongly even at low fiber concentrations to form network structures. Elastic fibers are either actively entangled with other fibers due to entrapment following a previous shear event, or simply hinder each other due to the high population. Stresses are set up in the network entities because many elastic fibers or fiber segments are unable to relax and straighten. Strong network cohesion caused by fiber to fiber repulsive forces results.

At low flow rates in pipe flow the peripheral shear zone is characterized by fiber and floc drag on the internal pipe wall *[8]*. As flow rate increases many flocs protruding from the non uniform plug network surface adjacent to the wall break away and roll back at a lower speed along the plug surface to be trapped again by stronger protruding floc asperities. Hence there is a strong fiber-wall interaction in addition to discontinuous laminar liquid shear. Some local secondary flows occur in the space between neighboring protruding flocs. In conventional pipe flow not many fibers are actively involved in the shear field as the suspension is essentially plug flow with a thin sheared annulus which is not laminar flow exclusively.

Empirical equations were obtained for several chemical and mechanical pulps for the same flow regime *[11]*. The equations have the form :

$$\Delta P/L = AC^{\alpha} U^{\beta} D^{\sigma} \tag{2}$$

For a specific chemical long fiber pulp *[11]*, as an example, α = 1.81, β = 0.31 and σ = -1.34, or expressed in terms of β, σ = -(1+ β) approximately. Hence:

$$\Delta P/L = 11.75\ C^{1.81} U^{0.31} D^{-1.34} \tag{3}$$

Equation (3) can be reworked into the form of the power law model (like Appendix equation (17)) although the medium is inhomogeneous, the bulk of the suspension is a coherent solid-like, unsheared plug, and all the shear is confined to the thin region adjacent to the wall :

$$\tau_w = \frac{\Delta P.D}{4L} = \frac{A.C^{\alpha}}{4 \cdot 8^{\beta}} \left[\frac{8U}{D}\right]^{\beta} = K'' \left[\frac{8U}{D}\right]^{\beta} \tag{4}$$

The model (equation (4)) has no direct mechanistic or rheological meaning *[8]* and simply has mathematical similarity to a power law equation (Appendix equation (17)). Many misunderstandings have resulted from extending the power law analogy too far so that the plug flow regime (> 95 percent unsheared plug) is regarded erroneously as laminar flow *[8]*.

(ii) Theoretical model

A theoretical model *[13]* was developed for elastic deformation of the plug undergoing a shear surface stress caused by an idealized thin annular layer of water in laminar shear. The level of shear stress over a range of shear rates produces in effect a subsequent retraction from the wall and a narrow fluid layer in laminar shear.

$$\frac{\partial P}{\partial l} = \left[4 \cdot 8^{1/3} G^{1/3} \mu^{1/3}\right] U^{1/3} D^{-4/3} \tag{5}$$

The shear modulus *G* was also directly related to fiber concentration *C* and the subsequent equation becomes :

$$\frac{\Delta P}{L} = \left(A' \mu^{1/3}\right) C^{1.85} U^{0.33} D^{-1.33} \tag{6}$$

The similarity between equations (6) and (3) is remarkable. For chemical softwood fiber pulps the exponents of most variables in equation (6) are almost identical to the empirical model (equation (3)) although the shear mechanisms are in reality quite different. The theoretical model *[13]* predicts the temperature-viscosity effect on frictional resistance quite accurately. The equation may also be expressed as a power law model (like equation (5)) but again it has the same interpretative restrictions as outlined above. In fact this model should apply to the flow regime between the maximum and minimum in the friction loss curve where a continuous thin laminar layer actually does exist.

At high flow rates similar expressions can be obtained for the relationship between shear stress and strain rate. However, inhomogeneity still exists and the flow is often non uniform. Flow is more complex because the onset of drag reduction, the onset of fully developed turbulence, and the point of maximum drag reduction are all strong functions of fiber concentration, fiber species and the method used to make the pulp.

It may be concluded from fiber suspension flow studies that a variable coefficient of proportionality can be obtained in terms of measured variables that relate wall shear stress and effective wall strain rate. The concept of shear factor λ can therefore be applied to incorporate the measurable and the unmeasurable effects.

APPLICATION OF SHEAR FACTOR

(a) Small diameter pipes

Systematic pipe friction loss data have been obtained by Duffy *[14]* for a wide range of chemical and mechanical, beaten and unbeaten pulps in pipes ranging from 25 to 200 mm diameter. Data have also been obtained from smaller pipe diameters (down to 6.4 mm) for some pulps, and for 3, 5 and 8 mm diameter pipes with one particular pulp type *[15]*. As pipe diameter was decreased it was found that the magnitude of the diameter exponent in equation (5) decreased from -1.34 to -1.10 for the same chemical pulp mentioned earlier as the diameter was reduced to 25 mm. Other data reported in the literature for small diameter pipes are less systematic and no clear conclusions can be made except that the radial network strength decreases with decreasing diameter.

