A mathematical method for determining fiber wall thickness and fiber width

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ABSTRACT: An exact mathematical expression for fiber wall thickness and fiber diameter has been deduced for combining wood density and fiber coarseness, under the assumption that the density of the fiber wall is constant. A literature survey has shown that this assumption holds true. The method was validated both by data found in the literature and data gathered from trials. For softwood kraft pulp, the fiber cross-sectional dimensions influenced sheet density, tensile index, and tear index. For TMP, the fiber wall thickness had a decisive effect on the light-scattering coefficient.

Application: An accurate, cost-effective way to calculate the fiber cross-sectional dimensions can predict several of the properties of paper made from softwood kraft pulp.

The properties of pulp and paper are highly dependent on fiber dimensions. Fiber length, fiber wall thickness, and fiber width are usually regarded as the most important fiber dimensions. Today, fiber quality analyzers can measure the fiber length in pulp in a swift and cost-effective way. Fiber width and fiber wall thickness in pulp can also be determined, but only the newest and more expensive analyzers have these capabilities. The accuracy of such measurements may be questioned, since it is possible that the fiber cross section may deform or collapse during pulping, leading to significant deviations from the true fiber dimensions.

Here, we describe a method for calculating the average values of the fiber wall thickness and the fiber width, based on measurements of fiber coarseness, pulp yield, and wood density. The formulas deduced express the fiber cross-sectional dimensions as measured directly on solid, dry wood. The fiber cross-sectional dimensions in pulp may deviate from the dimensions of dry wood, depending on the pulping process. These formulas do not involve the use of statistical models adapted to a set of coincidental experimental data. Rather, they are based on accurately deduced mathematical formulas.

Wood density is a property that expresses the amount of solid fiber wall compared to the amount of empty voids in the wood. Since the fiber wall density can be regarded as constant [1-3], the basic wood density expresses the ratio of the fiber wall area to the lumen area. The fiber coarseness can be used to distinguish between large-diameter fibers and small-diameter fibers originating from wood with equal wood density. When wood density is combined with fiber coarseness, a mathematical expression for fiber wall thickness and fiber width can be deduced. Another assumption is that the cross-sectional area of fiber and lumen should have a quadratic shape.

FIBER WIDTH AND WALL THICKNESS

Based on fiber coarseness, pulp yield, and wood density, the mathematically deduced formulas for fiber diameter, W_F, and fiber wall thickness, WT, are:

$$W_F = \sqrt{\frac{1000\,C_p}{PY\ D}} \tag{1}$$

$$WT = \frac{1}{2}\left(\sqrt{\frac{1000\,C_p}{PY\ D}} - \sqrt{\frac{1000\,C_p\,(1500 - D)}{1500\,PY\ D}}\right) \tag{2}$$

where

C_p = coarseness measured from chemical pulp fibers, μg/m
D = dry wood density, kg/m^3
PY = pulp yield from chemical pulp
W_F = average fiber diameter, μm
WT = average fiber wall thickness, μm.

The complete deduction of the formulas is given in the appendix. Dry wood density should be used in the calculations, not basic density. If basic density is measured but the dry density is not, the dry density can be calculated according to Eq. A16 in the appendix.

METHOD EVALUATION

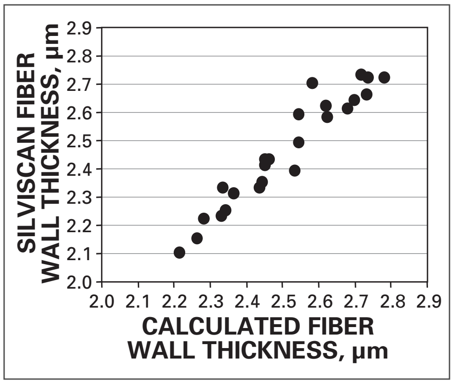
To evaluate the calculation method, it is necessary to compare calculated values with measured values. In this study, we used both data found in the relevant literature and data from our own trials.

