

Stress and strain for perforated tensile specimens, part 2: FEA simulations

RICHARD W. PERKINS, JR. AND GI JEUNG UM

ABSTRACT: The stress and strain distribution near a hole in a tensile strip was investigated using the finite element program ABAQUS, incorporating a constitutive model designed for use with paper materials. Our objective was to study the nonlinear behavior in the vicinity of the hole at a point close to the failure loading point. The strain distribution around the hole for a sequence of loading levels was measured using an image correlation method. This work was reported in part 1 of this report (*Tappi Journal*, March 2007). In this part, a finite element simulation of the hole test is compared with the experimental data that was previously presented. The results show an excellent correlation between the experimental data and the simulation. The simulation shows that the strain associated with failure at the hole surface is exceeded long before the hole test sample fails catastrophically. An estimate of the extent of the zone of localized damage in the vicinity of the point of failure at the hole surface is presented. The application of the point-stress failure criterion for the paper used in this study is presented.

Application: Experimental and analytical evidence is presented that provides an understanding of the conditions of stress, strain, and localized damage that exist in a perforated paper at the point of tensile failure. This study provides information related to the prediction of tensile failure in a two-dimensional state of stress.

Stress and strain components are much higher in a tensile strip with a hole than in a strip without the hole [1]. Consequently, failure of the plate resulting from high loads will be initiated at the edge of the hole. It is important to have a detailed understanding of the stress and strain in the vicinity of the hole surface where failure begins, to predict the loading of the strip that causes catastrophic failure. Part 1 of this study presented experimental strain measurements taken near the hole. Part 2 shows that it is possible to simulate accurately the results of the experiments using the finite element method in conjunction with a constitutive model developed for use with paper materials subjected to plane stress loading. The finite element analysis can be used to obtain a detailed understanding of the state of stress and strain associated with the failure of the material at the hole.

The finite element method can be implemented for the problem of a perforated strip using software packages

such as ANSYS or ABAQUS. While both of these popular software packages provide a wide variety of constitutive models that can be selected to model the material, neither one provides a constitutive model that has been expressly developed for paper. Paper is characterized as a highly nonlinear, inelastic, and strongly anisotropic material [2]. Furthermore, paper is generally believed to exhibit uniaxial stress-strain behavior in compression different from that in tension.

ANSYS does provide a constitutive model originally developed for metal behaviour that has these general attributes. [3]. The ANSYS model is three-dimensional and requires the input of material behavior parameters for both the in-plane and z-direction attributes. The ANSYS model requires the input of 24 separate parameters. The parameters associated with the z-direction loading are seldom available.

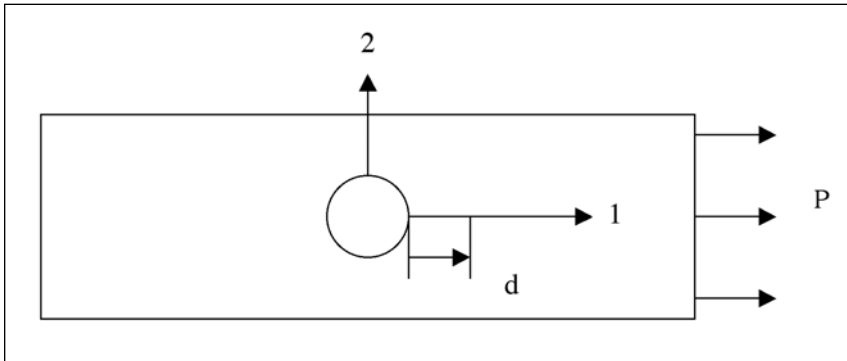
In this study, a model developed by Sinha and Perkins [4–6] was used because of its simplicity and efficacy. This

model requires only the input of parameters that are easily measured for paper using standard TAPPI procedures. The model is easy to implement in ABAQUS using a FORTRAN program that we developed to implement the paper constitutive model. The model requires only the input of eight quantities when it is assumed that the compression and tension properties differ. In the present problem, it was assumed that the differences in tensile and compressive properties were inconsequential, due to the fact that the loading is essentially tensile; therefore, only six quantities were used. The analysis results regarding the presence of compressive stresses is discussed in detail below.