Waterhouse *[16]* used a high pressure rheometer to obtain pressure drop data for a pulp suspension flowing through two annuli having very small clearances (0.137 and 0.204 mm). The slits were an order of magnitude larger than the fiber diameter and were close to the gap dimensions encountered in disc or conical refiners. The consistency range was from 0.5 to 5.0 percent. A graph of friction factor versus Reynolds number (using the viscosity of water) gave similar curves to those in the low Reynolds number range for larger diameter pipes with the curves being offset from the laminar flow water curve. The magnitude of offset increased with increasing consistency in both cases. However, plotting friction factor versus the generalized Reynolds number brought all the data onto a single curve which matched the laminar flow curve for non Newtonian fluids flowing through a single annular slot ($f = 12/Re'$ derived from the Rabinowitsch-Mooney equation (Appendix (18)). Waterhouse *[16]* commented that neither laminar flow nor plug flow were likely to exist in the small annular spaces. It is most likely that laminar shear would be interrupted by the presence of several adjacent fibers in the annuli. Also the annular gaps were smaller than the fiber length thus preventing coherent floc formation and the subsequent development of a coherent plug. The concept of shear factor λ would therefore be a reasonable way of describing the shear in very small shear gaps although the data fit the friction factor f - generalized Reynolds number Re' correlation for non Newtonian fluids where the apparent viscosity μ_a is used. The apparent viscosity μ_a could be replaced with shear factor λ in the generalized Reynolds number Re'.

(b) Heavily beaten pulps

Thirteen heavily beaten pulps were made from a mixture of different base pulps. They were beaten to a level between 87 and 91.5 degrees Schopper-Riegler °*SR* (28 to 20 CSF) and tested over a flow rate range from 0.5 to 1.5 m/s in a vertical 80 mm diameter pipe. The consistency range was from

3.0 to 3.75 percent the temperature ranged from 54.2 to 66.3 ºC. Pressure differential measurements were taken over a 3.0 m length of pipe.

A typical set of processed data for one of these pulps is plotted in a rheogram in **Figure 1** as wall shear stress τ_w versus the mean gradient at the wall [8*U/D*] (91.5 ºSR, *C* = 3.75 %, T = 64.9 ºC). The level of shear stress increases with shear rate. The ratio of the quantities at each point in the curve $\tau_w/[8U/D]$ gives the point magnitudes of the shear factor λ (equivalent to μ_a if the suspension was replaced with a liquid). The resultant plot of shear factor λ versus [8*U*/*D*] is presented in **Figure 2** showing that the value of shear factor decreases with shear rate in a manner similar to a fluid in laminar shear. If the data from Figure 2 are replotted on logarithmic coordinates, the linear portion of the curves can be fitted to obtain the *K* and *n* parameters according to equation (5). The values of shear factor λ however are one to two thousand times the magnitude for the viscosity of water alone in the range of the conditions investigated.

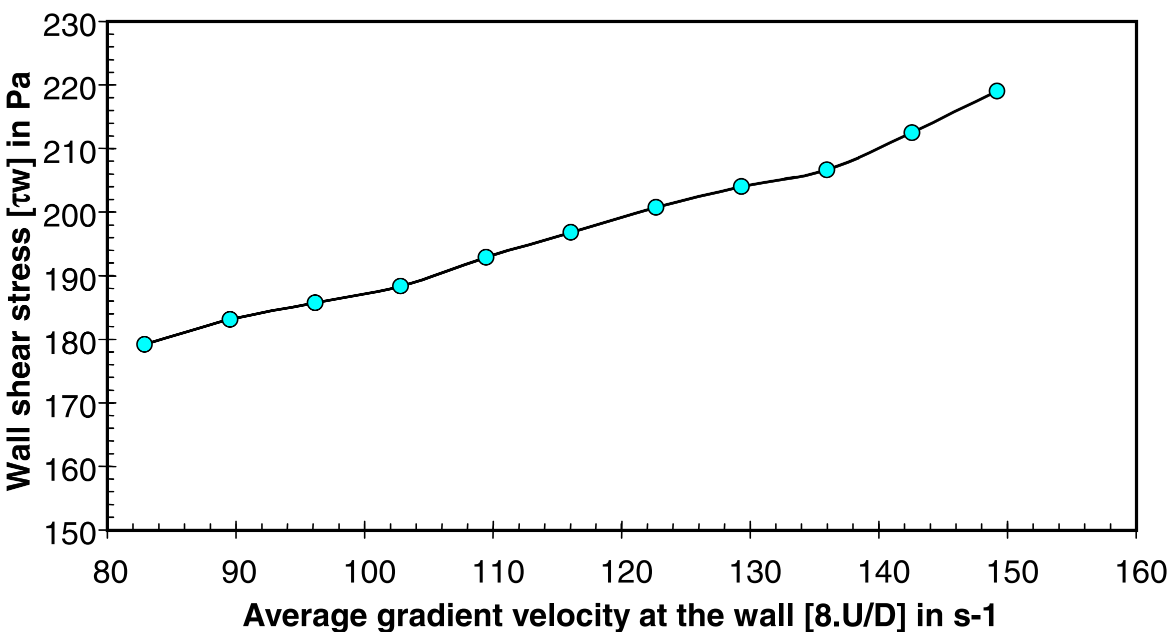

Figure 1. Wall shear stress versus average gradient velocity at the wall. Pulp characteristics : 91.5 ºSR - C = 3.75 % - T = 64.9 ºC.

