INFLUENCE OF GRAVITY ON CURL OF HORIZONTAL PAPER SHEETS

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ABSTRACT

The influence of gravity on the out-of-plane hygroinstability (curl) of horizontal paper sheets was studied analytically and experimentally. Nondimensional parameters describing the relative importance of moisture and gravity on paper deflections are defined. An approximative analysis based on cylindrical bending allows prediction of contact length, taking into account moisture and gravity. Linear and geometrically nonlinear finite-element analyses were performed on sheets resting on a flat horizontal surface. The contact problem was solved using an algorithm incorporated in the finite-element code. Gravity reduced the deflections and flattened the sheet over a contact region that decreases with increasing basis weight. The results indicate that the influence of gravity should be considered when evaluating the curl for horizontal papers of basis weight below 120 g/m^2 and size equal to or larger than A4 (210 mm × 297 mm). Two-ply laboratory sheets of 60- and 100-g/m^2 basis weight were manufactured from bleached chemical pulp. Vertical deflections were measured on horizontal sheets measuring 17 cm × 23 cm. It was found that gravity substantially reduced the deflections and that the sheets flattened in a central contact region. The predicted and experimentally observed deflections agreed within ±1 mm.

INTRODUCTION

Dimensional changes in paper sheets derive from the hygroscopic nature of fibers and their associated swelling or "hygroexpansion." When the constituent fibers are exposed to changes in humidity, they expand or contract, and their dimensional changes are transferred to the paper. If there is a gradient of hygroexpansion through the thickness of the paper, out-of-plane deformations will arise, and the paper will curl when subjected to a change in relative humidity.

Figure 1 illustrates how the "state of curl" is separated into three basic curvatures: κ_x, κ_y, and κ_{xy}, where κ_x and κ_y represent bending curvatures, while κ_{xy} is the twist curvature. Curl is typically maximized for sheets with a steep gradient of hygroexpansion through sheet thickness. The economic significance of the curl problem has prompted considerable research efforts to identify processing strategies to eliminate or reduce curl *(1–9)*. Additional efforts have been taken to experimentally characterize the curl shape of papers *(10–12)*.

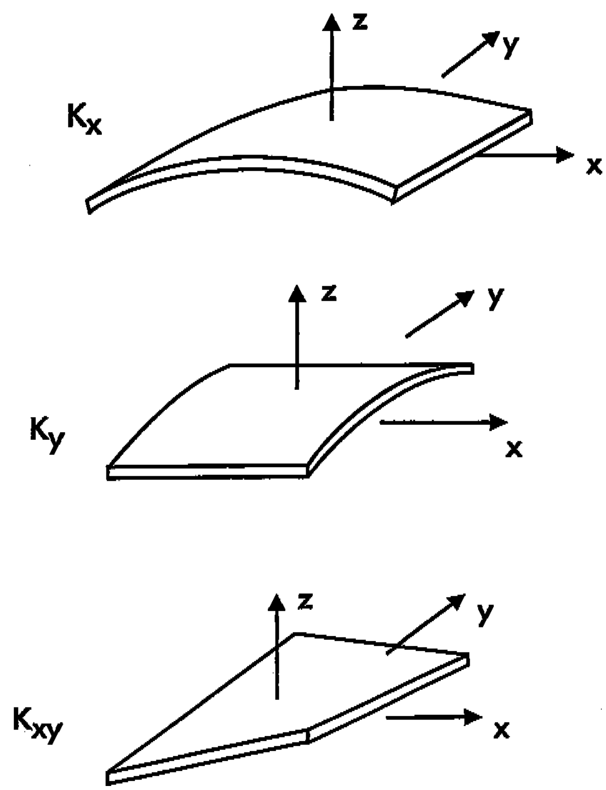

Figure 1. The state of curl is expressed by the curvatures κ_x, κ_y, and κ_{xy}.

One of the factors affecting the curl curvatures of low-basis-weight papers is gravity, although this has not received much attention in the literature. For example, if a low-basis-weight, curled copy paper is placed horizontally on a table, direct observation reveals that the state of curl and the deflections are influenced by gravity. Consequently, efforts to characterize the curl of low-basis-weight papers must (a) consider the influence of gravity on the out-of-plane deflection and (b) isolate the geometrical and material parameters that influence the deflection.

In this study, we present structural analyses and experiments on the influence of gravity on curl. An approximative analysis based on cylindrical bending is presented, and the finite-element method is applied using linear and geometrically nonlinear formulations of the strains. The predictions are compared with experiments on two-ply papers.

THEORETICAL BASIS

Nondimensional parameters

The governing plate equations including nonlinear kinematics to accommodate moderate rotations for paper sheets subjected to a change in moisture content were presented in our earlier work *(13)*. If gravity is also taken into account, the previous model *(13)* is slightly modified. Nondimensional variables, denoted by overbars according to the previous model *(13)*, are introduced first:

$$N_{\alpha\beta} = tE\left(\frac{t}{L}\right)^2 \overline{N}_{\alpha\beta}$$

$$M_{\alpha\beta} = t^2 E\left(\frac{t}{L}\right)^2 \overline{M}_{\alpha\beta}$$

$$x_\alpha = L\overline{x}_\alpha \qquad \textbf{(1)}$$

$$w = t\overline{w}$$

where $N_{\alpha\beta}$ denotes in-plane forces per unit length; $M_{\alpha\beta}$ is bending moments per unit length; x_α is the in-plane coordinate; w is the out-of-plane displacement; t is paper thickness; E is a typical elastic modulus; and L is a typical in-plane dimension. Following our previous work *(13)*, the governing differential equations for the out-of-plane displacements $\overline{w}$ can be expressed as

$$\overline{N}_{\alpha\beta,\beta} = 0$$

$$\overline{M}_{\alpha\beta,\alpha\beta} + \overline{N}_{\alpha\beta}\overline{w}_{,\alpha\beta} - p = 0 \qquad \textbf{(2)}$$

These equations are subjected to homogeneous boundary conditions except for the bending moment $\overline{M}_{\alpha\beta}\, n_\alpha n_\beta$ defined in Eq. 3

$$\overline{M}_{\alpha\beta} n_\alpha n_\beta = \omega q \qquad \textbf{(3)}$$

where n_α denotes a normal vector to the boundary and q a parameter (defined in our previous work *(13)*) of the order of magnitude of one. In Eqs. 2 and 3, the indices α and β can take the values 1 and 2, and repeated indices denote summation. Partial differentiation is denoted by a comma. The nondimensional parameters ω and p define the amplitudes of the moisture and gravity loads, respectively.

$$\omega = \left(\frac{L}{t}\right)^2 \Delta m\, \Delta\beta$$

$$p = \frac{\rho t g}{E} \cdot \left(\frac{L}{t}\right)^4 \qquad \textbf{(4)}$$

In Eq. 4, Δm denotes the change in moisture content; $\Delta\beta$ is the maximum difference in hygroexpansion coefficients in the thickness direction of the paper; ρt is the weight per unit area; and g is the gravitational constant ($g = 9.81$ m/s^2).