Calculations vs. Silviscan measurements

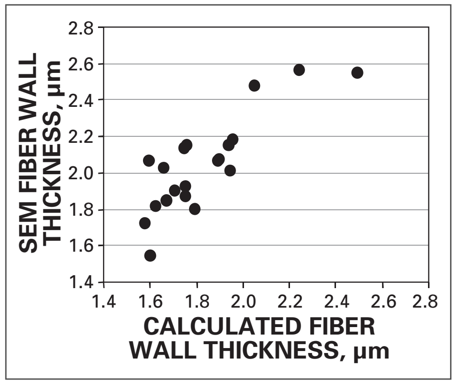
Evans *et al.* performed a study of different samples of Radiata pine (*Pinus radiata*) in which wood cores from the samples were examined using the Silviscan instrument developed at CSIRO [4]. Evans and coworkers measured fiber perimeter, fiber width, fiber wall thickness, and dry wood density. Wood chips from the samples were also cooked in a laboratory digester. Pulp yield and fiber coarseness, among other properties, were obtained from these pulps. Thus, we were able to compare the fiber perimeter, width, and wall thickness as measured with the Silviscan instrument with values of the same properties calculated from the formulas.

In **Fig. 1**, the 24 fiber wall thickness values measured with the Silviscan are plotted against the values calculated from pulp yield, dry density, and fiber coarseness. A simple linear regression, with the measured value as the response variable and the calculated value as the factor variable, showed an adjusted R^2 of .92, with a root mean squared error (RMSE) of 0.055 μm, representing the standard deviation from the regression line. The mean difference was 0.052 μm, with the calculated values as the higher values, but the difference disappeared for high values.

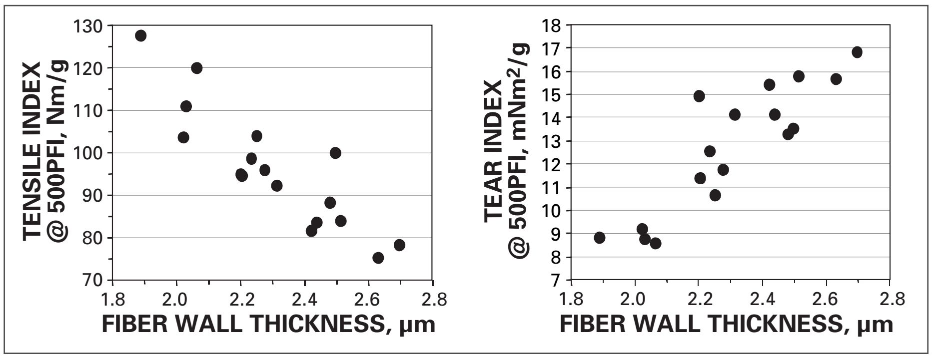
The same type of regression for calculated and measured fiber perimeter gave an adjusted R^2 of .77, with an RMSE of 3.0 μm. Here, the difference was 3.4 μm, with calculated values as the higher values. For fiber width, the mean difference between Silviscan and calculated values was 0.9



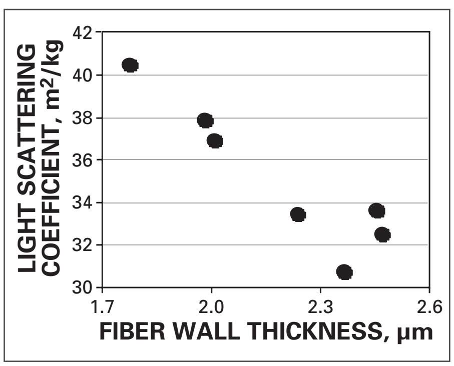
1. Measured [4] and calculated fiber wall thickness for samples of Radiata pine.



2. Measured and calculated fiber wall thickness for samples of Norway spruce.



3. Tensile index and tear index at 500 revs PFI vs. fiber wall thickness for 17 unbleached pulps from single trees of Norway spruce.



4. The relationship between fiber wall thickness and light-scattering coefficient for Norway spruce TMP fiber fraction (Bauer-McNett +50). R^2 = .88. The energy level is 2000 kW•h/o.d. ton.

µm, with calculated values as the higher. The adjusted R^2 was .77, and the RMSE was 0.8 µm.

Calculations vs. SEM measurements

The mathematical method was used to determine fiber properties in a study of 59 young Norway spruce (*Picea abies*) trees. Small samples of wood were cut out from the stem, and basic wood density and dry wood density were measured. The samples were cooked in a laboratory digester, and pulp yield was determined by the weight ratio of dry pulp to dry wood. Finally, we used an FQA instrument from OpTest to measure fiber length and fiber coarseness according to TAPPI T 271 cm-98. Fiber width and wall thickness were then calculated. Opdahl also studied some of the wood samples with SEM microscopy [5]. The fiber width and wall thickness values for these samples were found by the methods developed by Reme and Helle [6] and further developed by Opdahl [5].

From this research work, calculated and measured fiber cross-section dimensions could be compared for 20 wood samples. The calculated and measured fiber properties are shown in **Table I**. The results regarding fiber wall thickness are presented in **Fig. 2**.