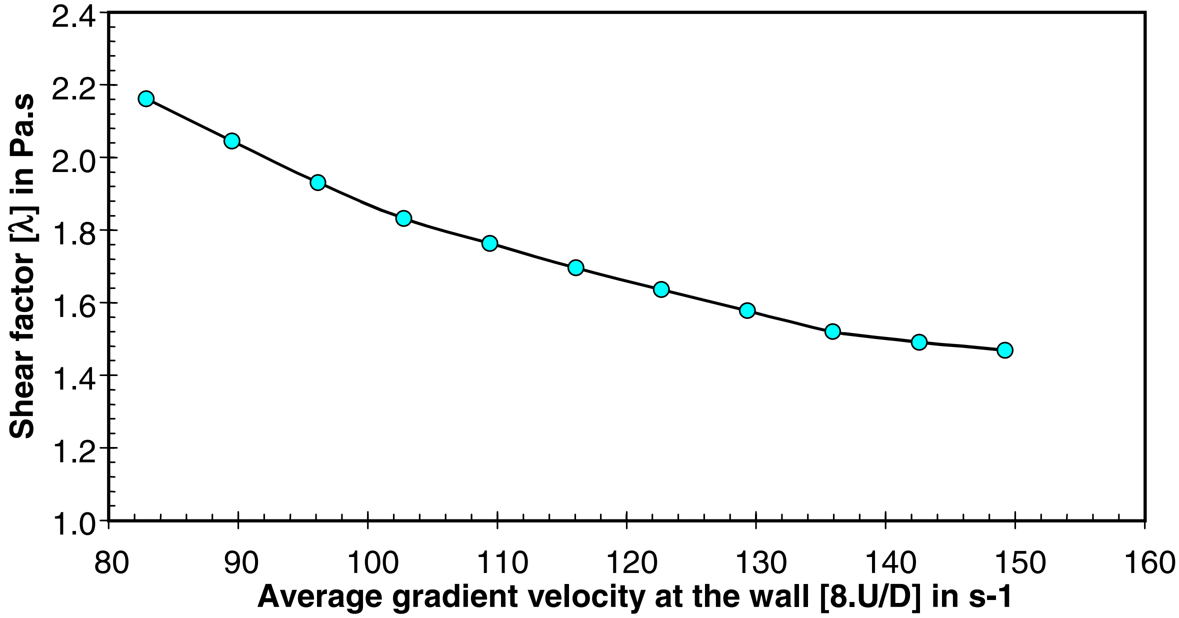

Figure 2. Shear factor versus average gradient velocity at the wall. Pulp characteristics : 91.5 ºSR - C = 3.75 % - T = 64.9 ºC.

(c) Influence of temperature

Typical data for shear factor versus mean gradient at the wall are presented in **Figure 3** for a 3.25 percent consistency pulp beaten to 91.5 º*SR* (20 CSF) at two different temperatures, namely, 54.6 ºC and 65.5 ºC. The curves converge to the same level of shear factor 1.23 Pa.s as the average gradient goes from 70 s^{-1} to 150 s^{-1}.

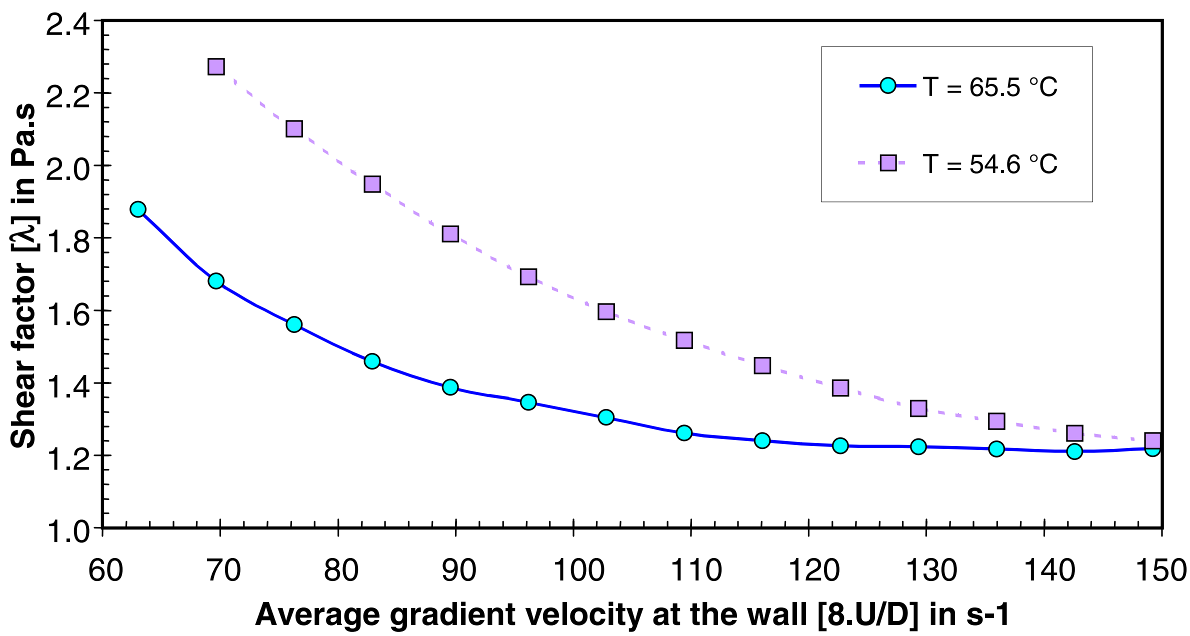

Figure 3. Influence of the temperature. Pulp characteristics : 91.5 ºSR - C = 3.25 %.