It can be seen that if either ω or p is large, nonlinear kinematics must be taken into account. Linear kinematics can be applied if both ω and p are small. Furthermore, if $\omega >> p$, then the gravitational effects can be neglected; and if $\omega << p$, the moisture expansion effects can be neglected. A new nondimensional number η can hence be defined as

$$\eta = \frac{\omega}{p} = \Delta m \Delta\beta \cdot \frac{E}{\rho t g} \cdot \left(\frac{t}{L}\right)^2 \qquad \textbf{(5)}$$

expressing the relative importance of moisture compared with gravity. Consequently, a large magnitude of η indicates that gravitational effects are less important than moisture expansion effects.

Approximative analysis

In applications, it may be useful to have an approximative closed-form expression by which the combined effects of moisture and gravity on the sheet deflections can be estimated. An initially flat paper is considered. It is first assumed that the paper is oriented so that an edge of the paper will bend upwards due to a moisture change. Without gravity, this moisture change Δm corresponds to a certain cylindrical curvature κ_y. An equivalent problem is then defined by bending moments (M) applied to the edges corresponding to the same curvature. It is first assumed that the bending length L resulting from bending moments and gravity is smaller than half the width of the paper (L_0), as seen in **Figure 2a**.

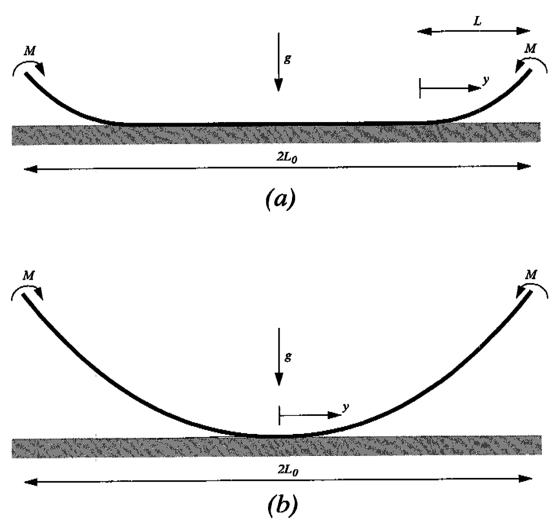

***Figure 2. Cylindrical bending for free edges with a tendency to bend upward; a: nonvanishing contact area,* $L < L_0$*; b: pointwise contact area,* $L \geq L_0$.**

The relation between bending moment M and moisture curvature κ_y is defined by Eq. 6:

$$M = \frac{Et^3}{12(1-\upsilon^2)} \cdot \kappa_y \tag{6}$$

The moment M_g due to gravity at $y = 0$ is given by Eq. 7:

$$M_g = -\frac{\rho g t L^2}{2} \tag{7}$$

The condition of vanishing bending moment at $y = 0$ gives the bending length (Figure 2a),

$$\left(\frac{L}{t}\right)^2 = \frac{Et}{\rho t g} \cdot \frac{\kappa_y}{6(1-\upsilon^2)} \tag{8}$$

where E denotes the effective bending modulus, which for nearly homogeneous sheets to a good accuracy can be estimated as the mean E-modulus over the paper thickness; υ the Poisson ratio; and t the paper thickness. From beam theory, the deflection w can be expressed as

$$w = \frac{\kappa_y L^2}{12}\left[4\left(\frac{y}{L}\right)^3 - \left(\frac{y}{L}\right)^4\right] \quad \textbf{(9)}$$

where L is given by Eq. 8. Hence, the length L and consequently the deflection w are nonlinear functions of the curvature κ_y. In the case that the length L estimated from Eq. 8 is larger than half the width of the paper considered (L_0), the contact length shrinks to a point, as seen in Figure 2b, and the beam expression for the deflection takes the form

$$w = \frac{\kappa_y L_0^2}{12}\left\{6\left(\frac{y}{L_0}\right)^2 + \left(\frac{L_0}{L}\right)^2\left[-6\left(\frac{y}{L_0}\right)^2 + 4\left(\frac{y}{L_0}\right)^3 - \left(\frac{y}{L_0}\right)^4\right]\right\} \quad \textbf{(10)}$$

where L still is defined by Eq. 8.

If the paper edge instead tends to bend downward due to moisture, any of the three scenarios defined in **Figure 3** can be realized.

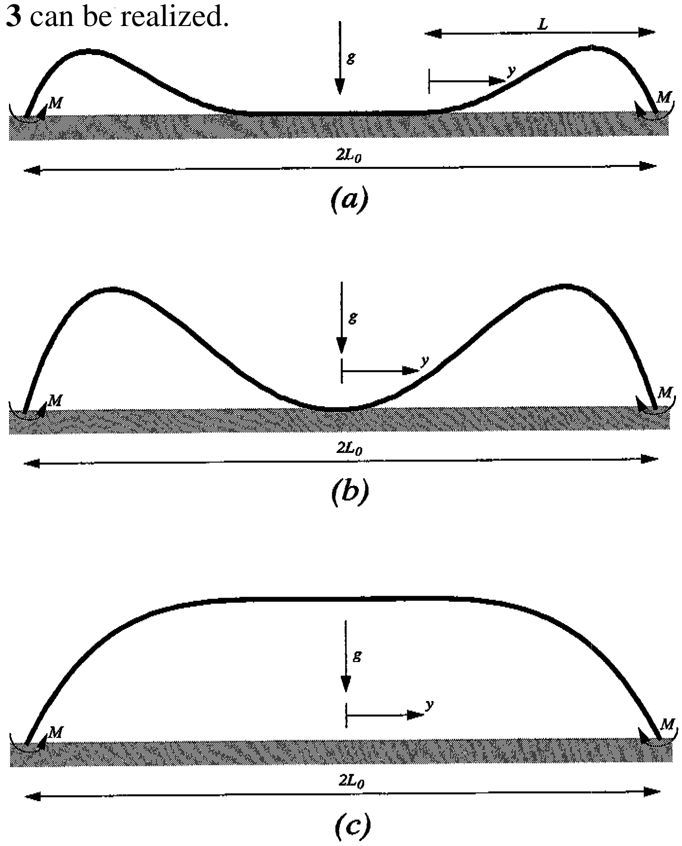

Figure 3. Cylindrical bending for free edges with a tendency to bend downward; a: nonvanishing contact area, $L < L_0$ ***; b: pointwise contact area at midpoint,*** $L_0 \le L \le \sqrt{5/3} \cdot L_0$***; c: solely edge contact,*** $L \ge \sqrt{5/3} \cdot L_0$.

