A novel device to measure pulp fiber hydrodynamics

BRAD T. B. WONG AND SHELDON I. GREEN

ABSTRACT: In order to understand the separation of different pulp fibers in a hydrocyclone, there is a need to study the hydrodynamics of pulp fibers. A novel device, called the rotating tank hydrodynamometer (RTH), has been developed to study the hydrodynamics of pulp fibers under conditions similar to those in hydrocyclones. The RTH is capable of achieving a wide range of Reynolds numbers (up to about 0.5 based on fiber diameter) with water. Use of the tank as a measuring device has been validated by testing glass spheres, copper wires, and nylon fibers, which all have known hydrodynamic characteristics. The RTH has also been used to test hardwood kraft pulp fibers. Long wood fibers travel faster than shorter fibers under identical hydrodynamic conditions. At the high centripetal accelerations present in a hydrocyclone (700m/s^2), the mean "radial settling velocity" of fibers is about 7mm/s.

Application: This paper describes a new device for measuring the hydrodynamics of pulp fibers. The device may be used to predict the fiber separation that occurs in hydrocyclones, given a knowledge of the flow field in the hydrocyclone.

The properties of paper depend on the structural characteristics of the wood fibers that comprise it. The cell wall thickness is one such important characteristic. In papermaking, fibers with thinner cell walls (earlywood) collapse readily into ribbons during sheet formation. Fibers with thicker cell walls (latewood) resist collapse and their smaller area of contact with other fibers means that they do not bond as strongly as fibers with thin cell walls. The thicker-walled fibers tend to produce an open, absorbent, bulky sheet with low burst/tensile strength and high tearing resistance. In contrast, thin-walled fibers produce a closed, thin, strong sheet. There is a strong desire in industry to separate pulp fibers with different properties depending on those desired in the finished product.

One industrial approach for achieving fiber fractionation is to use hydrocyclones. In a hydrocyclone, pulp slurry is fed tangentially to the wall at the top of the inverted conical vessel that comprises a hydrocyclone. This imparts a rotating motion to the pulp slurry. As the slurry travels in a spiral down the vessel, the centrifugal acceleration, which is typically several hundred times that of gravity, forces the fibers or particles that are denser than the fluid to migrate to the walls of the vessel. The motion may be characterized by the radial velocity of the fibers with respect to the surrounding fluid: the fiber "radial settling rate." This "radial settling rate" is an important parameter in hydrocyclone fractionation. Fibers with a higher radial settling rate become entrained in a downward flow that takes them to the reject outlet at the bottom of the vessel. Pulp fibers with a lower radial settling rate remain in the bulk of the swirling flow, where they are likely to be carried upward to the accept stream outlet at the center of the top of the hydrocyclone.

The hydrodynamic properties of pulp fibers in a centrifugal field have not been studied previously. Most numerical models and analytical studies of hydrocyclones tend to simulate the pulp fibers as simple rigid or flexible cylinders or chains of ellipsoids or spheroids. This paper presents an experimental procedure and the first experimental results documenting the hydrodynamics of pulp fibers in a simple rotating flow.

LITERATURE REVIEW

No one has previously measured the hydrodynamics of pulp fibers at elevated Reynolds numbers. Others have analytically and experimentally determined the drag on objects similar to pulp fibers. A rigid two-dimensional circular cylinder is similar to a pulp fiber. Lamb [1] derived a theoretical equation to predict the drag of such a cylinder placed in a stream of fluid with its axis normal to the flow. He showed that:

$$C_D = \frac{8\pi}{\mathrm{Re}(2.002 - \ln(\mathrm{Re}))}\ .$$

In this equation:

$$C_D = \frac{F_D}{\frac{1}{2}\rho_{fluid}U^2LD}$$

C_D is the drag coefficient, where F_D is the drag force, ρ_{fluid} is the fluid density, U is the flow velocity past the cylinder, L is the cylinder length, and D is the cylinder diameter. Re is the Reynolds number defined by:

