

Fiber interaction with a forming fabric

JINGMEI LI AND SHELDON I. GREEN

ABSTRACT: During sheet forming, the structure of the forming fabric leaves wire marks on the pulp mat. Paper nonuniformity caused by the wire mark can lead to ink nonuniformity in printing. We investigated wire mark numerically through simulations of the interaction of individual fibers with a forming fabric. In the simulations, the flow field through the forming fabric was taken to be that of single-phase water flow without disturbance of fibers. A particle level simulation method was applied to simulate the motion of fibers in the flow through a single layer sine-wave fabric. A hundred fibers of random initial distribution were placed into the flow above the fabric. Those fibers were advected onto the fabric, forming a fiber mat. The surface roughness of the resulting fiber mat was then calculated. The results show that during the initial formation, topographic wire mark is caused partially by fiber bending and partially by the geometry of the fabric. For the specific fibers and sinusoidal forming fabric considered, more than 50% of topographic wire mark is the result of geometry, with the remainder attributed to fiber bending. Fabrics with different geometries (e.g., different filament pitches or a nonsinusoidal geometry) will have different relative influences from geometry and fiber bending.

Application: This paper provides a theoretical basis for understanding topographic wire mark.

Wire mark is present on the wire side of the paper web and replicates to some degree the geometry of the forming fabric. It is especially noticeable in the surface topography and light transmittance characteristics of paper [1]. In the finished paper, wire mark may be either topographic (a three-dimensional paper surface caused by the knuckles and holes of the fabric) or hydrodynamic (nonuniform mass distribution in the sheet caused by the nonuniform mass flux through the fabric). Wire mark is known to affect the printing quality of paper [2,3] and is of interest for this reason. In this study, we mainly focus on the mechanism of topographic wire marking.

Wire mark can be characterized as a very weak systematic variation in the distribution and orientation of structural elements in paper [4]. Most research on wire mark and paper structure has used image analysis. Several image acquisition techniques have been used to assess printing paper quality, including desktop scanners, profilometry, X-ray microtomography [5,6], and scanning electron microscopy [7,8]. Danby [2,3] studied the effect of forming fabric structure on sheet formation and wire mark. He concluded that density variations on the sheet surface, resulting from the knuckles of the yarns in the forming fabrics, are the main cause of poor printing quality. He showed that the knuckles in a fabric create light areas in the printed sheet, and areas in the sheet that print darkly correspond to the holes in the fabrics. Helle [9] studied the initial paper web formation on a theoretical basis. The optimum geometric shape of the “frames” formed between crossing strands in the forming surface of the wire was calculated based on beam theory. Vakil et al. [10] studied hydrodynamic wire mark. They simulated the flow through forming fabrics and averaged the velocity field over areas comparable to the projected area of a fiber; the resulting average is related

to the amount of fiber that should accumulate in different regions of the fabric during the initial stages of dewatering.

Several parameters exist to characterize the roughness of a three-dimensional surface. The quadratic mean of the deviation from the mean S_q is one [8,11]:

$$S_q = \sqrt{\frac{1}{MN} \sum_{n=1}^N \sum_{m=1}^M (h_{(m,n)} - h_a)^2} \quad (1)$$

where h_a = the average height of the paper surface, and $h_{(m,n)}$ = the surface height at (m,n).

To study how fiber interacts with a forming fabric, the motion of flexible fibers must be simulated. Because of the inherent complexity of flexible particles, numerical simulations have been applied to study the motion of such particles in a flow field. One such numerical approach is particle-level simulation [12-17]. In particle-level simulations, flexible particles are modeled as a chain of rigid particles connected by flexible links. Yamamoto and Matsuoka [12,13] modeled the flexible particles as chains of interconnected spherical beads linked by springs, interacting with a prescribed fluid flow through viscous drag forces. Ross and Klingenberg [14] modeled fibers as a chain of spheroids connected by ball and socket joints, which reduced the computational time by eliminating the iterative constraints for connectivity of segments. In those works, fiber motions in a free flow field were simulated. Lindstrom and Uesata [16,17] simulated a semidilute suspension of fibers in shear flow. The contact force between fibers was prescribed based on the velocity and position of fibers in contact. Vakil and Green [18] studied fiber interaction with a cylinder in two dimensions. They determined the region of fiber “hang up” on the cylinder as a function of several dimensionless numbers.

In all these works, the creeping flow assumption was used to determine the hydrodynamic force on the fiber.