The case shown in Figure 3a is considered first. Beam theory with the condition of a vanishing bending moment at $y = 0$ then gives

$$\left(\frac{L}{t}\right)^2 = \frac{Et}{\rho t g} \cdot \frac{\kappa_y}{3(1-\upsilon^2)} \tag{11}$$

with the corresponding deflection

$$w = \frac{\kappa_y L^2}{6}\left(\frac{y}{L}\right)^3\left[1-\left(\frac{y}{L}\right)\right] \tag{12}$$

If the length L according to Eq. 11 is larger than half the width of the paper (L_0), two distinct cases can be defined. For $1 \le L/L_0 \le \sqrt{5/3}$, the paper has a pointwise contact at the center with a finite reaction force (Figure 3b); and for $L/L_0 > \sqrt{5/3}$, the paper only has contact with the flat surface at the edges (Figure 3c). The deflections for the three cases in Figure 3 can be derived from beam theory as

$$w = \frac{\kappa_y L^2}{6}\left(\frac{y}{L}\right)^3\left[1-\left(\frac{y}{L}\right)\right] \text{ for } L \le L_0$$

$$w = \frac{\kappa_y L_0^2}{12}\left\{3\left[\left(\frac{y}{L_0}\right)^2-\left(\frac{y}{L_0}\right)^3\right]+\left(\frac{L_0}{L}\right)^2\left[-3\left(\frac{y}{L_0}\right)^2+5\left(\frac{y}{L_0}\right)^3-2\left(\frac{y}{L_0}\right)^4\right]\right\} \text{ for } L_0 < L \le \sqrt{5/3}\cdot L_0 \tag{13}$$

$$w = \frac{\kappa_y L_0^2}{6}\left\{3-3\left(\frac{y}{L_0}\right)^2+\left(\frac{L_0}{L}\right)^2\left[-5+6\left(\frac{y}{L_0}\right)^2-\left(\frac{y}{L_0}\right)^4\right]\right\} \text{ for } L > \sqrt{5/3}\cdot L_0$$

where L is defined in Eq. 11.

Finite-element analysis

Analysis of the effect of gravity on curl of two horizontal papers was performed using the finite-element code MARC *(14)*. Papers laminated from two equal basis-weight sheets with planar dimensions of 17 cm × 23 cm were examined, as illustrated in **Figure 4**. Although somewhat smaller in size, the ratio between length and width is close to that of a typical copy paper. Each ply has different orthotropy ratios, with the top ply being almost isotropic and the bottom ply being more oriented, as seen in **Table I**. The machine direction (MD) of each ply coincided with the x-coordinate.

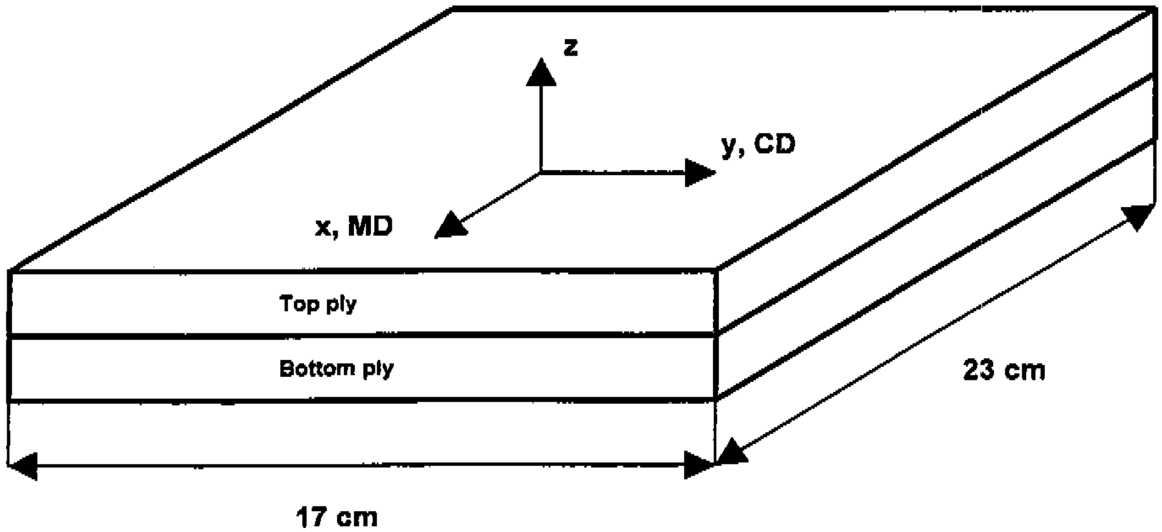

Figure 4. Geometry of the two-ply sheet considered in evaluation of the influence of gravity on curl.

Sheet	Anisotropy ratio (E_{MD}/E_{CD})	Basis weight, g/m^2	E_{MD}, GPa	E_{CD}, GPa	G_{12}, GPa	β_{MD}, $\times10^{-2}$	β_{CD}, $\times10^{-2}$	t, mm
Sheet 1								
Ply 1	1.2	30	4.48	3.60	1.55	10.0	12.7	0.053
Ply 2	2.6	30	5.73	2.24	1.39	8.0	19.3	0.053
Sheet 2								
Ply 1	1.2	50	5.18	4.34	1.84	8.0	10.7	0.082
Ply 2	2.4	50	7.04	2.92	1.76	6.7	15.3	0.082

Table 1. Mechanical properties, moisture expansion coefficients, and thicknesses of sheets investigated (averages from five replicates).

A half-symmetry section of the sheet was discretized using eight-node bilinear thin-shell elements, as seen in **Figure 5**. Along the symmetry line *AOB*, displacements in the *y*-direction and rotations around the *x*-axis were constrained to be zero. To prevent rigid body motions, the transverse displacements at *A* and *B* were also constrained to be zero. Swelling strains were incorporated in the analysis by replacing (a) the coefficient of thermal expansion with the coefficient of moisture expansion and (b) the temperature with moisture content.