$$\mathrm{Re} = \frac{\rho_{fluid}UD}{\mu}$$

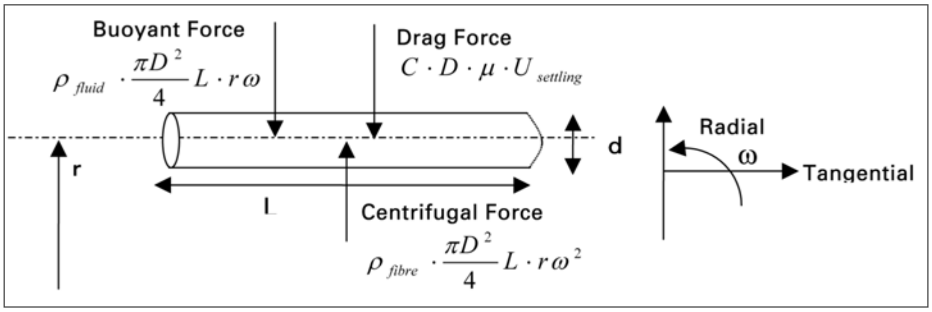
where μ is the fluid viscosity. Cox [2] considered the creeping flow around a long slender solid body that may or may not be straight and developed a general theory to find the hydrodynamic force exerted on such body when it is placed in an undisturbed flow field. He found that:

$$F_D = \frac{4\pi\mu LU}{\ln(\frac{2L}{D}) + 0.193}\ .$$

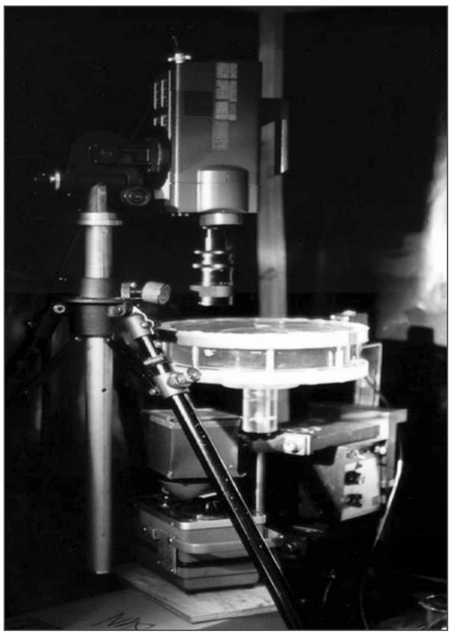
Aidun [3] pioneered the experimental study of the hydrodynamics of fibers. He carried out free settling experiments with different types of fibers (nylon, rayon, dacron, orlon, and cotton) and determined a general expression to predict the drag on fibers for aspect ratios L/D>90:

$$C_D = \frac{10.04}{\mathrm{Re}^{0.738}} \text{ for } 0.007<\mathrm{Re}<2,\ \pm 9\%\ .$$

The conventional definition of the drag coefficient, given above, is most useful at



1. Schematic of forces acting on a fiber. The fiber is a distance r away from the center of the tank and has diameter D, length L, and average density rfiber.



2. Rotating Tank Hydrodynamometer.

high Reynolds numbers where inertial forces are dominant and where, as a consequence, the drag coefficient, C_D, is nearly independent of Re. At low Reynolds numbers viscous forces, not inertial forces, are dominant, and therefore a different form of drag coefficient, named here the *creeping flow drag coefficient* (see, for example, reference 4), is more appropriate. This drag coefficient is defined by:

$$C = \frac{F_D}{\mu DU}$$

The relationship between the creeping flow drag coefficient and the more familiar inertial flow drag coefficient is:

$$C = \frac{C_D U L \rho_{fluid}}{2\mu}$$

As an example, the Stokes flow around a sphere of diameter D produces a drag force $F_D = 3\pi\mu UD$. With the definitions above, the inertial drag coefficient is therefore:

$$C_D = \frac{F_D}{\frac{1}{2}\rho_{fluid} U^2 D^2} = \frac{6\pi}{\mathrm{Re}}$$

whereas the creeping flow drag coefficient is:

$$C = \frac{F_D}{\mu DU} = 3\pi$$

The creeping flow drag coefficient is obviously much more appropriate for this flow than is the conventional drag coefficient, the latter of which implies a Reynolds number dependence of the flow where, in fact, none exists.

