A new model for maximizing the bending stiffness of a symmetric three-ply paper or board

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ABSTRACT: Based on the theoretical bending stiffness model for a symmetrical three-ply structure, researchers developed a model that could predict the maximum bending stiffness of a three-ply symmetric structure as a function of density and elastic modulus. The model was tested with experimental data either generated in the authors' laboratory or reported in the literature. The model accurately predicted the maximum bending stiffness based on the middle-layer share of basis weight. Model results indicate that it is possible to increase the middle-layer share without loss of bending stiffness, leading to reduced costs and lower virgin fiber requirements.

Results show that the model is a useful and rapid tool for papermakers to predict the maximum bending stiffness of either coated papers or three-ply boards for a given furnish. Furthermore, the model has potential to be developed further to predict maximum bending stiffness for boards with more than three plies.

Application: A model has been developed to predict maximum bending stiffness of three-ply board as a function of a single parameter, the middle-layer share of basis weight. Model results indicate that it is possible to increase the middle-layer share without loss of bending stiffness, leading to reduced costs and lower virgin fiber requirements.

Bending stiffness is a critical property of paper and board. Printed matter of low bending stiffness has poor appearance and is difficult to handle. Packaging boards require higher bending stiffness for good runnability and higher strength to resist bending [1]. Bending stiffness depends on the macroscopic thickness of the paper or board layers as well as the elastic modulus of the layered structure. The bending stiffness could be improved through optimization of the elastic modulus of the layers, the mid-ply bulk, or the basis weight distribution among the layers; the two latter options could play a significant role in optimizing the bending stiffness.

In practice, refining is used to modify elastic modulus, but thickness is modified by using bulky raw materials. To combine these two different strategies in paperboard, the authors suggest a multi-ply structure. Chemical pulp, which has higher elasticity, is used for surface layers, but bulky mechanical pulp is used for middle layers [2–4]. To optimize the contribution of these two parameters (i.e., elasticity and thickness), numerical methods have been tried, but a widely accepted theoretical method has yet to be found.

This work introduces a model to predict maximum bending stiffness in a symmetric three-ply board based on the middle-layer share of basis weight.

INTRODUCTION TO THE MODEL

Definitions

The bending stiffness of paper or board is defined by a standard expression for a uniform beam as follows [5]:

$$S_b = \int \left(E(z)z^2\right)dz \quad , \tag{1}$$

where z is the thickness and E the elastic modulus. For the homogeneous contribution of each ply, integration of Eq. 1 yields:

$$S_{b_i} = E_i\left[\frac{d_i^3}{12} + d_i\left(h_i - z_0\right)^2\right] . \tag{2}$$

The bending stiffness of an entire sheet of paper or board composed of N layers can be expressed in the following form:

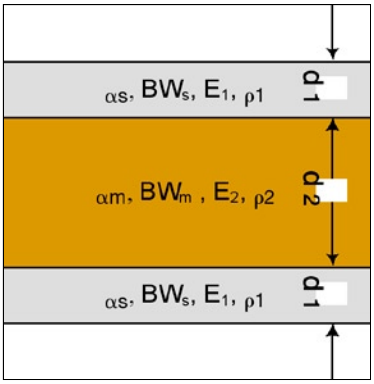
$$S_b = \sum_{i=1}^{N} S_{b_i} \quad . \tag{3}$$

Model development

Combining Eqs. 2 and 3 along with some readjustments yields the following equation for the bending stiffness of a symmetric three-ply paperboard [5]:

$$S_b = \frac{E_2 d_2^3}{12} + \frac{E_1\left(d^3 - d_2^3\right)}{12} \tag{4}$$

where E_1, E_2, d and d_2 are the elastic modulus of the surface



1. Schematic of symmetric three-ply sheet and notation.

layer, elastic modulus of the middle layer, total board thickness, and middle-layer thickness, respectively. It is reported that basis weight greater than 30 g/m^2 has a marginal impact on both elastic modulus and density [6-7]. Therefore, the middle-layer share of basis weight (α_m) could be used as an indicator of middle-layer thickness.