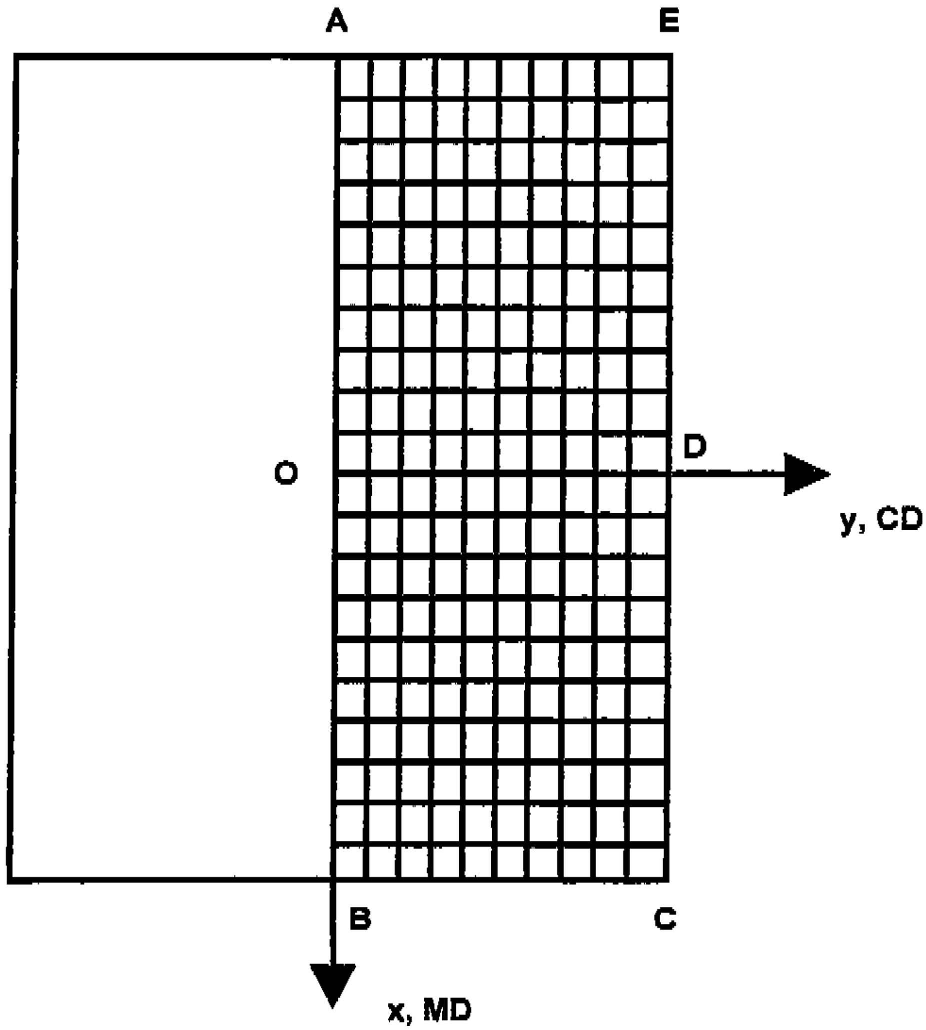

Figure 5. Finite-element discretization (10 × 20 elements) of a symmetry section (half) of the sheet.

The influence of gravity on an initially curled paper when placed horizontally on a flat surface is considered. Because the paper is flexible and the deflections can be influenced by gravity, contact conditions must be considered in the analysis of curl. The paper, discretized by a set of finite elements, is deformable, while the flat supporting surface was modeled as rigid. A nonpenetration constraint condition was used to determine if paper and support are in contact. If the distance between the deformable and the rigid surface is less than or equal to half the thickness of the shell element used, there is contact. Contact between the paper and the rigid surface is examined between each pair of nodes on the paper and rigid surface. MARC solves the contact problem using an iterative contact algorithm *(14)*. The contact region is defined by the node positions indicating zero vertical displacement. When the node displacement increases from zero, the paper is no longer in contact with the rigid surface. The node displacements from the MARC result file were thus inspected to evaluate the contact region. The size of the elements used to discretize the paper and supporting surface was 11.5 mm × 8.5 mm. The accuracy of the contact length determination could be improved by increasing the number of elements, but the obtained accuracy was considered to be sufficient.

The analysis of curl deflections was performed using both linear and geometrically nonlinear kinematics. To quantify the influence of gravity on a curled paper, the shape of the deformed paper in the absence of gravity has to be known. We assume here that the initial shape of the curled paper sheet was cylindrical, with the cylinder axis along the MD (machine direction). Such a curl shape is common for heavier papers, where gravity has only a minor influence on the deflections *(12, 13)*. As will be discussed later, nonlinear analyses of the sheets verify this assumption.

A cylinder is characterized by a circular shape transverse to the generator. The principal, transverse curvature is given by $\kappa_y = 1/R$, where R is the radius of the cylinder. The curvature can be calculated from the following expression derived for a two-ply sheet *(6)*,

$$\kappa_y = \frac{24\Delta m(\beta_2 - \beta_1)}{t\left(\frac{E_1}{E_2} + \frac{E_2}{E_1} + 14\right)} \quad \textbf{(14)}$$

where subscripts "1" and "2" represent the top and bottom plies, respectively; E denotes the plane-strain Young's moduli; β the coefficient of hygroexpansion; Δm the change in moisture content from the flat reference state; and t the total thickness of the sheet. We will first consider the 60 and 100 g/m^2 two-ply sheets examined experimentally. **Table II** lists experimentally evaluated "gravitation free" curvatures (to be discussed subsequently) and corresponding matching moisture contents calculated using Eq. 14 and ply data in Table I. The moisture contents calculated from Eq. 14 were subsequently verified using the MARC finite-element code.

Basis weight, g/m^2	κ_y, m^{-1}	Moisture content, %
60	5.6	0.60
100	3.5	0.84

Table II. Measured curvatures κ_y and moisture content required to achieve matching curvature of two-ply sheets.

Figure 6 shows curl shapes of a 60 g/m^2 laminated paper calculated from linear analysis without and with gravity. Without gravity, the paper assumes a double curvature saddle form, as seen in Figure 6a. The double curvature shape arises from the differences between the MD and CD coefficients of hygroexpansion for the plies (see Table I). When gravity is applied, the paper flattens and the saddle shape is suppressed, as seen in Figure 6b.