In comparing calculated and measured values, we followed the same procedure as that used by Evans *et al.* [4]. For fiber wall thickness, the simple regression model showed an adjusted R^2 of .69, with an RMSE of 0.15 µm. The mean difference between the calculated and SEM-measured wall thickness values was 0.20 µm, with the calculated values as the lower values. For the fiber perimeter, an adjusted R^2 of .47 was achieved, with an RMSE of 5.5 µm. On average, the calculated perimeter was 16.9 µm larger than the perimeter measured with SEM. The calculated fiber width explained 47% of the SEM fiber width, with an RMSE of 1.8 µm. On average, calculated values were 4.2 µm larger than measured values.

CROSS-SECTIONAL DIMENSIONS AND SHEET PROPERTIES

Unbleached kraft handsheets

The unbleached kraft pulp handsheets originated from 17 different Norway spruce trees. All pulps were beaten in a PFI mill to 500 revs PFI and were cooked to kappa no. 32±2. Handsheets were produced and tested according to relevant SCAN procedures.

Basic wood density was measured. Dry wood density was calculated from basic density according to Eq. A16, under the assumption that the fiber saturation point was 33% (0.33) for Norway spruce. Pulp fiber length and coarseness were measured with the Kajaani FS-200. Wood fiber width and fiber wall thickness were calculated from pulp yield, pulp fiber coarseness, and dry wood density according to Eqs. A5 and A9.

A simple regression model with fiber wall thickness as the x variable and sheet density as the y variable gave an adjusted R^2 of .31. The RMSE was 28.9 kg/m^3. A multiple regression model consisting of both fiber width and fiber wall thickness explained 44% (adjusted R^2) of the observed variance of the sheet density for the 17 pulps. When fiber length was included as well, the model explained 59% of the observed variance. The RMSE for the model containing fiber width and fiber wall thickness alone was 26.2 kg/m^3, compared to 22.3 kg/m^3 when fiber length was included. The sheet density increased with decreasing fiber length, increasing fiber width, and decreasing fiber wall thickness.

Variance in tensile index was quite well explained by fiber wall thickness alone (77%), and no other fiber properties could account for any significant additional variance. The RMSE was then 6.9 N•m/g. **Figure 3** shows that increasing tensile index was achieved with decreasing fiber wall thickness. The figure also shows that tear index increased with increasing fiber wall thickness. No other fiber properties explained any significant additional variance. The adjusted R^2 was .77, and the RMSE was 1.3 mN•m^2/g.

TMP handsheets

The TMP samples originated from seven batches of wood, refined in a double disk refiner to energy levels of 1700, 2000, and 2300 kW•h/o.d. ton. The wood samples consisted of three samples of thinnings, three samples of mature butt logs, and one sample of top logs from mature trees. Each wood sample originated from a different site in the southeastern part of Norway, except for the top log sample, which was taken from the same site as one of the mature butt log samples.

For each wood sample, unbleached kraft pulp was prepared to kappa no. 32±2. For the TMP samples, we tested for freeness, shives content, fiber length, and Bauer-McNett fractions of +30, +50, +100, +200, and –200. For the TMP handsheets, we tested for tensile strength, tear strength, elongation, sheet density, light absorption coefficient, and light-scattering coefficient. In addition, we prepared handsheets from pure +50 fraction (which means the +30 and +50 fractions) from the Bauer-McNett classifier. For these handsheets, we determined the light-scattering coefficient.

For the light-scattering coefficient, there was a strong correlation with the fiber wall thickness. Aside from this correlation, the results did not show a strong correlation between the fiber cross-sectional dimensions and the TMP handsheet properties. Generally, the major part of the observed variance in the handsheet properties was explained by the classification of the wood samples according to which part of the stem from which they originated.

Why does the fiber wall thickness have a consequential effect on the light-scattering coefficient? One theory is as follows [7]. Light scattering is a phenomenon that occurs when light hits a surface. The higher the surface area per weight unit of pulp, the higher the probability that light will be scattered. The relationship between fiber wall thickness and light-scattering coefficient probably arises from the fact that the fiber geometric specific surface area increases with decreasing fiber wall thickness.