In this paper, we study topographic wire mark by investigating fiber interaction with a forming fabric. Particle-level simulations have been used to describe the motion of flexible fibers through the fabric. Unlike the previous works, we did not use the creeping flow assumption. Rather, we calculated hydrodynamic force based on the lift and drag coefficient of cylindrical sections in a flow field. One-way coupling of fiber and flow was considered; the flow field was assumed to be the single phase flow through the fabric, and that fibers were assumed not to interact with one another. A sine-wave (2×2 shed) single-layer forming fabric was used in the simulations. Simulations of the flow through the fabric were conducted using ANSYS Fluent (Canonsburg, PA, USA) software. Single fibers with different orientations and positions were released in the flow upstream of the forming fabric; their deposition on the fabric surface created a simulated lightweight fiber mat. The surface roughness of the simulated fiber mat, which is related to the wire mark on the fiber mat, was then calculated as a function of the fiber rigidity, fiber length, and other parameters.

METHODS

In the particle-level simulation method, the fiber is modeled as a chain of particles connected by flexible joints (**Fig. 1**). For segment i , r_i is the vector pointing to its center, and c_i is the vector along its axis.

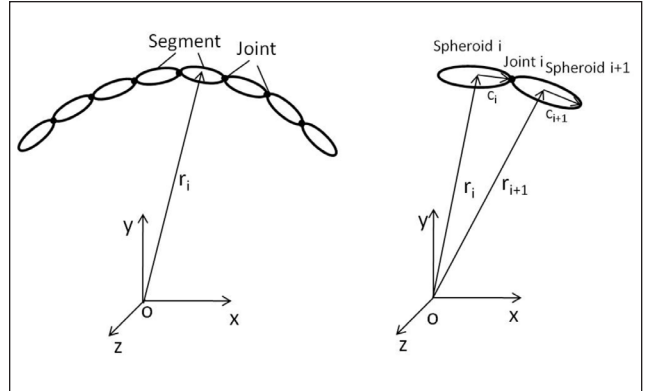
The governing equations of motion for the fiber include kinematic equations and dynamic equations. For each segment in the fiber, connectivity, linear momentum, and angular momentum equations were calculated. (Details of the governing equations are shown in the Appendix). By some manipulation of the governing equations, the translational velocity and angular velocity for each segment can be solved directly by the following equations:

$$\sum_{l=1}^{N_a} N_{il}^1 \omega_l = \sum_{j=1}^{N_a} N_{ij}^2 F_j^{(c)} + N^3 T_i^{(c)} + N_i^4 \quad (2)$$

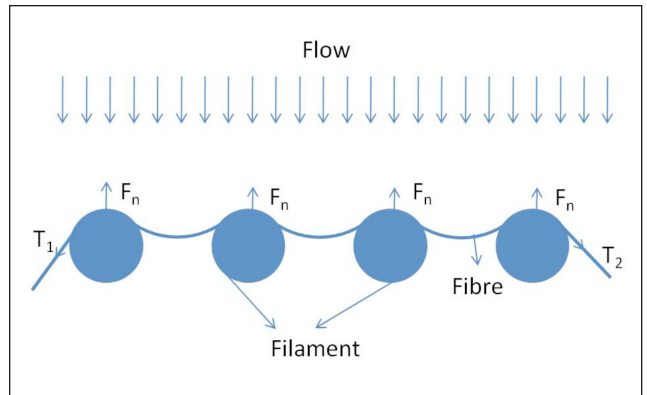
$$\dot{r}_i = \sum_{j=1}^{N_a} M_{ij}^1 \omega_j + \sum_{j=1}^{N_a} M_{ij}^2 F_j^{(c)} + M_i^3 \quad (3)$$

where ω_i is the angular velocity of segment i , and r_i is the translational velocity of segment i . $F(c)$ is the contact force, and $T(c)$ is the contact torque. N^1 , N^2 , N^3 , N^4 , M^1 , M^2 , and M^3 are matrices associated with the position and properties of the particles, as well as the velocity and properties of the flow field (see Appendix). When the fiber moves in a nonconstrained flow, the contact force is zero. In this case, the equations can be simplified as follows:

$$\sum_{l=1}^{N_a} N_{il}^1 \omega_l = N_i^4 \quad (4)$$



1. Schematic of a discretized fiber and neighboring segments i and $i+1$.



2. Schematic of a fiber in contact with a forming fabric.

$$\dot{r}_i = \sum_{j=1}^{N_a} M_{ij}^1 \omega_j + M_i^3 \quad (5)$$

When in contact, a forming fabric usually provides multiple support points over the length of the fiber (**Fig. 2**). Because of the hydrodynamic force, the contact force normal to the contact surface is high compared with the tangential force, so the tangential force cannot overcome the frictional force and make the fiber slide. Therefore, when the fiber is in contact with the forming fabric, the segments in touch with the surface are assumed to be nonsliding. In this case, the contact forces are added to the equations as unknown variables, and the segments in touch with the surface with zero velocity are added as boundary conditions.