MODELING

General aspects of the problem and the approach

The mechanics problem consisted of a tensile strip with a central hole (**Fig. 1**). The loading was applied at the right edge with displacement control provid-



1. Tensile specimen with central hole.

PARAMETER	MODEL 1
Sheet density	676 kg/m ³
Elliptical fiber orientation distribution, ζ	5.0
Mesoelement initial modulus	7 GPa
Mesoelement tensile yield	29 MPa
Mesoelement compressive yield	29 MPa
Mesoelement tensile tangent modulus	5e8 Pa
Mesoelement compressive tangent modulus	5e8 Pa
Drying restraint tangent modulus ratio	3.8

1. Effective mesoelement parameters.

ed by an Instron machine. The left edge was fixed. The hole diameter was 0.635 cm. The sample length was 26 cm. Sample widths of 1.6, 4.3, and 8.9 cm were investigated. The paper thickness was 1.8 mm. The specimen material consisted of a medium weight paper having a basis weight of 122 g/m² and a density of 676 kg/m³. Experimental measurement of the strains using an image correlation method was focused on the regions emanating from the hole along the 1 and 2 coordinate directions. The details of the experimental work are presented in Part 1.

We found that it was necessary to specify only six of the total of eight parameters for the paper constitutive model, as shown in **Table I**, since it was not necessary to assume different behavior in tension and compression in this case.

The physical implications of this set of parameters are as follows:

1. The ratio of the number of MD mesoelements to those oriented in the CD is given by the elliptical distribution using the orientation concentration factor of 5.
2. The effect of drying restraint is to harden preferentially the tangent modulus of the MD mesoelements. The amount of the increase in tangent modulus is determined by the ratio of the tangent modulus in the CD to that in the MD, equal to 3.8.
3. The average mesoelement stress is determined by the initial and tangent moduli.
4. The average number of mesoelements is given by the sheet density. Sheet density is a measured quantity and is not used in the process of selecting the parameters to fit the stress-strain behavior.

The mesoelement model described here is based on the assumption that it is possible to define a sin-

gle-valued function that provides the average mesoelement stress, depending on the total mesoelement strain. It has been used in problems that involve proportional loading and a single reversal of loading.

Finite element model

The tensile strip with central hole was modeled at $1/4$ scale because of the symmetry of the geometry and loading conditions. Loading was implemented by specifying the displacement of nodes on the boundary to correspond with the values used in the experiments. The plane stress ABAQUS model used 6-node triangle elements (CPS6). The UMAT routine [12] was used to program the effective mesoelement constitutive model.

Experimental characterization of the paper material

The paper material employed in the study is described in Part 1. Uniaxial tension stress-strain data were obtained from a tensile specimen (length=23 cm, width=3.9 cm) that was placed in the Instron test machine and loaded longitudinally using displacement control. The speed of the crosshead was 0.25 mm/min. Uniaxial tests were performed for samples having orientations MD, CD, and 45°. During the uniaxial loading and unloading period, images were taken at regular intervals of displacement for correlation using a Sony camcorder. The stress-strain curves obtained by image correlation and grip displacement yielded essentially identical results. The unloading curve was observed to exhibit a slope similar to that of the initial loading slope. The behavior is therefore strongly nonlinear, exhibits large plastic components and indicates that the material is strongly anisotropic.

As seen in **Fig. 2**, the model predicts the CD, 45° orientation, and MD curves very closely. **Figure 3** shows the experimental lateral contraction ratios (LCR) and the model simulations.

The model shows a slight decrease in LCR beyond the elastic limit. Similar

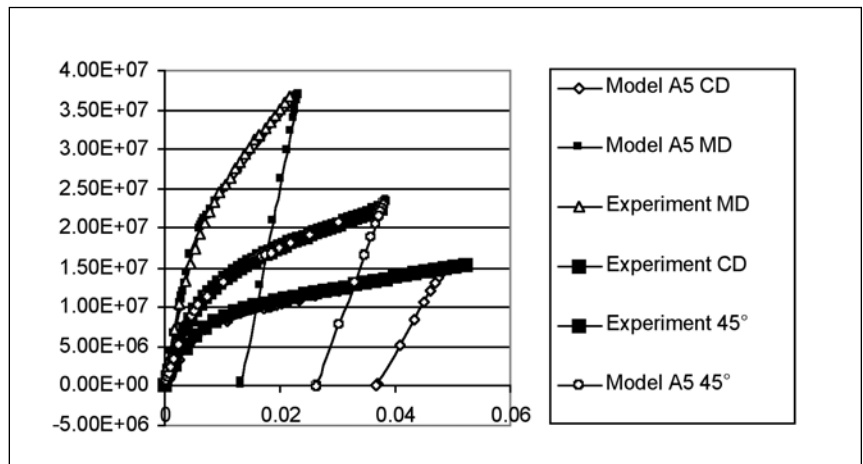
ly, the experimental data shows a decrease in LCR, but then is followed by an increase in LCR at higher strains. Comparison of experimental and simulation behavior both for the stress-strain curves and for the LCR may be useful in the process of selecting the best set of model parameters.