Based on the fact that fiber wall density can be regarded as constant (1500 kg/m^3) for all wood species [1–3], it can be shown mathematically that the area of the inner (lumen) and the outer surface per fiber weight is:

$$SSA = \frac{W_F \pi L_F + (W_F - 2WT)\pi L_F}{1.5\left(\frac{W_F}{2}\right)^2 \pi L_F - \left(\frac{W_F - 2WT}{2}\right)^2 \pi L_F} = \frac{2}{1.5\,WT} \quad (3)$$

where

SSA = specific surface area, m^2/g
W_F = fiber width, µm
WT = fiber wall thickness, µm
L_F = fiber length, mm.

Dry wood density, kg/m^3	Pulp yield, %	Fiber coarseness, µg/m	MEASURED (SEM) Fiber perimeter, µm	MEASURED (SEM) Fiber wall thickness, µm	CALCULATED Fiber perimeter, µm	CALCULATED Fiber wall thickness, µm
380	46.0	116.5	79.1	1.87	103.3	1.75
340	46.5	120.5	98.0	2.02	110.4	1.67
422	46.5	143.0	100.0	2.47	108.0	2.05
361	45.1	110.5	82.1	1.84	104.2	1.68
361	46.8	127.0	97.7	2.15	109.6	1.76
397	44.5	101.5	91.1	1.90	95.9	1.71
340	45.4	109.0	77.3	1.54	106.2	1.60
324	47.2	123.0	98.9	1.81	113.4	1.62
441	48.0	127.0	87.2	2.18	97.9	1.96
373	49.1	126.5	93.4	2.13	105.1	1.75
360	46.8	102.0	70.3	1.72	98.4	1.58
425	49.6	129.5	81.6	2.07	99.1	1.90
656	50.4	132.0	71.3	2.54	79.9	2.50
389	49.7	101.5	70.9	2.06	91.7	1.60
391	45.7	117.5	78.3	1.80	102.6	1.80
430	46.9	125.5	84.2	2.15	99.7	1.94
515	47.8	138.0	78.1	2.56	94.7	2.24
436	46.3	123.5	74.1	2.01	99.0	1.95
420	46.5	122.0	89.3	2.06	100.0	1.89
406	46.8	110.0	74.5	1.92	96.2	1.76

Fiber width = fiber perimeter ÷ 4.

I. Dry wood density, pulp yield, and fiber coarseness (our own data) and measured and calculated fiber perimeter and fiber wall thickness for samples of Norway spruce (SEM measurements by Opdahl [5])

As an example, a perfectly smooth fiber with a wall thickness of 2.5 µm will give a total specific surface area of 2/(1.5 × 2.5) = 0.53 m^2/g.

To test this theory more thoroughly, we prepared handsheets of pure +50 fraction. The results gave the relationship shown in **Fig. 4**.

Reme and Helle [8] and Fjerdingen *et al.* [9] have studied the relationship between fiber cross-sectional dimensions and the properties of TMP handsheets. The results of those studies showed that increased fiber wall thickness gave decreased light-scattering coefficient, surface smoothness, tensile strength, and gloss.

CONCLUSIONS

This method for calculating the fiber wall thickness and fiber width gives quite accurate values compared to those obtained with other methods (Silviscan and SEM). The calculated cross-sectional dimensions correlate quite well with several properties of handsheets made from softwood kraft pulp.

Sheet density was found to increase with decreasing fiber wall thickness, increasing fiber width, and decreasing fiber length. Tensile index increased with decreasing fiber wall thickness, and tear index increased with increased fiber wall thickness. Both tensile index and tear index correlated highly with fiber wall thickness.

In TMP handsheets, wood fiber wall thickness correlated strongly with the light-scattering coefficient. The reason for this outcome is probably that the fiber wall thickness determines the specific surface area of the fiber fraction and hence its light-scattering coefficient. **TJ**

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APPENDIX

Fiber coarseness C_p, pulp yield PY, and dry wood density D are measured. Based on a survey of several experiments, the cell wall density was found to be close to constant (1500 kg/m^3) when measured on dry, not swelled wood [1-3]. For Eqs. 1-9, it is assumed that a fiber cross section is quadratic and that the wall thickness is approximately equal to the average wall thickness. The lumen is also assumed to have a quadratic shape.

The wall ratio in wood can be defined by Eq. A1:

$$A_R = A_W / A_F = D/CD \qquad \text{(A1)}$$

where

- A_F = average area of fiber cross section in dry wood, mm^2
- A_R = fiber wall ratio
- A_W = average area of fiber wall cross section in dry wood, mm^2
- CD = density of cell wall in wood = 1500 kg/m^3
- D = dry wood density, kg/m^3.