The local slope n' of the plot ln (τ_w) versus ln [8*U/D*] (see Appendix equation (17)) can be obtained for both cases :

$n' = 0.18$ for T = 54.6 ºC
$n' = 0.52$ for T = 65.5 ºC,

Showing n' has a greater value at the higher temperature.

(d) Effect of beating level

Data are presented in **Figure 4** at two levels of refining investigated, namely, 87 and 91.5 ^{o}SR (28 and 20 CSF). It can be seen that shear factor λ decreases with increased beating and increasing shear rate over the range of experimental shear rates (70 s^{-1} to 150 s^{-1}). The difference between the two curves is approximately constant at 0.2 Pa.s over the entire range of shear rates.

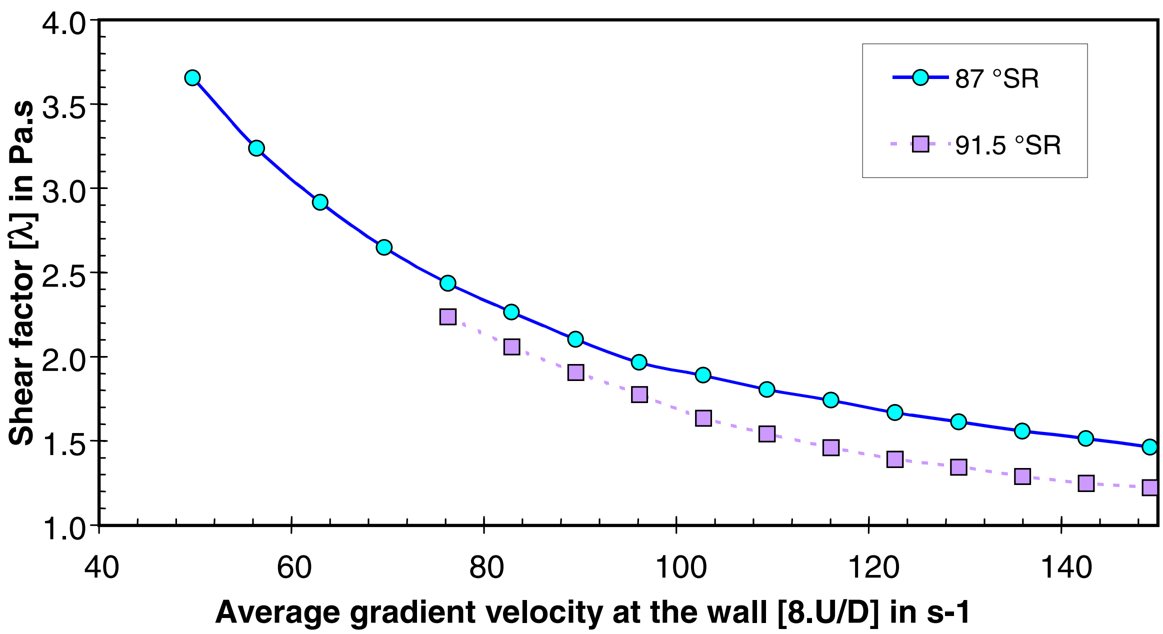

Figure 4. Influence of the beating. Pulp characteristics : C = 3.45 % - T = 63.5 ºC.

(e) Influence of fiber concentration

Pipe friction loss data were obtained for the heavily beaten pulps over a range of consistencies: 3.0, 3.25, 3.5, and 3.75 percent. **Figure 5** is a plot of shear factor λ versus average gradient at the wall [8*U/D*] for these data showing that shear factor decreases with increasing shear rate and decreasing consistency. Again it is seen that the decrease in shear factor is not linear as consistency decreases.