The hydrodynamic force of a spheroid in a creeping flow is a linear function of the relative velocity of the particle. In our simulations, the Reynolds number of the flow was much higher than that for creeping flow. Therefore, creeping flow equations were not suitable in these cases. Vakil and Green [19] simulated the flow field around a cylinder and calculated the hydrodynamic force on the cylinder at different angles and aspect ratios. A series of formulas for drag and lift coefficients at different conditions were obtained by curve-fitting the simulation results. The hydrodynamic forces at a moder-

ate Reynolds number were calculated using the following equations, based on the drag coefficient C_D and lift coefficient C_L :

$$F_i^h = A_{ij} (U_j^\infty - U_j) \quad (6)$$

$$A_{ij} = X_A d_i d_j + Y_A (\delta_{ij} - d_i d_j) \quad (7)$$

$$X_A = \frac{1}{2} \rho L D U_r (C_D - C_L \tan \theta) \quad (8)$$

$$Y_A = \frac{1}{2} \rho L D U_r (C_D + C_L \cot \theta) \quad (9)$$

where d is the unit directional vector along the axis of the cylinder; δ is an identity matrix of size 3; U^∞ and U are the velocity of the flow and the particle, respectively; U_r is the magnitude of the relative velocity of the particle; L and D are the length and the diameter of cylinder; and θ is the angle between the axis of cylinder and its relative velocity vector.

RESULTS AND DISCUSSION

Code validation

C++ code has been written to simulate the motions of a fiber and its interaction with the forming fabric. The code was verified by comparison with two analytical results.

Jeffery [20] studied the equations of motion of a neutrally buoyant ellipsoid suspended in a flow field, where the inertia force was neglected. According to Jeffery's theory, the axis of an isolated, rigid, neutrally buoyant ellipsoid in a uniform simple shear flow at low Reynolds number moves on one of a family of closed periodic orbits, the center of the particle moving with the velocity of the undisturbed fluid at that point. The dimensionless period of rotation of the ellipsoid in a simple shear flow depends merely on the aspect ratio of the particle (Eq. [10]):

$$T\dot{\gamma} = 2\pi \left(r_p + \frac{1}{r_p} \right) \quad (10)$$

where T is the period of rotation, $\dot{\gamma}$ is the shear rate, and r_p is the aspect ratio of the ellipsoid.

Numerical simulations of a spheroid motion in a shear flow were conducted to compare with Jeffery's equation. The hydrodynamic force for creeping flow conditions was used in the simulations. The periods of rotation calculated based on simulations and Eq. (10) were compared. The numerical results matched the theoretical solution well, with a difference of less than 0.03%.

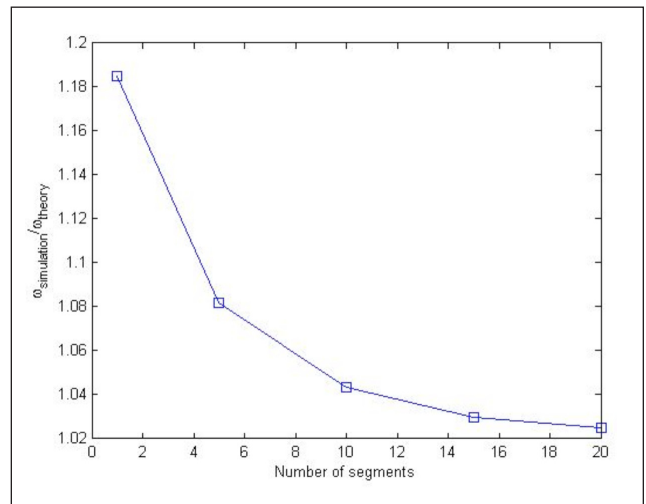
Small deflection beam theory describes the deflection of beams under a small force. For a cantilever beam with uniform load q , the maximum deflection ω is a function of the beam flexural rigidity EI and beam length L and q :