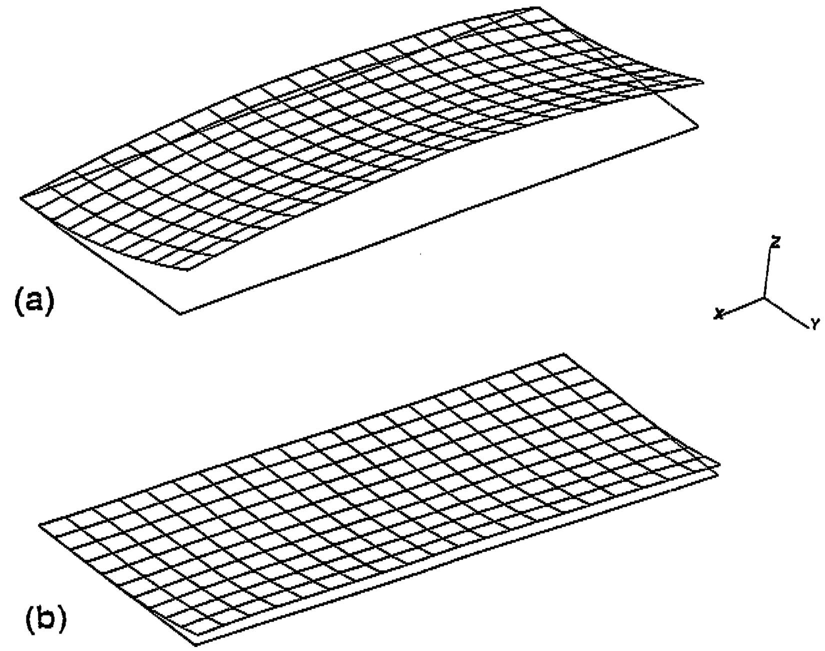

Figure 6. Deflection shape of a 60 g/m^2 two-ply sheet calculated using linear finite-element analysis: (a) without influence of gravity, (b) with influence of gravity.

Figure 7 shows the results for nonlinear analysis. The nonlinear results without gravity (Figure 7a) predict a uniform cylindrical shape in contrast to the saddle shape predicted with the linear analysis. This is a result of the differences between linear and geometrically nonlinear formulations of the strains. When gravity is incorporated in the nonlinear analysis, the sheet flattens, but it remains cylindrical, as seen in Figure 7b.

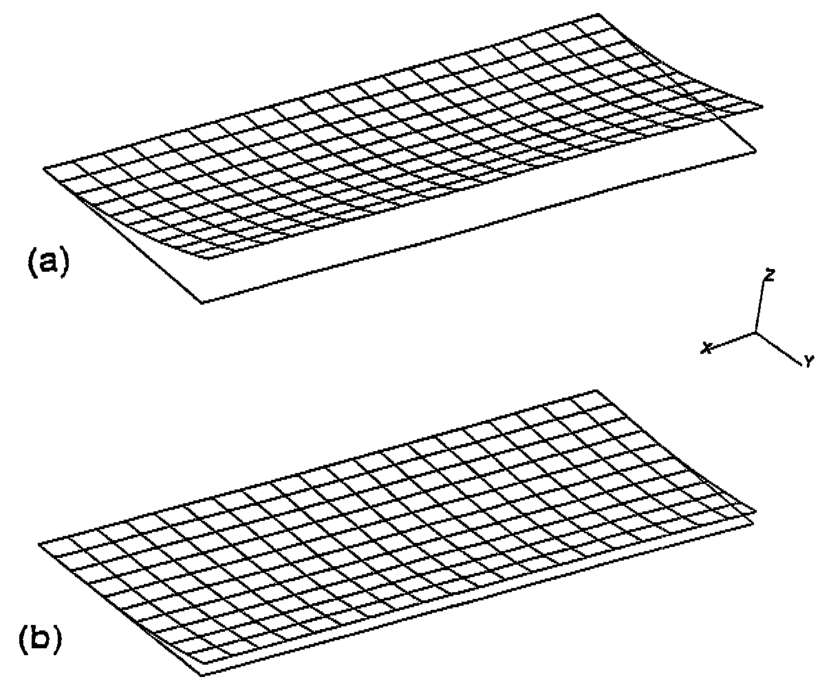

Figure 7. Deflection shape of a 60 g/m^2 two-ply sheet calculated using nonlinear finite-element analysis: (a) without influence of gravity, (b) with influence of gravity.

Figure 8 shows deflection curves at the midsection for a 60 g/m^2 paper calculated with linear and nonlinear analysis without gravity. The nonlinear analysis predicts smaller deflections than the linear analysis because the nonlinear analysis includes the stiffening action of membrane stresses induced by geometry changes, but the difference is quite small. At large out-of-plane displacements ($w \geq t$), it can be shown that a single curvature shape is energetically more favorable than a double curvature shape *(13, 15)*.

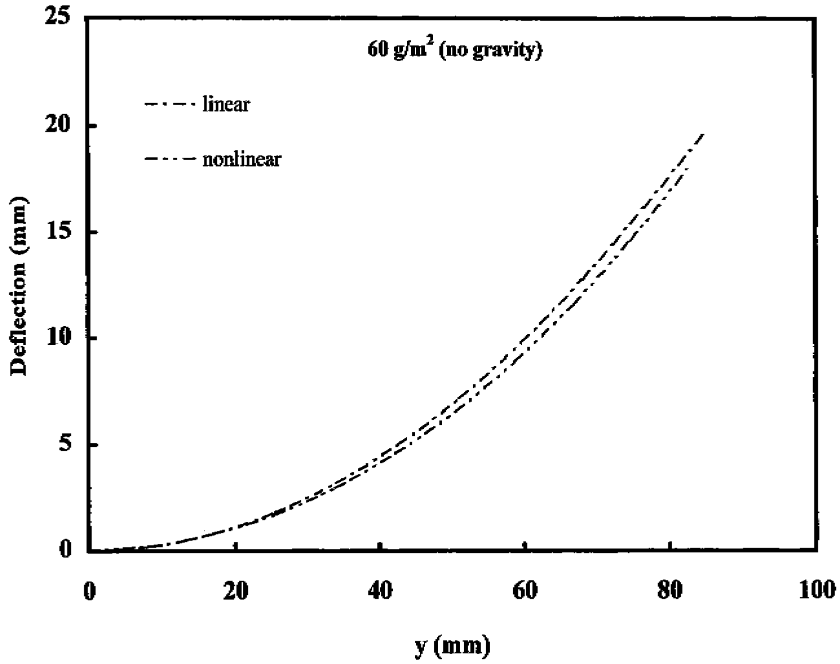

Figure 8. Deflection curves,* w *vs.* y, *for 60 g/m^2 sheet calculated using linear and nonlinear finite-element analysis without influence of gravity.

A parametric study on the influence of gravity on two-ply sheets with planar dimensions of 17 cm × 23 cm with various basis weights was performed. The gravitation-free curvature was set to 5 m^{-1}. The mechanical properties for the 30 g/m^2 papers listed in Table I were used. The total basis weight of the two-ply paper laminates was varied between 20 and 120 g/m^2 in steps of 20 g/m^2 by adjusting the thickness of each ply. The initial moisture content for κ_y = 5 m^{-1} was calculated by using Eq. 14. The basis weights (20, 40, 60, 80, 100, and 120 g/m^2) and the assumed initial moisture content correspond to nondimensional loading parameters for moisture (ω = 693, 173, 77, 43, 28, and 19, respectively) and gravity (p = 2259, 281, 83, 35, 18, and 10, respectively) according to Eq. 4 and the data in Table I, with L_0 = 85 mm. From the magnitudes of these parameter values, we conclude that nonlinear kinematics should be considered and that the gravity effects dominate for small basis weights.