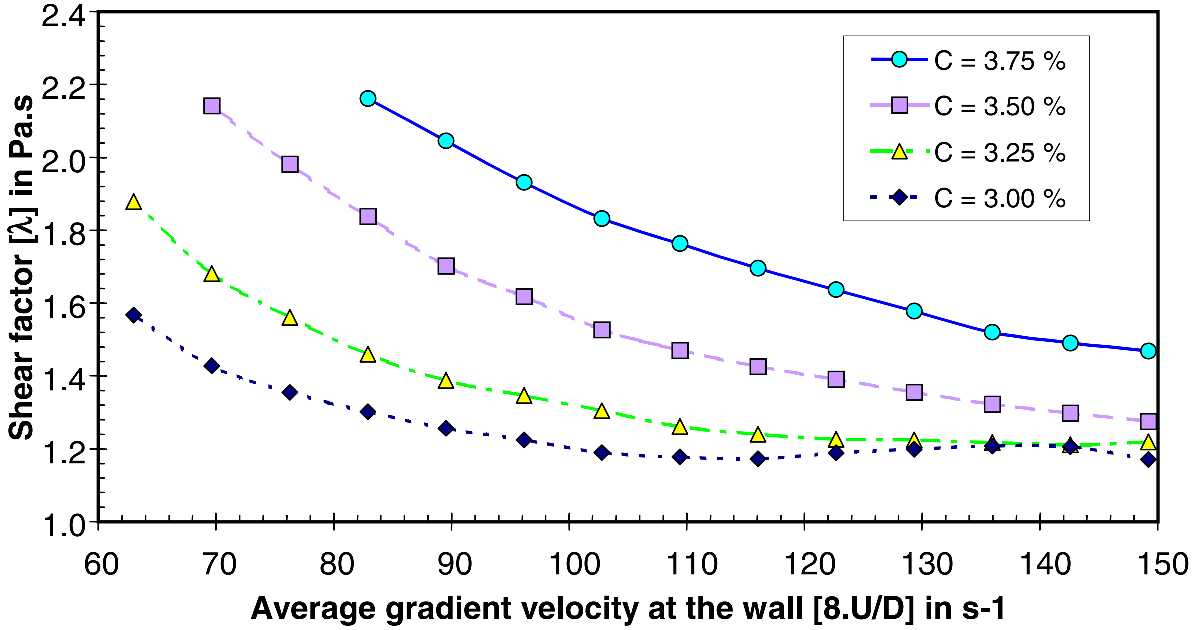

Figure 5. Influence of the consistency. Pulp characteristics : 91.5 ºSR - T = 65.6 ºC.

Appendix equations (26) and (24) can be combined and rewritten by replacing apparent viscosity μ_a with shear factor λ :

$$\lambda = K\left[\frac{3n+1}{4n}\right]^n\left[\frac{8U}{D}\right]^{n-1} \qquad (7)$$

The two parameters (K, n) can be deduced from the values of K' and n' over the range of velocity gradients investigated. The experimental values of shear factor λ are obtained from the modified equation (1) (similar to Appendix equation (22)):

$$\lambda = \frac{\tau_w}{\frac{8U}{D}} \qquad (8)$$

Then the predicted values of λ can be determined from the equation (8). The comparison in **Figure 6** between the experimental and predicted values of shear factor shows excellent agreement with a correlation coefficient r^2 of 0.99. The result is remarkable as it covers the whole range of data in this investigation for several highly beaten pulps, a range of temperatures and fiber consistencies.

All the rheological data obtained in this investigation (namely, 3% < C < 3.75%, 87 < °*SR* < 91.5, 14.5 ºC < T < 66 ºC) were reprocessed and graphed in **Figure 7** as friction factor versus generalized Reynolds number *Re'* using the shear factor λ values as the 'equivalent' apparent viscosity. The correlation coefficient r^2 of 0.996 clearly shows an excellent relationship for the data obtained in this investigation. The relationship of f = *16/Re'* is exactly what Waterhouse *[16]* found for flow through small annular slots (f =*12/Re'*). In this case the highly beaten state of the fiber suspension has less interlocking strength than conventional pulps but plug flow still exists in the 80 mm diameter pipe. Consequently, although the typical friction factor–generalised Reynolds number correlation for laminar flow of non Newtonian fluids holds for this non-laminar case for heavily beaten pulps in the plug flow regime with only shear near the wall, shear factor λ is a suitable way of describing the fiber suspension.