Tensile tests were conducted to determine the tensile strength of the paper material used in this research. The results are reported in part 1.

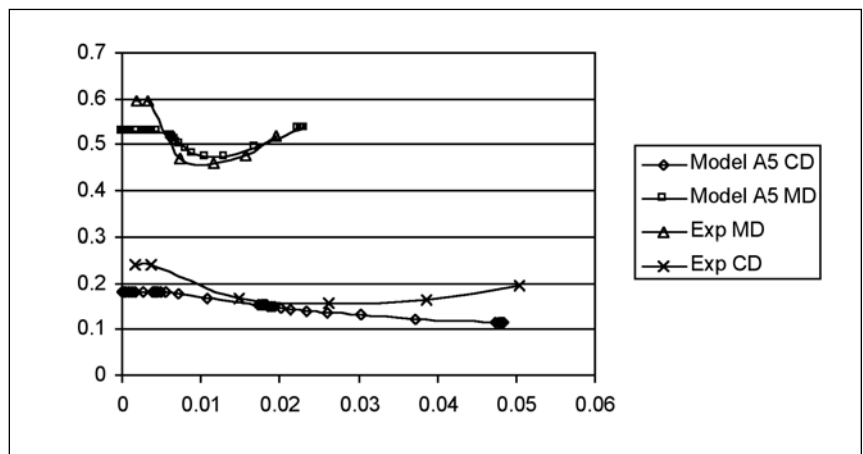
Comparison of experimental results and FEA simulation

Comparisons have been carried out for the strain components along the coordinate axes for a wide variety of samples and loadings. Figure 4 shows the comparison between analysis and experiment for the strain component ϵ_{11} along the x_2 axis for the case of the CD direction loading. The results show that good comparisons are obtained as long as the axial loading is sufficiently low, so that the strain does not exceed the failure limit at the hole surface. This is the case for loading level 1 corresponding to an apparent strain level of 0.89% strain (Fig. 4). Note that in this case, the strain at the hole surface is more than 3%, resulting in a strain concentration factor of about 3. The failure strain for the CD direction is about 4.5% (Cf. Table I, part 1), therefore, the maximum strain at the hole surface is below that associated with failure.

Figure 5 illustrates the experimental and analysis results for the case of the highest applied load level (Level 3, apparent sample strain of 2.1%) and for the case of the residual deformation following total removal of the load (Level 4, apparent sample strain level of 1.27%). It is observed that the local strain at the hole surface for strain level 3 greatly exceeds the 4.5% strain associated with uniaxial paper failure. It is assumed, therefore, that there must be extensive localized failure in the vicinity of the hole surface. As a result of this localized damage, the analysis results must not be reliable since the constitutive model cannot predict be-



2. Experimental stress-strain curves for MD, CD, 45° orientation and the results of the model simulations.



3. Lateral contraction ratios for MD and CD.

havior for strain levels higher than those associated with failure of the material. The residual strains remaining after load removal are expected, since the extensive inelastic response and the localized damage prevent the material from recovering completely.

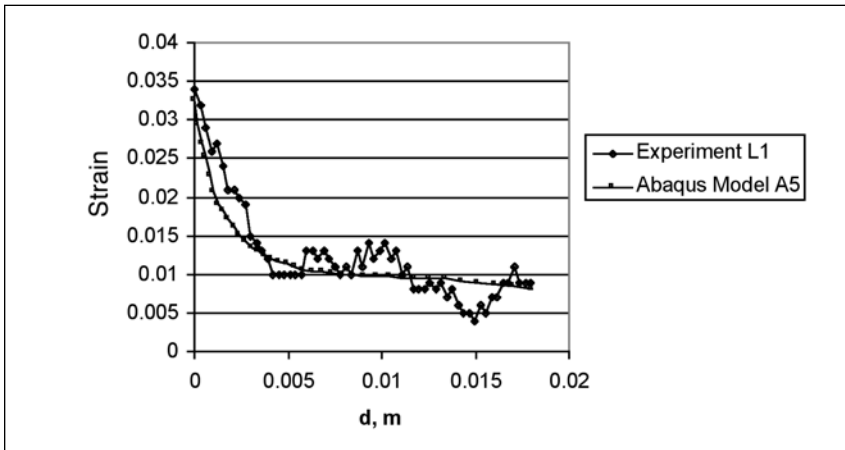
FEA simulation of stress around the hole at the point of failure