$$\omega = \frac{qL^4}{8EI} \quad (11)$$

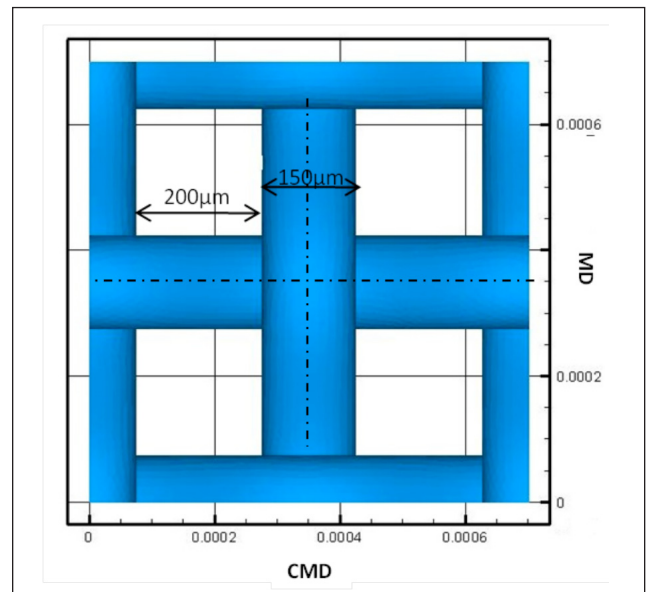
The deflection of beams under uniform force was calculated in simulations. The beam was modeled as a number of rigid particles connected with flexible joints. The force was applied to the center of each segment. **Figure 3** shows that as the number of segments increases, the numerical results approach the theoretical results. The numerically computed beam deflection was within 3% of the small deflection beam theory result when the number of segments was 20 or more.

Flow through the forming fabric

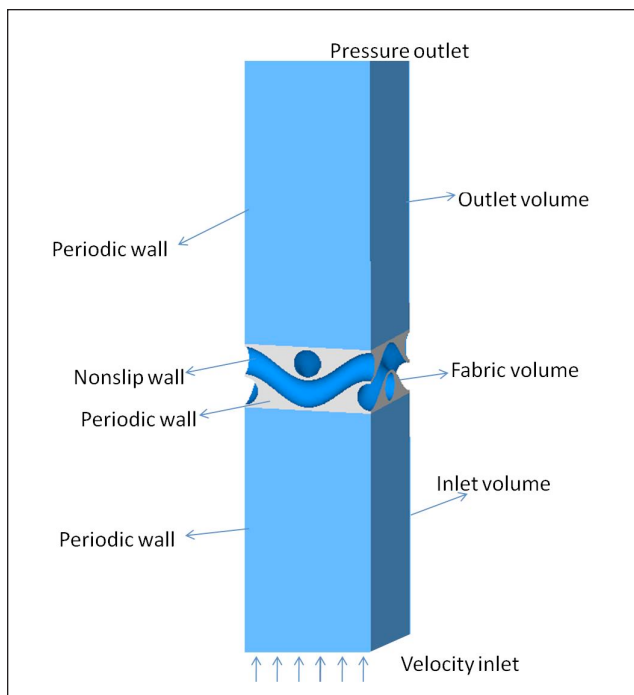
Figure 4 shows the sine-wave single layer forming fabric geometry that we studied. The diameter of the filaments, d ,



3. Comparison of simulation and theory for the maximum deflection of a cantilever beam.



4. The geometry of the sine-wave forming fabric.



5. The computational domain and boundary conditions.

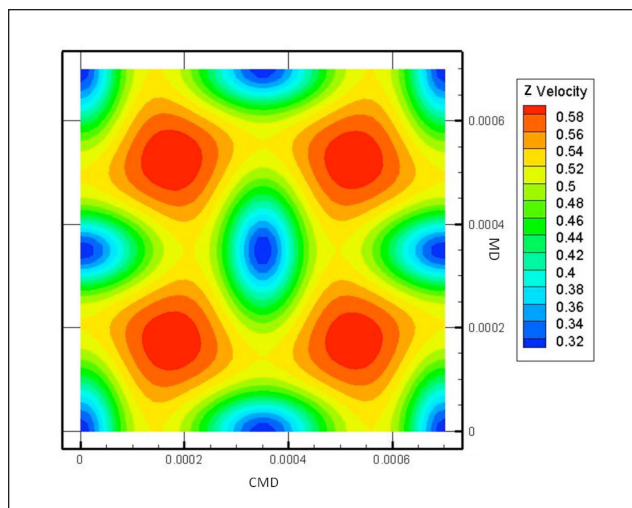
is 150 μm , and the gap between two filaments is 200 μm . We also studied a second sine-wave fabric with twice the gap (400 μm). The flow field through the forming fabric was simulated using Fluent software.