ROTATING TANK HYDRODYNAMOMETER

In order to achieve a range of Reynolds numbers in the water flow around a fiber, a novel device, called the *rotating tank hydrodynamometer* (RTH), has been developed to study the hydrodynamics of pulp fibers. The tank has been validated with glass spheres, copper wires, and nylon fibers. The basic concept of the RTH is to generate a simple solid-body-rotation fluid motion. Once this solid body rotation is developed, fibers or other objects for testing can be inserted in the flow, and their motion observed. Since the flow field is known, measurements of the fiber motion may be used to infer the fiber hydrodynamic characteristics. In particular, a fiber inside the rotating tank is subject to a centrifugal force, a drag force, and a buoyant force (**Fig. 1**). The radial equation of motion of a fiber in the rotating tank will be exactly the same as during gravitational settling, except the centrifugal acceleration replaces the gravitational acceleration:

ORIENTATION ANGLE, degrees	AVERAGE SETTLING VELOCITY, cm/s
0-30	1.96
31-60	2.36
61-90	2.66

I. Effect of Orientation Angle on Settling Velocity.

$$m_{fibre} \cdot \frac{d^2 r}{dt^2} = m_{fibre} r\omega^2 - C\mu D\left(\frac{dr}{dt}\right) - r\omega^2 \rho_{fluid} \frac{\pi D^2}{4} L$$

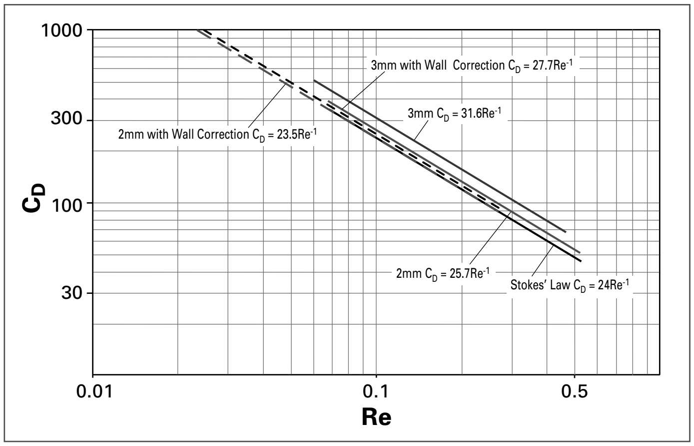
During testing in the tank, the fiber acceleration term is three orders of magnitude less than the other terms. The equation of motion of the fiber can therefore be rewritten to express the drag coefficient at "radial terminal velocity" as:

$$C = \frac{\pi \cdot D \cdot L \cdot \left(\rho_{fibre} - \rho_{fluid}\right) \cdot r \cdot \omega^2}{4 \cdot \mu \cdot U} \qquad (1)$$

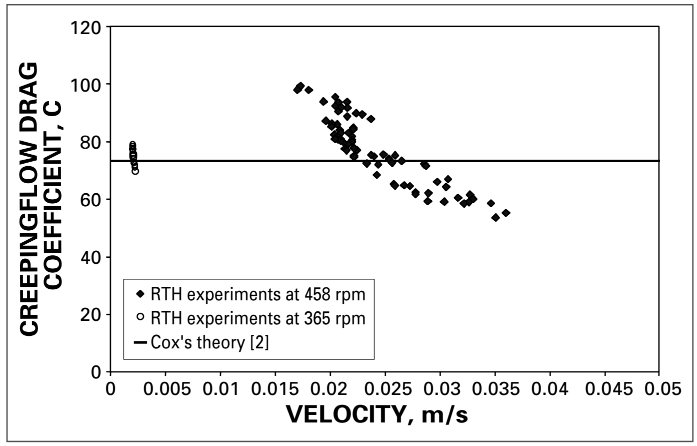
In the above expressions, D is the fiber diameter, ρ_{fluid} is the fluid density, ρ_{fiber} is the fiber density, r is the radial position of the fiber, Ω is the angular speed of the tank, μm is the fluid viscosity, and $U=dr/dt$ is the fiber velocity, which we will refer to as the radial settling velocity, or simply the "settling velocity."

The rotating tank we constructed is a cylindrical, transparent container driven by an electric motor. The rotating tank is constructed from two circular plexiglas plates and a plexiglas tube (**Fig. 2**). The tank is 30.5 cm in diameter and 9.5 cm high, with a vertical axis. The cavity inside for fluids is 25.4 cm in diameter and 4.4 cm in depth. The tank was assembled and balanced in a lathe to minimize vibration during high-speed rotation. A one inch diameter hole was drilled in the center of the top plate for the introduction of fluids and objects. We marked concentric circles 1 cm apart on both the top and bottom plates for measuring distances and aligning the camera.