As noted above, the evidence shows that there must be localized failure of the paper in the vicinity of the hole surface. As a result, the FEA analysis cannot be expected to predict accurately the state of stress or strain at the hole surface.

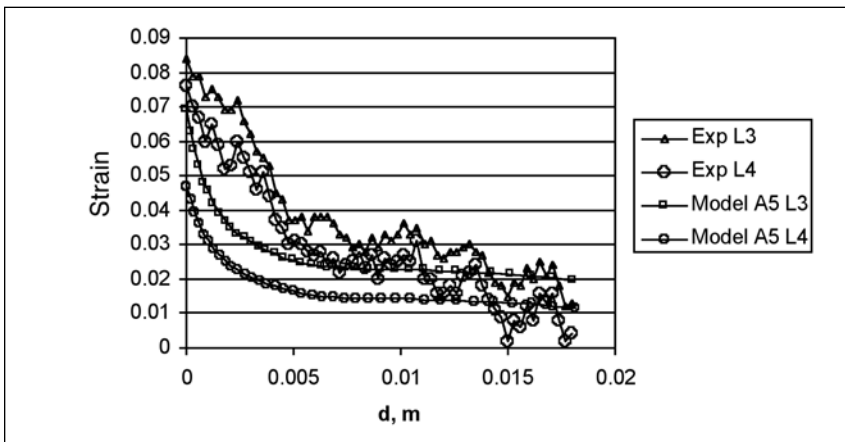
A failure theory based on the con-

cept that the material experiences stresses in excess of the uniaxial tensile strength in the vicinity of a hole was developed for composite materials and subsequently applied to paper failure by Kortschot and Trakas [1]. This theory hypothesizes that localized failure near the hole at the point of maximum stress develops and spreads outward until a sufficiently large region has been damaged before catastrophic failure takes place. The theory, called the point-stress criterion of failure, requires the specification of both the applied stress and the characteristic distance of damage for the failure criterion.

The FEA analysis can be used to esti-



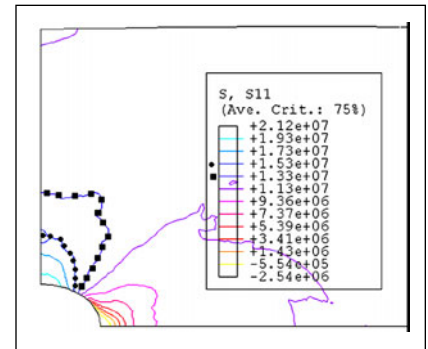
4. Strain component ϵ_{11} along the x_2 axis, d represents distance from the hole edge. CD loading at apparent strain level of 0.89%. Sample c10, subset size 30 and step size 5 pixels.



5. Strain component ϵ_{11} along the x_2 axis, d represents distance from the hole edge. CD loading at apparent strain levels of 2.1% (L3) and 1.27% (L4). Sample c10, subset size 30 and step size 5 pixels.

mate the region where there is localized damage. The extent of the estimated localized damage, along with the uniaxial tensile strength, can be used to predict the load associated with catastrophic failure. This scenario is investigated for the three sample widths used in the present study. The uniaxial tensile strength and the load corresponding to catastrophic failure of the hole samples has been presented in part 1. In the following, we have used the experimental failure loads in the FEA simulation. The results of the analysis are then used to estimate the extent of the localized damage.