Figure 5 shows the computational domain and boundary conditions. The computational fluid dynamics mesh was generated in ANSYS Gambit (Canonsburg, PA, USA) and imported into ANSYS Fluent. **Figure 6** shows the z-velocity contour in a plane $d/4$ upstream of the forming fabric. In this figure, the bulk velocity through the fabric is 0.5 m/s. The flow field data was imported into the written code to simulate the motion of fibers in the vicinity of the fabric.

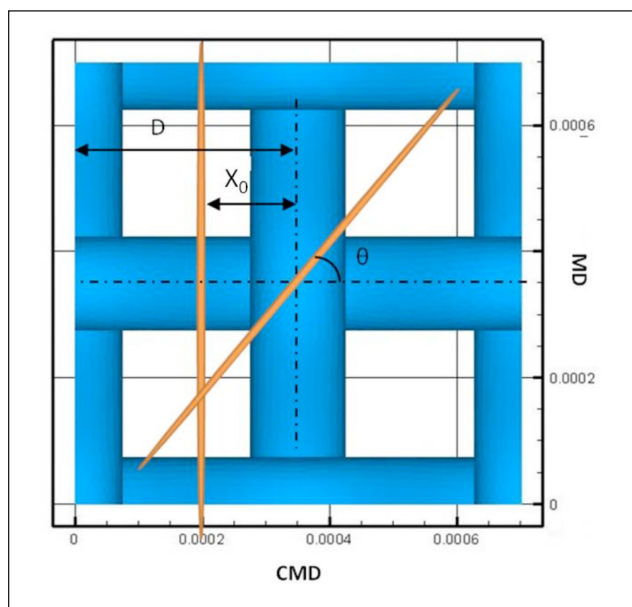
Individual fibers interacting with the fabric

In the simulations, a single fiber with a given initial position was released into the flow upstream of the forming fabric. Different initial positions, X_0 , and orientations, θ , were considered (**Fig. 7**). Unless otherwise stated, fibers used in the simulations had lengths of 1 mm and diameters of 40 μm . Previous researchers have measured the bending stiffness, EI , of pulp fibers to be between 10^{-10} and 10^{-13} Nm^2 [21-23]. For this reason, we considered bending stiffnesses in this range (and as small as 10^{-14} Nm^2) in the simulations.

After the fiber is released, it rests atop the forming fabric. From the fiber geometry, the average height of the fiber can be computed, measured relative to the plane flush with the top of the fabric. **Figure 8** shows that the average height of the fiber did not vary much with EI ; the three curves for different EI have similar trends. Therefore, the geometry of the forming fabric is primarily responsible for the fiber mat topography. The standard deviation of fiber height decreases as the



6. Z-velocity contour in a plane $d/4$ above the forming fabric.



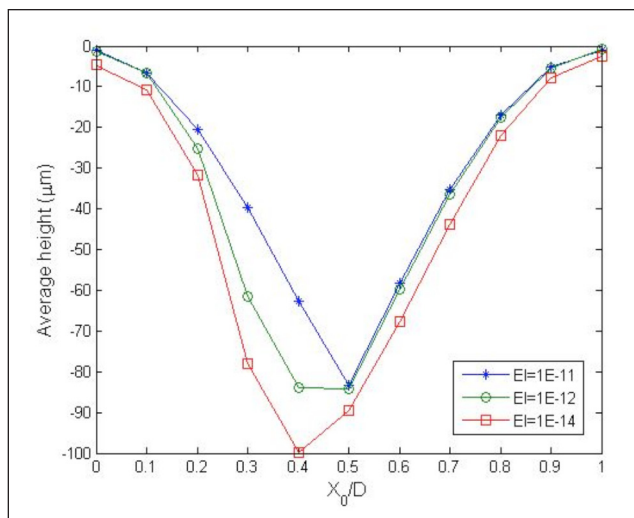
7. A schematic of the fiber position.

fiber becomes more rigid (**Fig. 9**). For fibers centered on one fabric knuckle (i.e., $X_0 = 0$), but with variable θ , because the height in the center is fixed, the average height of fibers varies substantially with EI and θ , except for very rigid fibers (**Figs. 10 and 11**).