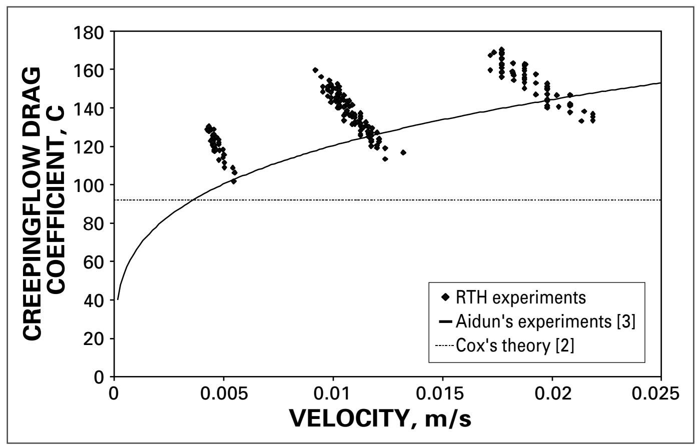
In order to freeze the motion of the tank, we placed a strobe light (StroboTac® 1531-AD General Motor) underneath the tank and aligned it with the camera. The strobe light was synchronized with the tank by a retro-reflective optical sensor (MPL1 Honeywell®).



3. Drag coefficients of glass spheres in the RTH. The best fit curves for the 2 mm and 3 mm spheres, when corrected for wall interference effects, are in close agreement with Stokes' Law. The raw data are not shown, but the error in each measurement of CD is ±15% and in each measurement of Re is ±10%.



4. Creeping flow drag coefficient versus velocity for copper wires measured in the RTH. The experimental points are scattered along a line of slope -1, owing to the definition of the creeping flow drag coefficient (see text).



5. Creeping flow drag coefficients of nylon fibers tested in the RTH at 400, 600, and 900 rpm. As explained in reference to Fig. 4, the results for each rpm cluster along a hyperbola.

diffusion box were needed to give the right exposure. We positioned a high-speed video camera (HSV-1000 NAC, Inc.) with a macro lens above the tank and aligned it with the strobe. We adjusted the camera to be exactly perpendicular to the top surface of the tank in order to accurately measure the distance traveled by an object.

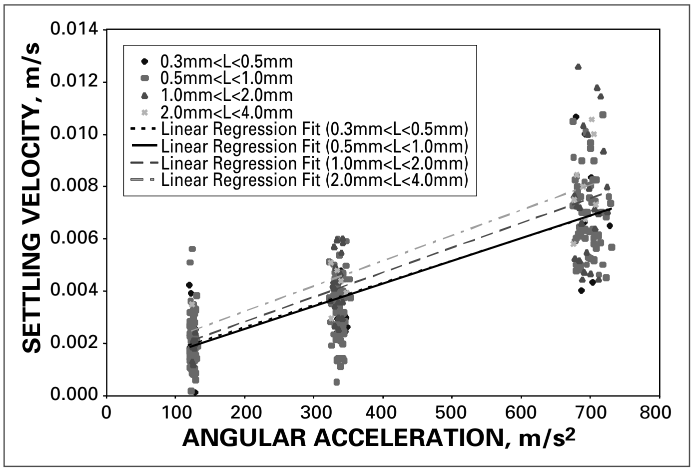
We focused the camera on a region 7 cm radially from the center of the tank and at the mid-depth of the tank, with a depth of field of ±5 mm. The camera image was calibrated by focusing it on a millimeter scale. The area in the flow that could be viewed by the camera was 1 cm x 1.5 cm. We qualitatively checked the flow in the tank by dropping dye into the center of the tank rotating at a fixed angular speed. With the strobe set to freeze the motion of the tank, we observed the pattern of dye moving in the tank to be a radial straight line. This observation implies that the fluid was in solid body rotation (the dye moves radially owing to its slightly elevated density). We found that 15 min of constant speed rotation was sufficient for the tank to achieve solid body rotation.