Figure 9 shows deflection curves (w vs. y) at the midsection calculated from nonlinear analysis without and with the influence of gravity as well as from the approximate analysis presented in Eqs. 8 and 9. Without gravity, all deflection curves coincide. With gravity, the 20 g/m^2 paper is almost flat. This behavior is in accordance with the nondimensional parameter η in Eq. 5, which takes the value 0.31, indicating that the bending response is dominated by gravity effects. As the basis weight increases from 20 to 120 g/m^2, the contact length decreases. Even if the contact lengths are small for the 100 and 120 g/m^2 papers, the deflection curves are influenced by gravity. Gravity would thus influence the contact length and the curl shape for horizontal papers of similar size and properties at basis weights below 100 g/m^2. The curl shape of smaller sheets is expected to be less influenced by gravity.

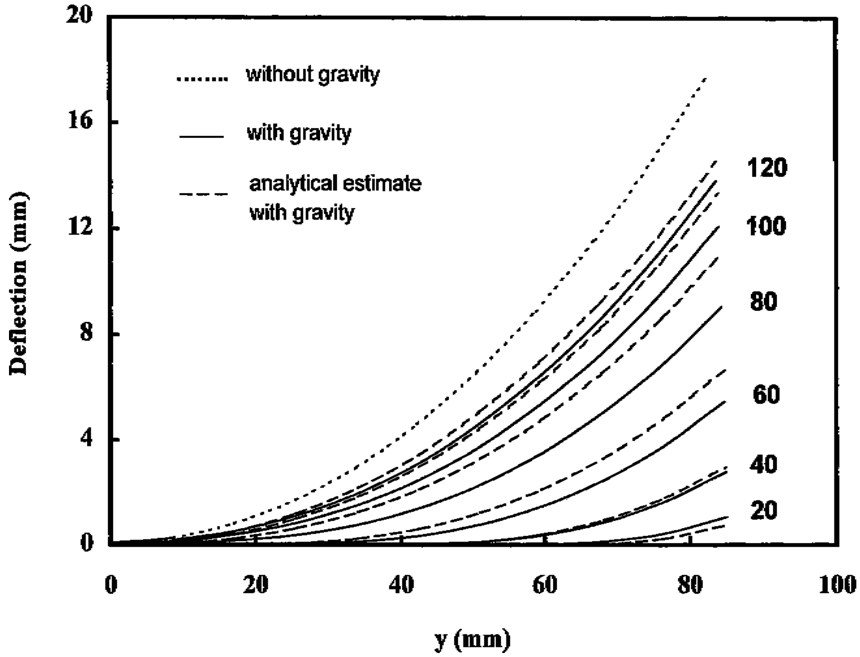

Figure 9. Deflection curves for sheets of basis weights 20, 40, 60, 80, 100, and 120 g/m^2 with and without influence of gravity; comparison between finite-element calculations and analytical estimates.

The finite-element results are in general agreement with the analytical predictions. The discrepancies are attributed to the approximate two-dimensional analysis and neglection of geometrically nonlinear strains in the analytical model.

EXPERIMENTAL

Experiments were performed on two-ply laboratory sheets to examine the curvature response and validity of the analysis. A dried bleached chemical pulp was refined at 2.5% consistency to a beating degree of 26°SR in an Escher–Wyss R1L laboratory refiner. A two-component retention aid system (Hydrocol) consisting of bentonite clay microparticles and a cationic high-molecular-weight polyacrylamide was added to the suspension prior to sheet manufacture. Single-ply sheets with two orthotropy ratios (Table I) and basis weights of 30 and 50 g/m^2 were made on a Formette Dynamique laboratory sheet former *(16)*.

Laminates were prepared from the single-ply sheets with coinciding principal directions (wire side-to-wire side), as seen in Figure 4. The laminated sheets were then pressed in a laboratory press according to SCAN Standard C 26:76. After pressing, each of the sheets (i.e., single-ply sheets and laminate) was placed in a heater and clamped between a metal frame and a porous metal plate resting on a base, as seen in **Figure 10**. The 20-mm-wide metal frame constrained shrinkage of the sheet in its plane during drying *(17)*. To avoid asymmetry introduced by onesided drying, the heater in the base was switched off. Only the heating elements in the clamp were turned on during drying. The temperature and relative humidity (RH) in the sheet-preparation laboratory were kept at 22°C and 29%. When the sheets were completely dry, they were conditioned at 23°C and 33% RH. Young's moduli E_{MD} and E_{CD} (tension) and thickness of the single-ply sheets were measured at 23°C and 50% RH according to SCAN Standards P 67:93 and P 7:75. The in-plane shear modulus was estimated from measured Young's moduli using the empirical relation of Baum *et al. (18)*, $G_{12} = 0.387(E_{MD}E_{CD})^{1/2}$. Poisson's ratio, ν_{12}, was assumed to be 0.3 *(18)*. Ten replicate specimens were used in the evaluation of E_{MD} and E_{CD}.

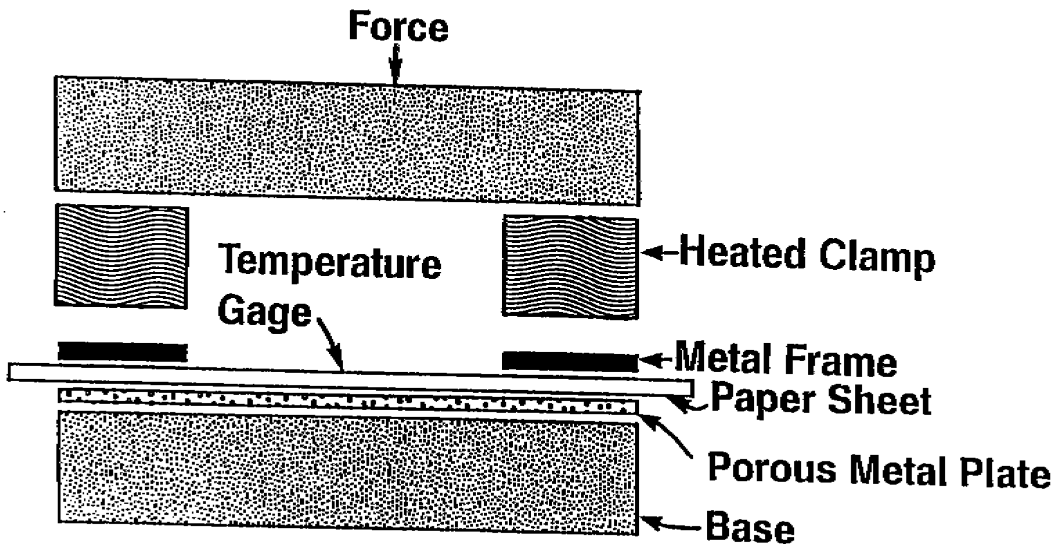

***Figure 10. Schematic of STFI plate dryer* (16).**

The hygroexpansion strains in the MD and CD were measured using the VARIDIM computer-controlled system *(19)* after conditioning the sheets at 66% RH and then subjecting them to 33% RH. Five replicate specimens were used. The percent moisture content, m, is defined by

$$m = \frac{\text{weight }(H_2O)}{\text{weight }(H_2O) + \text{weight (dry paper)}} \circ 100 \tag{15}$$