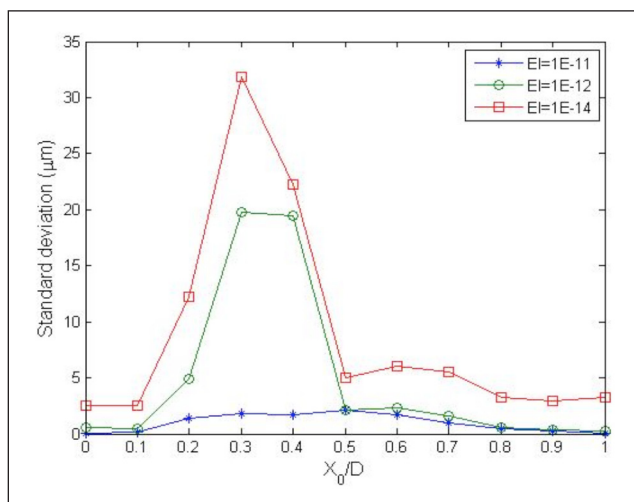
The cases shown in Fig. 8 and Fig. 10, respectively, are situations in which the geometry of the forming fabric and fiber bending contribute most to the change of the fiber average height. Therefore, in general, fiber bending and the geometry of the forming fabric are responsible for the topographical wire mark.

Lightweight fiber mat

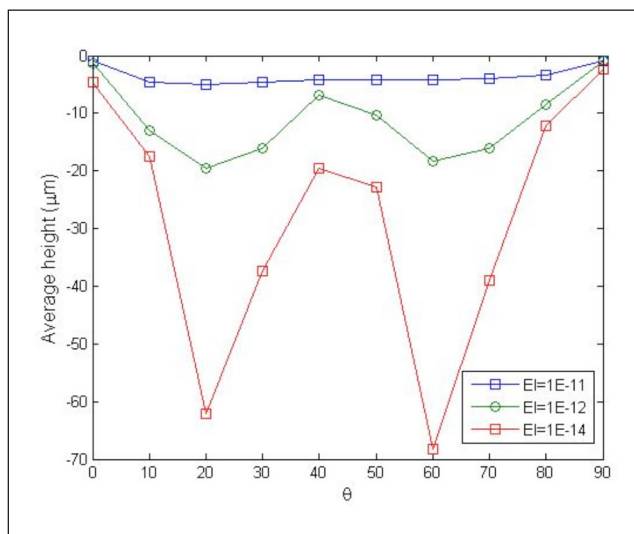
To understand topographical wire mark in papermaking, we studied the initial stage of sheet forming. Simulations were conducted to simulate a number of fibers deposited on the



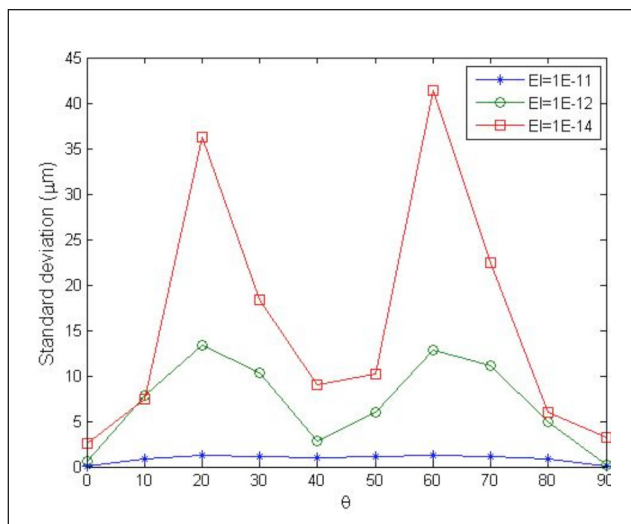
8. Average height of fibers at different initial positions (X_0).



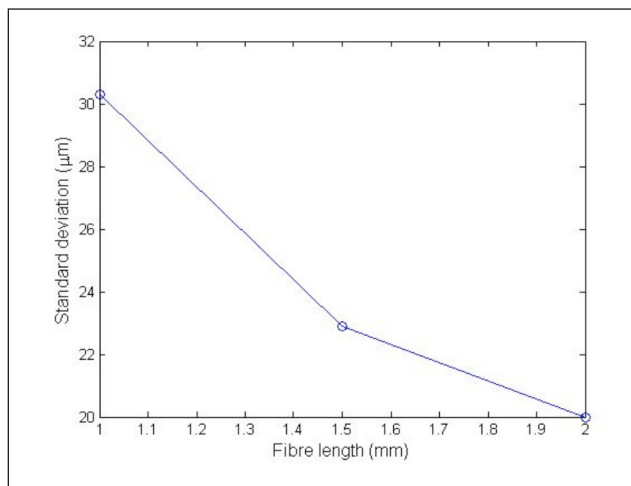
9. Standard deviation of fiber height at different initial positions (X_0).