The middle-layer share of basis weight, α_m, can be defined as (**Fig. 1**):

$$\alpha_m = \frac{BW_m}{BW_t} \tag{5}$$

and

$$d_2 = \frac{BW_m}{\rho_2} = \alpha_m \frac{BW_t}{\rho_2} \tag{6}$$

$$d_1 = \frac{(1-\alpha_m)BW_t}{2\rho_1} \tag{7}$$

where BW_t and BW_m are the total and middle-layer basis weight of the paper or board, ρ_1 and ρ_2 are the densities of the surface and middle layers, and d_1 and d_2 are the thicknesses of the surface and middle layers. Substituting $d=\Sigma d_i=2d_1+d_2$ for d_1 and d_2 in Eqs. 6 and 7 yields:

$$d = \left(\frac{\rho_2}{\rho_1}\left(\frac{1-\alpha_m}{\alpha_m}\right)+1\right)d_2 \quad . \tag{8}$$

By combining Eqs. (4), (6), and (8), we can express the bending stiffness as a function of the middle-layer basis weight share, α_m, in the following form:

$$S_b = \frac{1}{12}\left(\frac{BW_t}{\rho_2}\right)^3\left(E_2 + E_1\left(\left(\frac{\rho_2}{\rho_1}\left(\frac{1-\alpha_m}{\alpha_m}\right)+1\right)^3 - 1\right)\right)\alpha_m^3 \ , \tag{9}$$

or

$$S_b=f(\alpha_m) \ , \tag{10}$$

where $f(\alpha_m)$ is a polynomial function of α_m. $f(\alpha_m)$ is maximum if:

$$\frac{\partial f}{\partial \alpha_m} = 0 \quad , \tag{11}$$

assuming that the middle-layer share is not zero. Solving Eq. 11 for α_m^* yields:

$$\alpha_m^* = \frac{\dfrac{\rho_2}{\rho_1}}{\dfrac{\rho_2}{\rho_1} - 1 \pm \sqrt{\dfrac{E_1 - E_2}{E_1\left(1-\dfrac{\rho_2}{\rho_1}\right)}}} \quad . \tag{12}$$

As Eq. 12 shows, the optimum middle-layer share depends only on the pulp properties (i.e., the density and the elastic modulus of the middle and surface layers), which are constant for a given paper or board. Therefore, the middle-layer share is the only variable which controls the maximum bending stiffness.

The second derivative of Eq. 10 gives the correct value of α_m^* and shows that it is independent of basis weight. Bending stiffness at the maximum should satisfy the relationship:

$$\frac{\partial^2 f}{\partial \alpha_m^{*2}} \langle 0 \, . \tag{13}$$

Then the acceptable value of α_m^* would be:

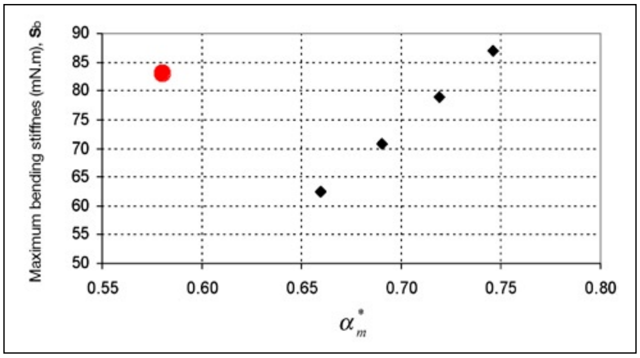
$$\alpha_m^* = \frac{\dfrac{\rho_2}{\rho_1}}{\dfrac{\rho_2}{\rho_1} - 1 + \sqrt{\dfrac{\left(1-\dfrac{E_2}{E_1}\right)}{\left(1-\dfrac{\rho_2}{\rho_1}\right)}}} \quad . \tag{14}$$

Substituting for α_m^* in Eq. 10 then yields the maximum bending stiffness:

$$S_b^m = f(\alpha_m^*) \, . \tag{15}$$

Predictions of the model

Papermakers are producing three-ply paper with 58% middle-layer share, but the model shows that it is possible to obtain the same bending stiffness with 75% middle-layer share. **Figure 2** shows that at 58% middle-layer share (indicated by the circle), the bending stiffness is equal to 83.2 mN, but the same bending stiffness could be achieved at a middle-layer share of 75%. This is a valuable discovery because it offers an outstanding economic breakthrough to the board industry and also can contribute to reducing deforestation.

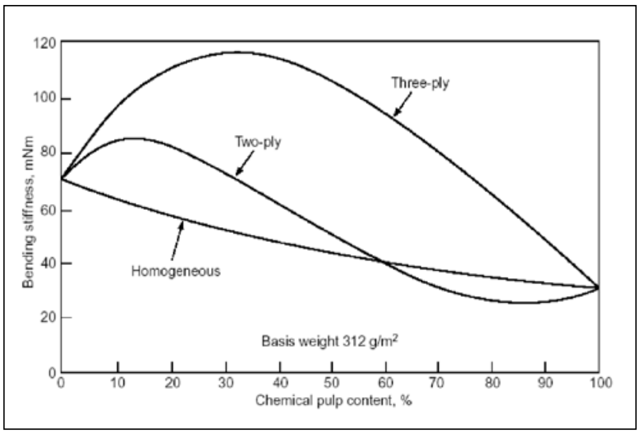


2. Maximum bending stiffness vs. middle-layer basis weight share.

Predictions of the model compared to the numerical results

Case 1

Figure 3 illustrates the optimum share of middle and surface layers. Its maximum, which is at 33% (α_m^* = 100% – 33% = 0.67), fits well with the prediction of the model (α_m^* = 0.66).