The average fiber cross-sectional wall area, length, and cell wall density give the mass of the average fiber:

$$M_F = C_W \times L_F$$

$$M_F = A_W \times L_F \times CD \qquad \text{(A2)}$$

where

- L_F = average fiber length, mm
- M_F = average mass of one fiber, mg

The average area of the fiber cross section is the square of the average fiber width:

$$A_F = (W_F)^2 \qquad \text{(A3)}$$

By using Eqs. 1 and 2, we can also express A_F as

$$\begin{aligned} A_F &= A_W / A_R \\ &= A_W \times CD/D \\ &= [M_F/(L_F \times CD)] \times CD/D \\ &= [C_w/CD] \times CD/D \\ &= [C_p/(PY \times CD)] \times CD/D \\ &= C_p/(PY \times D) \end{aligned} \qquad \text{(A4)}$$

where C_w is the fiber coarseness in wood in units of mg/m.

By combining Eqs. 3 and 4 and correcting for units of measurement, we get the average fiber width W_F in dry wood:

$$W_F = \sqrt{\frac{1000\, C_p}{PY\ D}} \qquad \text{(A5)}$$

The external average fiber perimeter in dry wood, P_F, can be calculated from Eq. 5 and is expressed (in µm) as:

$$P_F = 4 \times W_F \qquad \text{(A6)}$$

The average lumen area in cross section dry wood, A_{Lu}, is expressed by

$$A_{Lu} = A_F \times (1 - A_R) \qquad \text{(A7)}$$

The average lumen width, W_{Lu}, in dry wood (in µm) is then

$$W_{Lu} = \sqrt{(W_F)^2 \left(1 - \frac{D}{1500}\right)} = \sqrt{\frac{1000\, C_p\, (1500 - D)}{1500\, PY\ D}} \qquad \text{(A8)}$$

The average fiber wall thickness WT in dry wood can be calculated from Eqs. 5 and 8

$$W_T = (W_T - W_{Lu})/2 = \frac{1}{2}\left(\sqrt{\frac{1000\, C_p}{PY\ D}} - \sqrt{\frac{1000\, C_p (1500 - D)}{1500\, PY\ D}}\right) \qquad \text{(A9)}$$

If the dry density D is unknown, we can calculate it from the basic density BD (green density, in kg/m^3) and the fiber saturation point U_F. The volume swelling and shrinkage and the relationship between them are given by

$$a = (V_{100}/V_0) - 1 \qquad \text{(A10)}$$

$$b = 1 - (V_0/V_{100}) \qquad \text{(A11)}$$

$$a = [1/(1 - b)] - 1 \qquad \text{(A12)}$$

where

- a = rate of volume swelling
- b = rate of volume shrinkage
- V_0 = volume of wood, absolute dry, m^3
- V_{100} = volume of wood, absolute saturated with water, m^3.

According to Kollmann and Côte [1], swelling and shrinkage can also be expressed as

$$a = U_F \times D/1000 \qquad \text{(A13)}$$

$$b = U_F \times BD/1000 \qquad \text{(A14)}$$

By definition and by using Eq. 11, we also get

$$\begin{aligned} D &= (m_0/V_0) \\ &= (m_0/V_{100}) \times (V_{100}/V_0) \\ &= BD \times (a + 1) \\ &= BD \times 1/(1 - b) \end{aligned} \qquad \text{(A15)}$$

where m_0 is the mass of wood, absolute dry, in kg units.

By using Eq. 13 in Eq. 14, we finally get

$$D = BD/(1 - (U_F \times BD/1000)) \qquad \text{(A16)}$$

Kollmann and Côte have stated that the fiber saturation point U_F is about 0.28 over a range of wood species [1], but the saturation point for a specific species may deviate from this average. Therefore, in calculating fiber dimensions as described, we can achieve better results by using real values of dry density D instead of values based on basic density and general assumptions of swelling.

This paper is also published on TAPPI's web site <www.tappi.org> and summarized in the February *Solutions! for People, Processes and Paper* magazine (Vol. 87 No. 2)

INSIGHTS FROM THE AUTHORS

Traditionally, the determination of fiber cross-sectional dimensions has involved expensive equipment that is hard to use, such as electron microscopes. We found this problem to be a constraint on our research on fiber properties. We needed to simplify the measurement of the fiber cross-sectional dimensions. In short, a better solution had to be found.

Actually, it was not difficult to find an exact mathematical formula for determining the fiber cross-sectional dimensions. We plan to continue to use this method in our research. Armed with this formula, mill personnel should also be able to determine the cross-sectional dimensions of fibers in an accurate and cost-effective manner.

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