VALIDATION

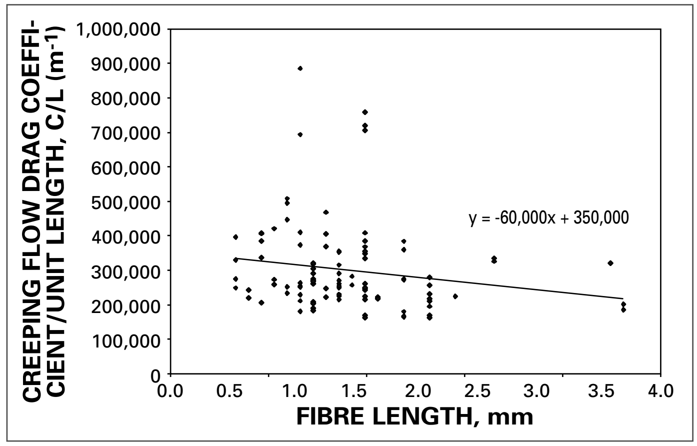
In order to validate the use of the tank, glass spheres with diameters of 2 and 3 mm were tested in the RTH when it was filled with pure glycerin. We performed the experiments at three different angular speeds to produce a range of Reynolds numbers from 0.1 to 0.5, which is in the Stokes' flow regime for a sphere. As shown in **Fig. 3**, the results agree well with Stokes' Law, which confirms that the tank is operating according to expectations. In the Stokes regime, even boundaries 10 or more diameters away affect the flow. In Fig. 3 we have applied the wall correction due to Ladenburg [5] to correct for the presence of the top and bottom walls of the tank. No correction to allow for the cylindrical outer wall of the tank has been applied, although owing to the relatively greater distance of the spheres from this outer wall, the correction should be small.

A long copper wire with uniform diameter of 0.25 mm was cut into pieces 6 mm long to give an aspect ratio of 23. We chose this aspect ratio because it is similar to that of pulp fibers.

Figure 4 shows the results of testing the wires in the RTH filled with glycerin and spinning at 365 and 458 rpm. The mean creeping flow drag coefficient, C, is 76.6, which is only 4.4% different from that (73.3) predicted by Cox's (1970) theory for 3-D cylinders. Two further observations about this figure warrant mention. Since the settling velocity appears in the denominator of the creeping drag coefficient definition ($C=F_D/\mu UD$), for identical cylinders and rotation rates (and thus identical drag forces), any variation in settling velocity causes a plot of C versus U to be hyperbolic in shape rather than to be randomly scattered. Furthermore, much of the variation of settling velocity was due to the settling orientation of the copper wire, as wires did not settle solely with their axes perpendicular to the direction of radial motion. Rather, the settling orientation can be anything from totally perpendicular (0°) to totally parallel (90°) to the direction of radial motion. The settling orientation of slender bodies in the rotating tank was dependent on their initial orientation upon release and was largely unchanged as they settled from the center of the RTH to the outside. **Table I** demonstrates that fibers with their axis parallel to the direction of



6. Radial settling rate of hardwood kraft pulp in the RTH. Regression lines through the results show that the settling rate increases with fiber length. The regression lines for the two shortest groups of fibers lie atop one another.



7. Creeping flow drag coefficient per unit length versus fiber length for hardwood kraft pulp fibers tested in the RTH. The drag coefficient was computed by assuming in eq. 1 that all fibers have the same diameter (22 µm) and density (1330 kg/m^3). The fiber length and settling velocity were measured in the tank.

radial motion travel faster than those perpendicular to the radial direction. These findings are consistent with the theoretical results of Happel and Brenner [6], who show that for a very large aspect ratio cylinder the drag with the cylinder axis parallel to the flow is half that when the axis is perpendicular to the flow.

Dyed nylon fiber, 8 denier in thickness and 1 mm long, were tested in the RTH filled with distilled water, at angular speeds of about 400, 600, and 900 rpm. **Figure 5** shows a comparison with both the experiments of Aidun and the theory of Cox. The agreement with the results of Aidun (who carried out experiments at a different L/D ratio) is very good (within 10%), except at the lowest velocities, where the data trend toward the theoretical prediction of Cox. We would anticipate that at the low velocities (V=0.005 m/s corresponds with a Reynolds number of 0.17) the creeping flow theory of Cox would be more accurate; Aidun's empirical correlation obviously ceases being valid at very low Reynolds numbers because it predicts an unreasonable dependence of C on Re. The generally good agreement with both Aidun's results at higher Reynolds numbers and Cox's theory at low Reynolds numbers lends further credence to the use of the RTH for hydrodynamic measurements.