3. Calculated bending stiffness vs. share of chemical pulp (chemical pulp (surface layer): E/ρ=7.69×106 Nm/kg, ρ=826 kg/m³ and mechanical pulp (middle layer): E/ρ=3.02×106 Nm/kg, ρ=400 kg/m³) [8].

Case 2

Elastic modulus and density are related [9,10]. The relationship is given as [11]:

$$\frac{E_2}{\rho_2} = 0.014\rho_2 + 2.3\ ,\ (\frac{kNm}{g}) \qquad \text{(for middle layer – mechanical pulp)}$$

$$\frac{E_1}{\rho_1} = 0.013\rho_1 + 3\ ,\ (\frac{kNm}{g}) \qquad \text{(for surface layer – chemical pulp)}$$

For a symmetric three-ply board, the following densities can be assumed:

$$\rho_1 = 650\ (\frac{kg}{m^3}) \quad \text{and} \quad \rho_2 = 400\ (\frac{kg}{m^3}).$$

For a board with total basis weight of 180 g/m², the numerical method yields a middle-layer share of 73% (α_m^*= 0.73), which matches well with the prediction of the model (α_m^*= 0.734).

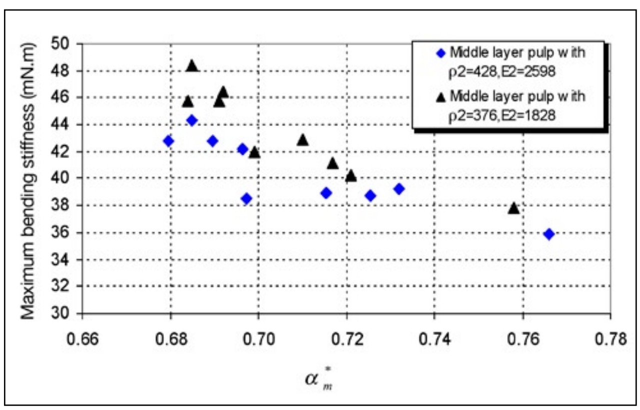
Case 3

Elastic modulus and density were measured for a mixture of hardwood and softwood at different refining levels and reported in **Table I** [12]. The bending stiffness for a hardwood: softwood ratio of 80:20 at a refining level of 3000 revolutions per minute suggested a layer structure of 40+150+40 g/m². The prediction of the model is almost the same (35+160+35 g/m²).

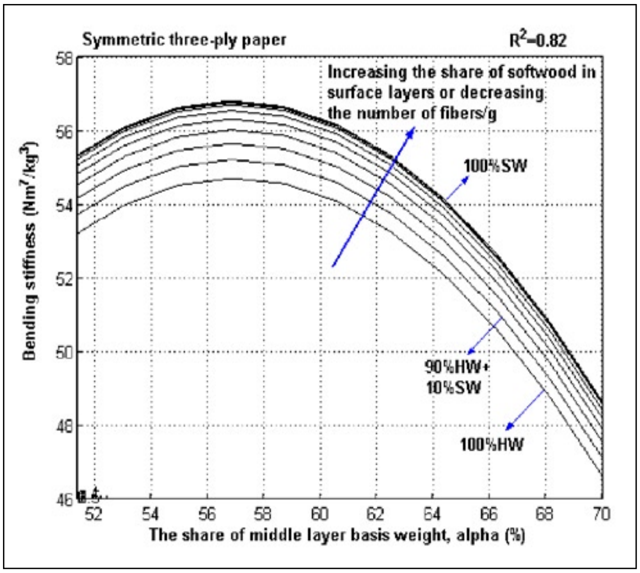
Figure 4 shows the influence of middle-layer basis weight share on the maximum bending stiffness of a symmetric three-ply board with 230 g/m² basis weight for two different pulps used in the middle layer. An increase in the middle-layer share reduces the maximum bending stiffness.