The reduction in moisture content between 66% and 33% RH was 1.5%. The coefficients of hygroexpansion in MD and CD were calculated by dividing the incremental swelling strain $\Delta\varepsilon$ by the change in moisture content (Δm)

$$\beta_{MD} = \frac{\Delta\varepsilon_{MD}}{\Delta m} \tag{16a}$$

$$\beta_{CD} = \frac{\Delta\varepsilon_{CD}}{\Delta m} \tag{16b}$$

Table I summarizes measured Young's moduli and calculated shear moduli, measured thicknesses, and coefficients of hygroexpansion for the single-ply sheets of 30 and 50 g/m^2 basis weight. The standard deviation was less than 6% for the moduli and less than 9% and 6% for the moisture expansion coefficients and thicknesses, respectively. The moderate variations observed indicate that the sheets were adequately prepared.

Prior to measurement of out-of-plane deflections, the sheets were transferred from the dryers to a lab conditioned at 33% RH and 23°C. Sheets of dimensions 10 cm × 10 cm were prepared for evaluation of the curvatures (κ_x, κ_y, and κ_{xy}). The sheets were arranged vertically in order to minimize influence of gravity on the deflections. The curvatures κ_x, κ_y, and κ_{xy} were evaluated from the measured deflections over an 8 cm × 8 cm area using a method and evaluation procedure detailed in an earlier work *(12)*. To examine the influence of gravity on curl, sheets were cut to 17 cm × 23 cm size (CD × MD) and placed horizontally on a flat table, and out-of-plane displacements were measured using the instrument for curl evaluation *(12)* turned vertical. The measuring area was 15 cm × 21 cm, and deflections were measured over the area in *x* and *y* increments of 10 mm.

COMPARISON OF MEASURED AND PREDICTED DEFLECTIONS

The curl shapes of the vertically arranged sheets were approximately cylindrical with the *x*-axis along the MD ($\kappa_x \approx 0$ and $\kappa_{xy} \approx 0$). The major, nonzero curvature κ_y, however, is evaluated assuming moderate deflections *(12)*. For large deflections, the error in curvature can be appreciable, and this error increases with increasing measurement area *(12)*. For an 8 cm × 8 cm area, it was found that the apparent curvature was 27% larger than the actual *(12)*. Although the correction is expected to depend on the curvature, the curvatures measured herein using an 8 cm × 8 cm area were corrected by dividing the measured curvatures by a factor of 1.27. The resulting curvatures are presented in Table II.

Figure 11 shows half-symmetry sections of the curl shapes measured for horizontally arranged 60 and 100 g/m^2 17 cm × 23 cm sheets under the influence of gravity. The shapes are approximately cylindrical, although there is some unevenness in the deflected shapes as a result of nonuniformities in the sheet structure. A flattening of the central region is evident, indicating contact. The 60 g/m^2 paper has a larger contact area than the 100 g/m^2 paper.

As shown in Figure 11, the measured shapes of the horizontal papers were somewhat nonuniform. To simplify a comparison between experimental and analytical results, average deflection curves (*w* vs. *y*) based on the experimental results for 22 equally spaced positions along the *x*-axis for the two sheets were calculated. **Figure 12** shows average experimental deflection curves, analytical model, and deflection curves obtained from the linear and nonlinear finite-element analyses for the 60 and 100 g/m^2 sheets. Deflection curves from the analyses represent the line *OD* in Figure 5. The predicted deflections for the 60 g/m^2 sheet in Figure 12a agree with the experiments, although the measured deflections exceed the nonlinear predictions away from the central contact region. The analytical model predicts larger deflection over the whole range. The difference, however, is quite small, less than 1 mm. The measured contact length (23 mm) is consistent with the linear and nonlinear predicted length, while the analytical model predicts a contact length of 16 mm. For the 100 g/m^2 sheet, Figure 12b shows that the experimentally measured contact length is around 15 mm, which is less than that for the 60 g/m^2 sheet because of the higher bending stiffness of the 100 g/m^2 sheet. But the predictions of contact length (8 mm) underestimate the experimental value, and the predicted shape is different than the measured average. However, as with the 60 g/m^2 sheet (Figure 12a), the deviations between predictions and observations are quite small, less than 0.7 mm.