10. Average height of fibers at different orientations (θ).



11. Standard deviation of fiber height at different orientations (θ).

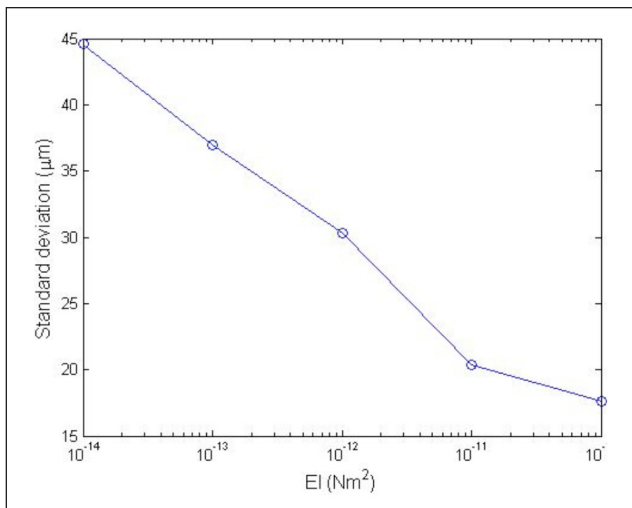


12. Standard deviation of the surface height of the fiber mat made from fibers with different lengths.

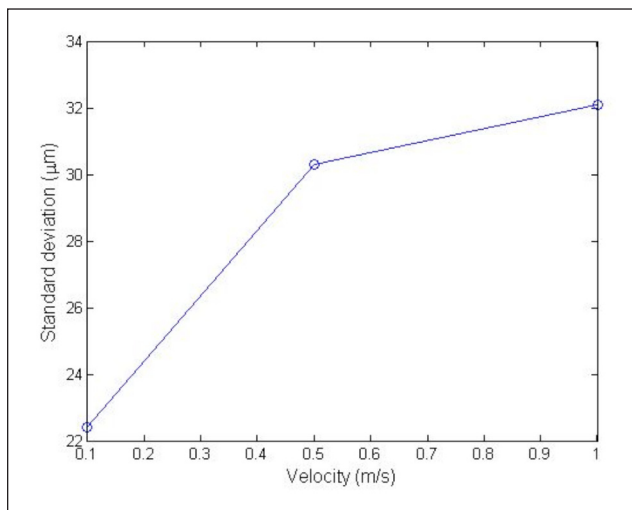
top of the forming fabric. Fibers with initially randomly distributed positions and orientations were simulated. The fibers move independently of other fibers. The surface roughness of the fiber mat is represented by the standard deviation of the surface height, given by Eq. (1).

The standard deviations of the surface height of the fiber mat made from different numbers of fibers were compared. The standard deviation changes only slightly (2.5% difference) from 100 fibers to 200 fibers; therefore, in the simulations, 100 fibers give statistically robust values of mat surface roughness.

Fibers with different lengths were simulated to study the effect of fiber length on surface roughness. The EI of each fiber was fixed at 10^{-12} Nm². **Figure 12** shows that as the fiber length increased, the value of standard deviation decreased. However, in actual papermaking, paper made from short fibers is smoother than paper made from longer



13. Standard deviation of the surface height of the fiber mat made from fibers with different bending stiffness (EI).



14. Standard deviation of the surface height of the fiber mat made at different drainage velocity.

fibers. The reason for the difference might be that, in the simulations, only the initial formed fiber mat is calculated, without accounting for the pressing and drying sections. We believe that pressing might change the topography of short fiber mats more than long fiber mats because short fibers might be less constrained by other fibers in the mat. Furthermore, it is very difficult to vary just one pulp property, independent of other properties; thus, for example, fiber length is correlated with fiber stiffness, so the smooth paper made from short fibers might be a function of their different stiffness.

Fiber mats made from fibers with different initial orientations (random and machine-direction oriented) were compared. The standard deviation did not change much between the two orientations (30.3 μm for randomly oriented and 33.0 μm for machine-direction oriented). For the sine-wave forming fabric, fiber orientation did not affect the surface smooth-

ness significantly. We believe this result would not apply for nonsymmetric forming fabrics.