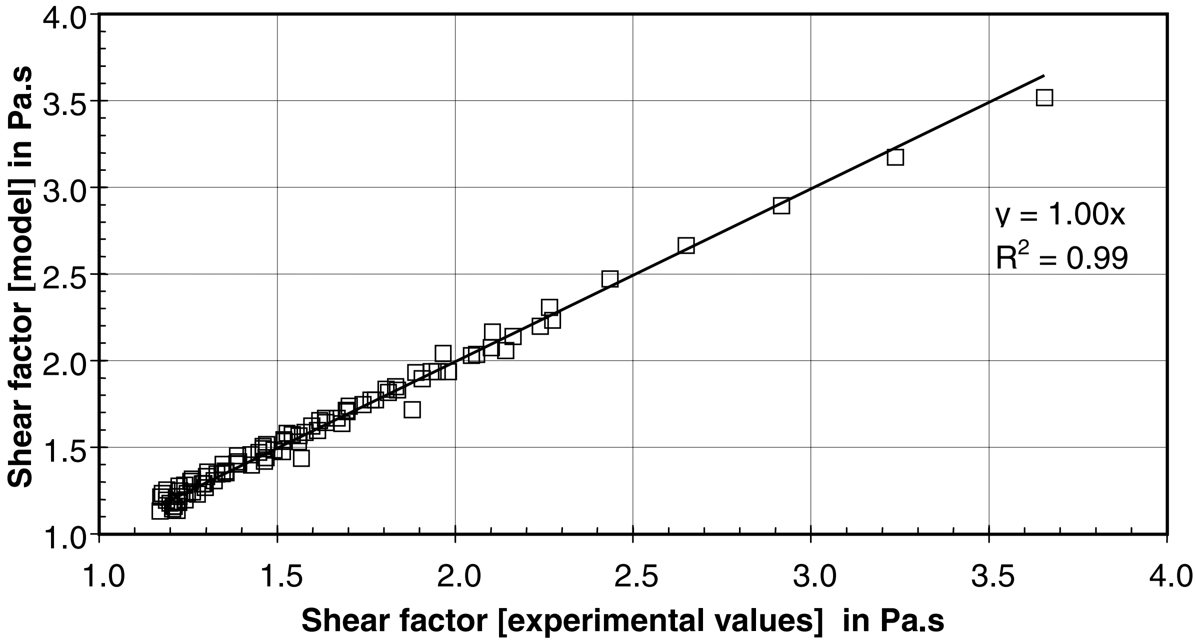

Figure 6. Comparison between model and experimental values of the shear factor.

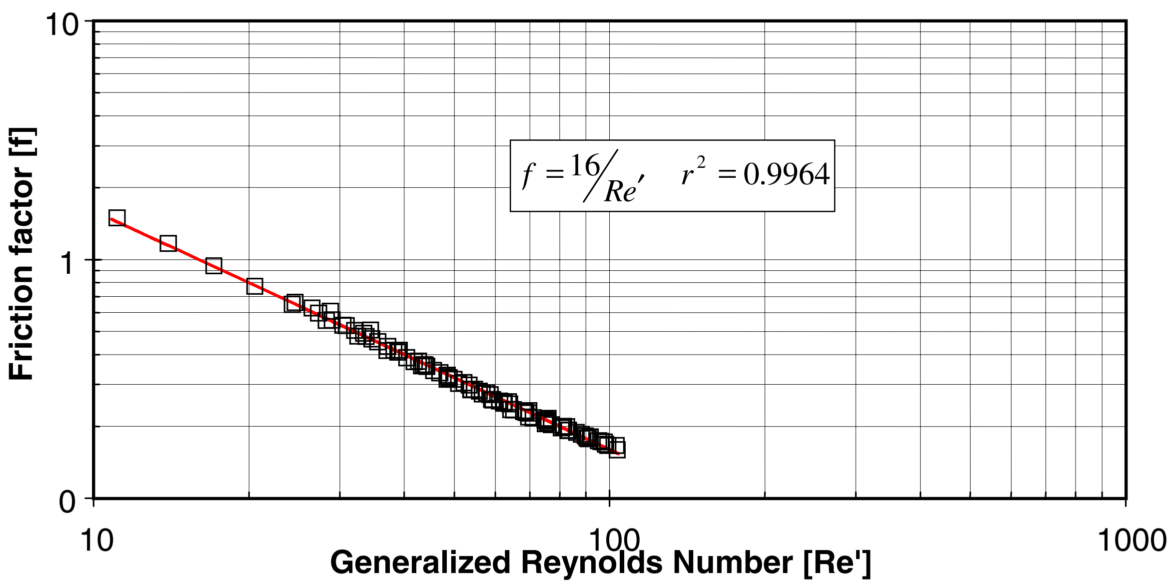

Figure 7. Friction factor versus the generalized Reynolds number.
3.0% < C < 3.75%
87 < ºCSR < 91.5
54.6 ºC < T < 66 ºC.

DISINTEGRATOR PERFORMANCE PREDICTIONS FROM SHEAR FACTOR

It has been pointed out that in any close tolerance shear gap there are acceleration and deceleration forces, solid surface-fiber, and fiber-fiber interactions in addition to fluid shear. It has also been shown that in the limiting cases of small shear gaps or slots, or heavily beaten continuum-like pulps, the friction factor-generalized Reynolds number holds even though laminar flow does not exist throughout. Hence it is better for these cases to replace apparent viscosity μ_a with shear factor λ, and then extend the concept to include the other factors in addition to liquid shear.

Shear factor λ has been used (1) to predict the desintegrator performance and it is remarkable that a master curve can be obtained if the power number is plotted against the product of Reynolds number and Froude number to the third power , on logarithmic coordinates.

SHEAR FACTOR APPLIED TO REFINING

By studying the physics of the process in disc refiners, the axial force exerted on the shaft is found to have two contributions : a hydraulic force, and a normal reaction force exerted on the fibrous pads in each confined zone of the gap clearance. This compressive force is inversely proportional to the mean gap clearance during the refining process whatever the refining trial and the state of the pulp. This result can be theoretically obtained by considering the shear factor λ. The variation of shear factor is highly process dependent. Two important results come from this :

- A theoretical equation for the net power consumed in a disc refiner can be derived similar to Dalzell's empirical equation already published almost forty years ago *[17]* using the shear factor λ,
- A Sommerfeld number for the refining operation in disc machines that reinforces a hydrodynamic description of the refining process *[18]*.

Pulp and paper properties can also be related to the refining operation through shear factor as this has already demonstrated in *[19]*.