HARDWOOD KRAFT PULP

After validation of the RTH, we performed some initial tests on hardwood kraft pulp fibers. FQA measurements showed that the fibers had a mean length of 0.795 mm with a standard deviation of 0.4 mm from a sample size of 10,000 fibers. The average diameter measured with a microscope was 22.3 mm. The average wet density of the fibers was determined by immersing them in sugar solutions of varying density, until the majority were neutrally buoyant. Based on these tests, we estimated the wet density of the fibers to be 1330 kg/m^3. It is possible in the RTH to measure the length of fibers as they pass through the device. One may therefore determine the settling velocity of the fibers both as a function of fiber length and angular acceleration ($r\omega^2$). There is an enormous spread in the experimental results plotted in **Fig. 6** for several reasons. The fiber diameters measured for this pulp varied by a factor of 8. With everything else held constant, the diameter variation alone would explain a factor of nearly 8 spread in the radial settling velocity. A second source of the spread in results is the varied orientation of fibers in the tank, which we have shown has a significant influence on the settling velocity of fibers. There are also likely to be significant differences in wetted fiber density, which will affect settling velocity. For all fiber lengths, the settling rate increases with angular acceleration, as one would anticipate.

The regression lines of Fig. 6 show there is a statistically significant effect of fiber length on settling velocity. Long fibers settle about 15% faster than very short fibers. **Figure 7,** which is a plot of drag coefficient per unit length (*C/*L) against fiber length, more clearly illustrates this fiber-length dependence of settling velocity. The significant negative slope of the regression curve through the data confirms that longer fibers settle more rapidly than shorter fibers. Whether this effect is a true one-to-one dependence, or a reflection that fiber length might be correlated with, for example, fiber diameter, is not known. The decrease in *C/L* is consistent with the Stokes flow theoretical results [6]. However, the magnitude of *C/L* is about 2 times larger experimentally than for the theoretical results (about 300,000 m^{-1} versus 130,000 m^{-1}). This difference may be indicative of the fact that wood fibers are not simple cylindrical bodies, but rather contain fibrils and other structures that enhance the drag. Alternatively, it may be a consequence of the fact that at these Reynolds numbers (about 0.3) the flow does not have the negligible inertia required by the theory.

CONCLUSION

A novel instrument for measuring the hydrodynamics of fibers, called the *rotating tank hydrodynamometer* (RTH), has been successfully developed and validated with spheres, copper wires, and nylon fibers. The validation tests are in excellent agreement with previous work. The RTH can be used with confidence to study wood fibers under similar centrifugal force conditions (several hundred times that of gravity) to those existing in a hydrocyclone. Hardwood kraft pulp has been tested with the RTH, and the results show an enormous spread in the radial settling rate of pulp fibers, which is caused by the large variation in both length and diameter from fiber to fiber. For angular accelerations of 120 m/s^2 in water, the fibers have a mean radial settling velocity of about 2 mm/s. At higher centripetal accelerations (700m/s^2) the mean settling velocity increases to about 7mm/s. The settling velocity increases (i.e., the drag per unit length decreases) with increasing fiber length. **TJ**

ACKNOWLEDGEMENTS

The authors would like to thank Seppo Karrila of IPST for helpful comments about an early draft of this report that lead to a substantial improvement in the paper.

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Received: January 16, 2002
Revised: March 13, 2002
Accepted: March 18, 2002

This study was presented at the 2001 TAPPI Engineering Conference

This paper is also published on TAPPI's web site <www.tappi.org> and summarized in the March *Solutions! for People, Processes and Paper* magazine (Vol. 86 No. 2)

INSIGHTS FROM THE AUTHORS

We chose this research topic because we wished to understand how pulp fibers move in a hydrocyclone.

It is the first study of the hydrodynamics of pulp fibers under conditions simulating those that exist in a hydrocyclone.

The research was quite challenging as it involved complex experiments. Developing a "paradigm flow field" resembling that in a hydrocyclone and conducting experiments on individual pulp fibers were both very difficult.

The most interesting finding for me was that pulp fibers behave quite differently from cylinders of identical density, length, and diameter in creeping flow. We believe that both microfibrils attached to the fiber and a Reynolds number dependence may cause this difference.

This research is of a very basic character. It will help hydrocyclone manufacturers develop better devices, and guide mills on how best to use their hydrocyclones.

Now that we have established a "proof of concept," the next step will be to test different types of pulp to see what sort of variations show up between, for example, hardwood and softwood pulps.

— Sheldon Green

Wong and Green are with the Department of Mechanical Engineering, University of British Columbia 2324 Main Mall, Vancouver, BC, V6T 1Z4, Canada. Email Green at green@mech.ubc.ca

Wong

Green