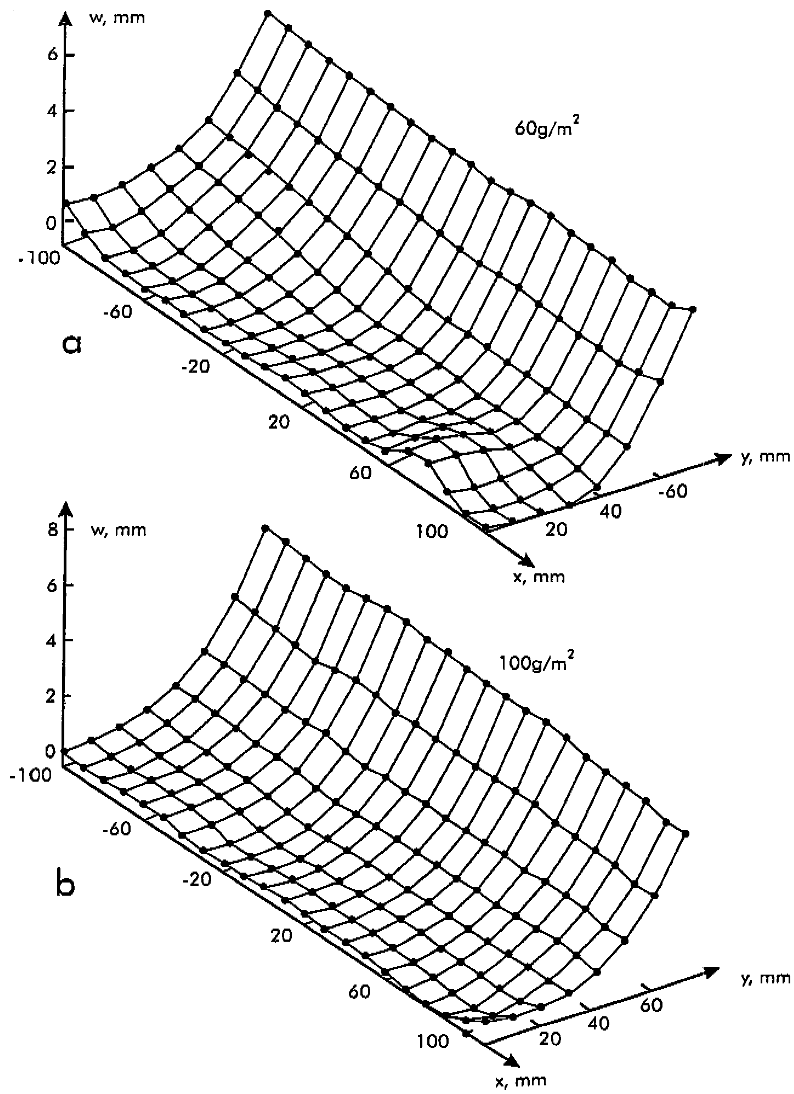

Figure 11. Measured out-of-plane deflections for half-symmetry section of horizontal sheets: (a) 60 g/m^2, (b) 100 g/m^2.

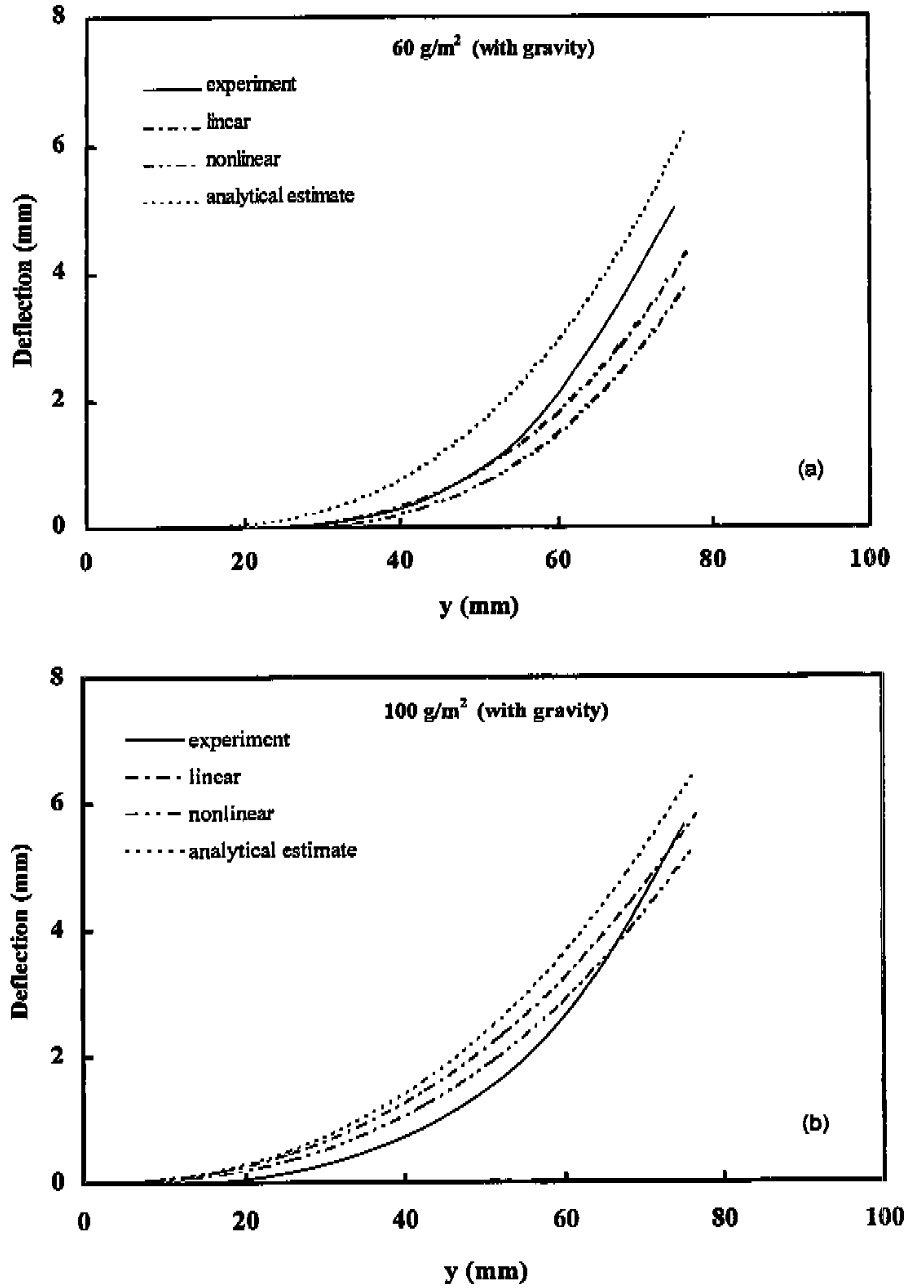

Figure 12. Experimental and calculated deflection curves (*w *vs.* y*) for two-ply sheets with influence of gravity: (a) 60 g/m^2, (b) 100 g/m^2 (calculated results obtained using analytical model and both linear and nonlinear finite-element analysis).

CONCLUSIONS

The influence of gravity on the curl of horizontal papers was studied. The nondimensional parameters ω, p, and η presented in Eqs. 4 and 5 may be useful in judging the relative importance of hygroexpansion and gravity in sheet curl. The results derived from cylindrical bending analysis of the combined effects of hygroexpansion, gravity, and contact are valuable for quickly estimating paper deflections and contact lengths.

Linear and geometrically nonlinear finite-element analyses were conducted, and the results were compared with experiments on two-ply papers. Without gravity, nonlinear analysis predicts a cylindrical shape in contrast to the saddle shape predicted with the linear analysis. Such a difference is attributed to the nonlinear formulation of the strains giving a more accurate modeling. The linear finite-element analysis consistently predicts larger deflections than the nonlinear analysis because the nonlinear analysis considers the stiffening action due to membrane stresses induced by the geometrical changes.

A parametric study on horizontal sheets measuring 17 cm × 23 cm indicates that gravity significantly influences deflections and curvatures at basis weights below 120 g/m^2. The results also show that the approximative analytical model based on cylindrical bending provides reasonable estimates of contact length and deflections. Although the influence of sheet dimensions was not analyzed, the analytical model reveals that the deflections of small-size sheets are expected to be less influenced by gravity. Comparisons between experiments and analysis on 60 and 100 g/m^2 sheets showed a predicted curl shape in close agreement with the experiments. The measured and predicted contact lengths also were in reasonable agreement.

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