CONCLUSIONS

Fiber suspensions in processes such as disintegrators, refiners as well as pipes, often flow in conditions far from the laminar regime. In these conditions, using an apparent viscosity is not a satisfactory way of describing shear phenomena. However, it is always possible to relate the wall shear stress to the wall strain rate even if the flow is not laminar and/or inhomogeneous. In this paper, it is demonstrated that the coefficient proportionality of between the shear stress and the strain rate at the wall, called shear factor, is not a fluid property but integrates all the friction effects that occur to the pulp suspension in various situations and geometry's. It is proven that for highly beaten pulp suspensions that simulate a fluid-like continuum and for suspension shear in small gaps, the shear factor reverts to the pulp apparent viscosity μ_a.

NOMENCLATURE

$(du/dy)_w$: strain rate at the wall, s^{-1}
ΔP : pressure differential, Pa
A : constant in Eq. (4)
A' : constant in Eq. (6)
C : fiber concentration, %
D : diameter pipe, m
G : shear modulus
K' : constant in Eq. (17)
K : parameter in Eq. (7)
L : length of pipe, m
l : mean distance over which eddies exist, m
n' : local slope in Eq. (17)
n : parameter in Eq. (9)
T : temperature, °C
U : bulk velocity, $m.s^{-1}$
y : distance from the internal pipe wall, m

dimensional numbers :

f : friction factor $= \tau_w \Big/ \rho \frac{U^2}{2}$
Re' : generalized Reynolds number $= \rho UD/\mu_a$

Greek symbols:

$\dot{\gamma}_w$:gradient at the wall, s^{-1}
ρ : density of water, kg/m^3
η : eddy viscosity, Pa.s
λ : shear factor, Pa.s
α, β, σ : constant in Eq. (2)
τ_w : wall shear rate, Pa
μ : dynamic viscosity, Pa.s
μ_a : apparent viscosity, Pa.s

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APPENDIX

A RHEOLOGICAL ANALOGY FOR INHOMOGENEOUS FIBRE SUSPENSION FLOW

(i) Laminar flow of non Newtonian Fluids

The flow of a homogeneous, steady, incompressible, non Newtonian fluid in fully developed laminar flow in a pipe is often simply modeled by the Ostwald de Waele power law equation. It relates the wall shear stress τ_w to the strain rate $(du/dy)_w$ at the wall :

$$\tau_w = K\left[\left(\frac{du}{dy}\right)_w\right]^n = K[\dot{\gamma}_w]^n \qquad (9)$$

where $\dot{\gamma}_w$ is the shear gradient at the wall, y is the distance from the internal pipe wall, and K and n are two constant fluid properties. When the flow behavior is known, n is a measure of the non Newtonian character of the flow (when $n = 1$, K becomes the Newtonian or dynamic viscosity μ). Both τ_w and $\dot{\gamma}_w$ are derived from measurable quantities : pressure, flow rate and pipe diameter. Wall shear stress is given by :

$$\tau_w = \frac{\Delta P \cdot D}{4L} \qquad (10)$$

And the strain rate (or shear rate) at the wall is :

$$\left(\frac{du}{dy}\right)_w = \dot{\gamma}_w = \left(\frac{3n+1}{4n}\right)\left[\frac{8U}{D}\right] \qquad (11)$$

Combining equations gives :

$$\tau_w = K\left[\frac{3n+1}{4n}\right]^n\left[\frac{8U}{D}\right]^n \qquad (12)$$

The parameters K and n can be obtained from a plot of $ln(\tau_w)$ versus $ln[8U/D]$ where the flow characteristic $[8U/D]$ is the average gradient at the pipe wall. The apparent viscosity μ_a defined in equation (13) as the stress-shear rate ratio $\tau_w /[8U/D]$ can easily be expressed in terms of the other parameters:

$$\mu_a = \frac{\tau_w}{[8U/D]} = K\left[\frac{3n+1}{4n}\right]^n\left[\frac{8U}{D}\right]^{n-1} \qquad (13)$$

where $[8U/D]$ is precisely the strain rate at the wall for a Newtonian fluid (equation (12) when $n = 1$). The assumption of laminar flow of a fluid can be verified a-priori if the friction factor f versus Metzner-Reed generalized Reynolds number Re' is plotted.
Hence :

$$f = \frac{\tau_w}{\rho U^2/2} \tag{14}$$

and the generalized Reynolds Re' number is :

$$Re' = \frac{\rho U D}{\mu_a} \tag{15}$$

where ρ is the density of water and μ_a is the apparent viscosity according to equation (13). For laminar flow the friction factor f generalized Reynolds number Re' relationship is:

$$f = \frac{16}{Re'} \qquad (Re' < 2100) \tag{16}$$

Equation (9) calls for both K and n to be constant fluid properties. When this is not the case, point values K' and n' can be used with a simplified empirical and power-law equation that relates wall shear stress τ_w and the expression $[8U/D]$ (strain rate at the wall only for the case of a Newtonian fluid):

$$\tau_w = K'\left[\frac{8U}{D}\right]^{n'} \tag{17}$$

The interrelationships can be found *[20]* from the following using the Rabinowitsch-Mooney equation which is valid for of any time independent fluid in steady, laminar flow conditions :

$$\left[\frac{8U}{D}\right] = \frac{4}{\tau_w^{\,3}} \int_0^{\tau_w} \tau^2 \cdot \dot{\gamma} \cdot d\tau \tag{18}$$