The calculated distribution for the stress component σ_{11} is shown in the contour plot of **Fig. 6**. As shown in Table II of part 1, the apparent failure strain for a CD loading and 4.3 cm width is 2.15%. Therefore, the grip displacement at failure for the edge located 13 cm from the hole center for the $1/4$ model is $0.0215 \times 0.13 = 2.795 \times 10^{-3}$ meters. This loading was used to generate the contour plot shown in Figure 6. Referring to Table I of Part 1, the uniaxial failure stress for the CD direction is 14.4 MPa. In accordance with the contour plot, the zone of estimated failure



6. Contour plot of stress component σ_{11} (S11) near the hole surface. CD loading for sample width of 4.3 cm at the failure point.

lies between the 13.3 MPa and 15.3 MPa contours. The damage zone is estimated to extend a distance of approximately the radius of the hole (0.318 cm) from the hole surface. Note that the only nonzero stress component at the hole surface is σ_{11} .

For purposes of estimating the zone of damage extending along the x_2 axis, it is advantageous to inspect the path plot for the stress component σ_{11} along x_2 , which is shown in **Fig. 7**. The figure shows the path plots at the estimated failure point for the three sample widths of 1.6, 4.3, and 8.9 cm where in each case the failure point is taken from Table II in Part 1.

The point-stress criterion for failure requires that the intersection of the uniaxial strength curve and the simulated stress curve should take place at the same value of the distance from the hole surface, d , regardless of sample width. In the case of the CD loading (Fig. 7), the 8.9 cm, and 4.3 cm width curves intersect the uniaxial strength curve at the same point that is, for the value of d , equal to about 6 mm. On the other hand, the 1.6 cm curve intersects the strength curve at a smaller value of d , equal to about 3 mm.

DISCUSSION

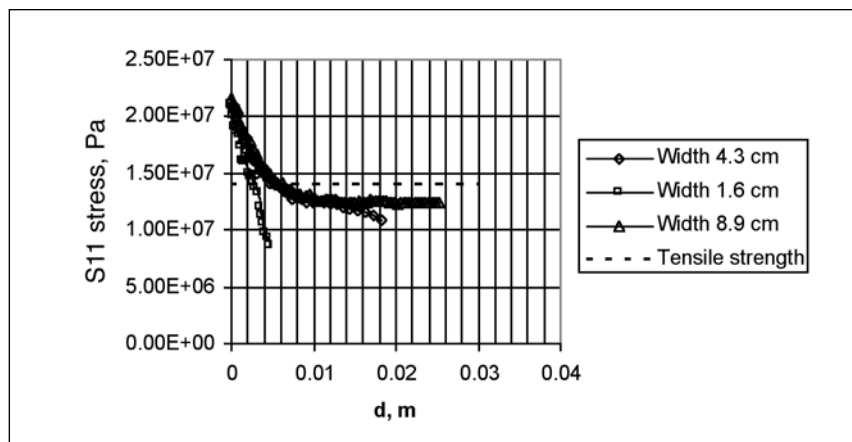
The results of the simulations show that there is a region extending outward from the hole surface along the x_2 axis where the stress has exceeded the

strength of the material. This leads to the expectation that there is extensive damage in this region that precedes the load corresponding to catastrophic failure of the holed sample. In the context of the point-stress criterion, the damage region appears to be on the order of 3 mm to 6 mm from the hole surface. This finding agrees with results reported by Kortschot and Trakas [1]. The experimental results of strain shown in Fig. 5 appear to confirm that damage occurs in the region extending about 6 mm from the hole surface. Of course, this result is for a load that is slightly lower than that associated with failure.

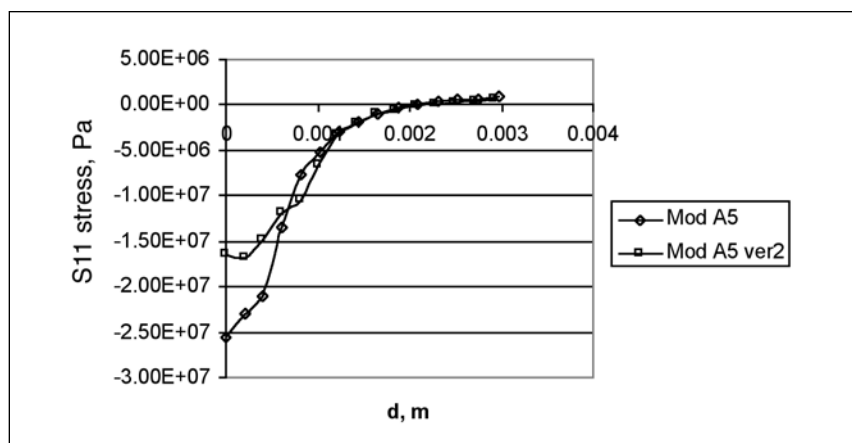
The procedure for simulation of the failure of the holed samples should probably be considered from a more probabilistic point of view. In the simulations above, we have used the average values of the apparent strain at the point of failure. Furthermore, the constitutive model is based on the average stress-strain curve. The actual stress-strain curve varies from sample to sample and the failure point for the holed sample also is subject to variation from sample to sample. Even the process of making the hole in the paper may lead to variable damage at the hole surface.