	%SW/HW	Pulp property	1000 Rev.	3000 Rev.	5000 Rev.
Middle layer E_2=1.828 (GPa), ρ_2=376 (kg/m³)	0/100	ρ_1 (kg/m³)	519	582	622
		E_1(GPa)	3.140	4.126	4.746
	20/80	ρ_1 (kg/m³)	559	622	671
		E_1(GPa)	3.802	5.181	5.811
	40/60	ρ_1 (kg/m³)	576	648	696
		E_1(GPa)	4.111	5.450	6.417
Surface layer ρ_2=428 (kg/m³), E_2=2.598 (GPa)	0/100	ρ_1 (kg/m³)	574	641	686
		E1(GPa)	4.21	5.43	6.01
	20/80	ρ_1 (kg/m³)	625	682	690
		E_1(GPa)	5.31	6.74	6.85
	40/60	ρ_1 (kg/m³)	575	659	718
		E_1(GPa)	4.71	6.46	7.54

I. Maximum values of elastic modulus and density for a mixed surface layer at different refining levels.



4. Maximum bending stiffness vs. α_m^* using two different pulps in the middle layer.



5. Bending stiffness vs. middle-layer share.

Pulp with higher density and higher elastic modulus, but the same middle-layer share, produces a board with lower bending stiffness. Interestingly, Fig. 4 shows that the bending stiffness is more sensitive to the surface-layer properties (density and elastic modulus) than to those of the middle layer.

Case 4

For further validation of the model, the authors conducted a laboratory experiment. A set of three-ply boards was made according to the method provided in Appendix I. Surface-layer furnish was formulated using Eucalyptus camaldulensis and pine bleached kraft pulps in different proportions. Chemithermomechanical pulp (CTMP) was used for the middle layer. All pulps were market pulps. The hardwood and softwood pulps were mixed in different ratios, then refined to 4000 revolutions using a PFI mill. About 2% (g/g) modified starch was also added to the refined pulp as a co-binder in the surface layers. The total basis weight of the symmetric three-ply paperboard was 240 g/m^2. The data suggest that the maximum bending stiffness occurs at 58% middle-layer share, as shown in **Fig. 5**. The model prediction of the maximum bending stiffness is almost the same (64%). The average values of ρ_1, ρ_2, E_1, and E_2 (600 kg/m^3, 355 kg/m^3, 2.789 GPa, and 0.808 GPa, respectively) were used in the model. **TJ**

CONCLUSIONS

A model was developed that maximizes bending stiffness of a symmetric three-ply board by controlling middle-layer basis weight. The predictions of the model fit well with experimental results.

The findings of the model represent a potential savings to paperboard mills as well as a forest resource conservation opportunity.

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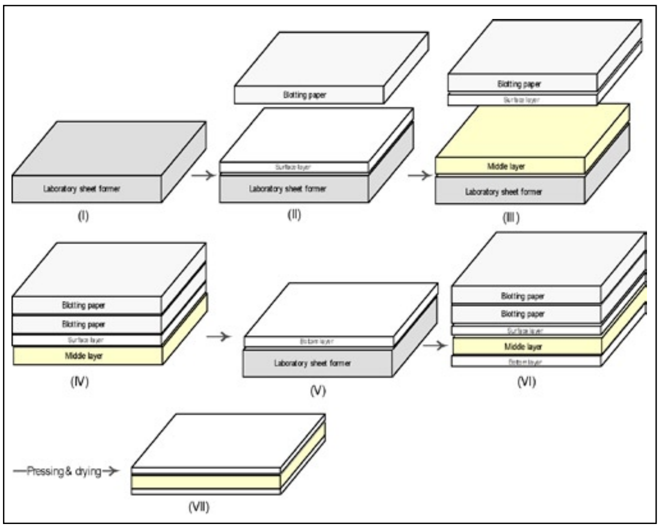
APPENDIX I

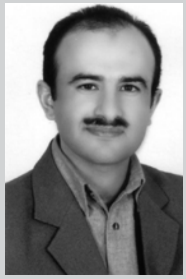


A-1. A procedure for producing the laboratory-scale symmetric three-ply board.

INSIGHTS FROM THE AUTHORS

We chose this area of research because we were looking for a clue about how to control bending stiffness with a single parameter. This work takes a mathematical approach to previous work, and provides an extension of the previous mathematical model. Developing a practical formula from a general mathematical model was the most difficult aspect of the project.

Our main finding was that a single variable could control the bending stiffness. Having the role of middle layer share as a single controlling variable was an interesting finding for us. We feel that by using the present finding, mills could save raw materials, increase bending stiffness of the product, and reduce the production cost for paperboard mills.



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