The general expression for wall shear rate can then be derived :

$$\dot{\gamma}_w = \left[\frac{8U}{D}\right] \cdot \left[\frac{3}{4} + \frac{1}{4}\frac{d(ln[8U/D])}{d(ln\,\tau_w)}\right] \tag{19}$$

Data can be plotted as $\ln(\tau_w)$ versus $\ln[8U/D]$, and the local slope n' at any point on the abscissa can be defined as :

$$n' = \frac{d(ln\,\tau_w)}{d(ln[8U/D])} \tag{20}$$

Hence, the local shear rate $\dot{\gamma}_w$ can be obtained by substitution of equation (20) in equation (19):

$$\dot{\gamma}_w = \left(\frac{3n'+1}{4n'}\right)\left[\frac{8U}{D}\right] \tag{21}$$

The apparent viscosity can also be obtained from the same plot by analogy with a non Newtonian fluid (equation (13)), according to its definition:

$$\mu_a = \frac{\tau_w}{8U/D} \tag{22}$$

If n' is constant at a point or can be regarded as constant over a narrow range of experimental data, equation (20) can be integrated in that case to lead to a power law type relationship as given by equation (17).

Combining equations (21) and (17) gives :

$$\tau_w = K'\left(\frac{4n'}{3n'+1}\right)^{n'}(\dot{\gamma}_w)^{n'} \tag{23}$$

In this case the local slope n' equals n and the constant K can be obtained by identification between the previous and the equation (9):

$$K' = K\left(\frac{3n'+1}{4n'}\right)^{n'} \tag{24}$$

Equation (17) may always be expressed as:

$$\tau = K'\left[\frac{8U}{D}\right]^{n'-1}\left[\frac{8U}{D}\right] \tag{25}$$

Comparing equation (25) with equation (22) shows that:

$$\mu_a = K'\left[\frac{8U}{D}\right]^{n'-1} \tag{26}$$

Hence the apparent viscosity μ_a can be determined independently by two methods:

- one using direct measurements of pressure drop, bulk velocity U and pipe diameter D , equations (10) and (13)
- one using U and D and the two parameters K' and n', equation (17).

The express development of these equations is important because if a highly beaten pulp behaves as a homogeneous fluid in steady and laminar flow conditions (limiting case), then the analogy is not only a mathematical one but also a rheological one. Hence the apparent viscosity of the heavily beaten pulp suspension can be determined. The parameters K' and n' which in turn are a function of concentration, temperature, state of refining, and the nature of the slurry. For cases where laminar flow does not exist, the basic mathematical relationship between τ_w and [$8U/D$], of equation (22) can always be used and shear factor λ can replace μ_a.

(b) Extension to turbulent flow

A simple approach to modeling turbulent flow has been to add an extra term in Newton's equation for viscosity μ by assigning an eddy viscosity η to describe the general state of turbulence :

$$\tau_w = (\eta + \mu)\frac{d\bar{u}}{dy} \tag{27}$$

Unlike laminar flow eddy viscosity η is not a fluid property. One approach to give η some physical significance has been the application of Prandtl's mixing length theory. The mean distance l over which eddies exist, fluid density ρ, and velocity gradient $(d\bar{u}/dy)$ provide a simple way of describing the state of turbulence and hence eddy viscosity.

$$\eta = \rho l^2 \left(\frac{d\bar{u}}{dy} \right) \tag{28}$$

At high shear rates when the eddy viscosity η is greater than fluid viscosity μ this parameter may be regarded as the coefficient of proportionality between wall shear stress τ_w and the velocity gradient $(d\bar{u}/dy)$:

$$\tau_w = \eta \frac{d\bar{u}}{dy} = \left[\rho l^2 \left(\frac{d\bar{u}}{dy} \right) \right] \frac{d\bar{u}}{dy} \tag{29}$$

In our case the eddy viscosity η equation (28) can be expressed as the shear factor λ.

For both turbulent and laminar cases wall shear stress is proportional to shear rate but the coefficient of proportionality (a variable) is often difficult to determine. A coefficient of proportionality could be used to relate shear stress and shear rate for fiber suspensions in shear zones of small dimensions the size of a fiber or floc if the suspension is close to being homogeneous (like a continuum) or fluid-like.

For small-gap, rotor-stator devices the shear rate is of the order of 10^4 s^{-1}. At low consistencies and at these high shear rates the flow would tend to be turbulent. At higher consistencies the fiber populations would be greater and there would be a tendency toward plug-like shear flow, although generally only one solid surface would be in motion. Hence, either a pseudo-eddy viscosity or a pseudo-apparent viscosity could be used. However, it is usually more complex in that there are additional fiber-fiber, fiber-solid surface interactions, and acceleration and deceleration effects, as well as the influence of vessel geometry and bulk flow pattern effects. To avoid confusion, a variable coefficient of proportionality called the shear factor λ is proposed in this paper to incorporate all these effects.