In spite of these uncertainties, it is reasonable to use the point stress criterion (Fig. 7) as a means for predicting the point of failure for a paper sample with a hole.

The simulations presented above were all based on the assumption that the stress-strain curve for paper would be the same for uniaxial tensile and uniaxial compression loading. It is well known [2] that the tensile and compression behaviors can be significantly different; however, it is extremely difficult to measure the compression behavior. We attempted to measure the compression stress-strain curve using the cylinder test. The results were judged to be unsatisfactory due to the fact the stress-strain curves for tension and compression at the origin were different. We opted to neglect the differ-



7. Stress component σ_{11} (S_{11}) versus distance from the hole surface (d) along the x_2 axis. CD loading at the failure point for three different sample widths.



8. Stress component σ_{22} along the x_1 axis for the CD at the point of failure for width 4.3 cm. Comparison of constitutive models.

ence in tension and compression behavior, because for the present problem it was believed that the only compression region in the hole test is confined to a small region where the x_1 axis intersects the hole surface.

To test this hypothesis we modified the constitutive model to incorporate a difference in tension and compression behavior. Instead of the mesoelement tensile and compression yield points considered as equal at 29 MPa, we arbitrarily assumed that they were 35 MPa for tension and 15 MPa for compression. We have observed that a reduction in the compressive yield requires an in-

crease in the tension yield point to obtain essentially the same stress-strain curve for uniaxial tensile loading. **Figure 8** shows the results of the CD direction simulation for the case of the 4.3 cm width. The curve labeled Mod A5 corresponds to the case of assuming that the mesoelement yield points in tension and compression are the same. The curve labeled Mod A5 v. 2 corresponds to the assumption that the tensile and compression mesoelement yield points are different as described above. It is apparent from Fig. 8 that the predicted compressive value of stress has a smaller absolute value in the region extend-

ing about 1 mm from the hole surface for the case of model Mod A5 v. 2. For greater distance away from the hole surface, the two constitutive models give essentially the same result. Furthermore, the region for which σ_{22} is negative extends only a distance of about 2 mm from the hole surface. A study of the predicted result for the stress component σ_{11} along the x_2 axis showed no difference for the two assumed constitutive models. We conclude, therefore, that the assumption of a difference in tension and compression behavior is unimportant for the problem examined in this report.

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This paper is also published on
www.tappi.org
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INSIGHTS FROM THE AUTHORS

Our goal in conducting this study was to obtain a better understanding of how to model the mechanical behavior of paper when loading exceeds the elastic limit and approaches the failure point.

Previous work by Kortshot and Trakas involving the stress and strain in the vicinity of a hole in a tensile specimen was based on the assumption that paper behaves elastically. The present work extends the analysis to account for the nonlinear inelastic behavior of paper beyond the elastic limit.

The most difficult aspect of our research involved the direct measurement of the strain distribution in paper at loads that approach the failure point. We used the digital image correlation method to measure the strain distribution around the hole in the tensile strip.

We learned that considerable damage takes place in the paper before the material actually fails. Although we don't know the physics of what takes in paper preceding the failure point, our results seem to indicate that the paper experiences a mechanical degradation as the loading approaches the failure point.

It is expected that holes and other defects in paper

cause paper breaks during manufacture. The ability to predict the failure point may result in fewer breaks.

The next step is to carry out further experimental studies that show the physics of what happens in the vicinity of a hole in a tensile strip as the failure point is approached and to extend the modeling process to predict the degradation that occurs. Information of this type will lead to an improved theory for the prediction of paper failure.



Gi Jeung Um



Richard W. Perkins, Jr.

Perkins, Jr., is a professor emeritus of the Department of Mechanical and Aerospace Engineering, Syracuse University, Syracuse, New York, USA; Um is now project manager at the Chang Gang Paper Research Institute, Kangwon National University, Hyoja2-Dong, Chunchon, Kangwon Do, South Korea; email Um at gjum_1@yahoo.co.kr.