Figure 13 shows the surface roughness of the fiber mat for randomly distributed fibers, as a function of the fiber bending stiffness. When EI was 10⁻¹⁰ Nm² or larger, the fiber was very rigid and the deflection of the fiber was negligible. Therefore, a standard deviation of surface height of 18 μm results solely from the geometry of the forming. Fibers used in paper-making have EI in the range of 10⁻¹⁰ Nm² to 10⁻¹³ Nm², for which the corresponding standard deviation of surface height varies from 18 μm to 37 μm. Thus, for the simple sine-wave forming fabric and realistic fibers, 50% or more of the surface roughness is caused by the geometry of the forming fabric.

Fiber mats also were formed at different drainage velocities. In these simulations, the stiffness of fibers was fixed at 10⁻¹² Nm². When the velocity increased from 0.1 m/s to 1 m/s, the standard deviation of the mat surface height increased (**Fig. 14**). The increased deviation is a consequence of the greater fiber bending, caused by the increased hydrodynamic force acting on fibers at higher flow velocities.

CONCLUSION

For this report, we studied topographical wire mark numerically. We simulated the physical interaction between the fibers, the flow field, and the forming fabric. The numerical methods used were verified by comparison with Jeffery's equation and beam theory. The simplest fabric geometry, a sine-wave single layer fabric, was used in the simulations.

We simulated the motion of a single fiber with different given initial positions and orientations. A lightweight fiber mat was formed in simulations from 100 fibers, which initially were randomly distributed on top of the forming fabric. The surface roughness of the fiber mat was calculated and compared as a function of fiber EI and length. For fixed EI, increasing the fiber length decreased pulp mat surface roughness, owing to the multiple support points over the length of each fiber. For fixed fiber length, increasing the EI decreased the surface roughness, owing to the reduced bending of fibers into fabric surface holes. Surface roughness also was calculated as a function of drainage velocity. As drainage velocity increased, so did fiber bending, and therefore, mat surface roughness increased. We concluded that topographical wire mark is caused partially by fiber bending and partially by the geometry of the fabric. For the simple sine-wave single layer fabric considered, more than 50% of topographical wire mark was attributed to the geometry of the forming fabric, and the remainder resulted from fiber bending.

Because of the complexity of the fiber mat interaction with the forming fabric, numerous simplifications were made in the simulations, including uniform fibers, simple fabric geometry, negligible interaction between fibers, etc. We plan to follow up on this research by carrying out measurements of the topography of fiber mats, and comparing the surface roughness of the formed mat with numerical results. **TJ**

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APPENDIX

As shown in Fig. 1, if the fiber is inextensible, system connectivity for the fiber can be described as:

$$(\mathbf{r}_{i+1} - \mathbf{c}_{i+1}) - (\mathbf{r}_i + \mathbf{c}_i) = \mathbf{0} \quad (12)$$

In Eq. (12), $\mathbf{r}$ is the vector to the center of spheroids, and $\mathbf{c}$ is the vector along the major axis of spheroids.

The equation of momentum for particle i is given as:

$$m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i^{(h)} + \mathbf{F}_i^{(c)} + \sum_{a=1}^N S_{ia} \mathbf{X}_a \quad (13)$$

where m is the mass of the segment, $\mathbf{F}^{(h)}$ is the hydrodynamic force, $\mathbf{F}^{(c)}$ is the contact force, $\mathbf{X}_a$ is the internal force at the joint a , S is the connectivity matrix, and N is the number of segments.

The equation of angular momentum is:

$$\dot{\mathbf{H}}_i = \mathbf{T}_i^{(h)} + \mathbf{T}_i^{(c)} + \sum_{a=1}^N S_{ia} [(\mathbf{c}_{ia} \times \mathbf{X}_a) + \mathbf{Y}_a] \quad (14)$$

where $\mathbf{T}^{(h)}$ is the hydrodynamic torque, $\mathbf{T}^{(c)}$ is the contact torque, and $\mathbf{c}$ is connectivity vector. $\mathbf{Y}_a$ is the internal torque at the joint a , which includes the twisting torque and bending moment. $\dot{\mathbf{H}}$ is the time derivative of angular momentum, and can be extended as:

$$\dot{\mathbf{H}}_i = \dot{\mathbf{I}}_i \cdot \boldsymbol{\omega}_i + \mathbf{I}_i \cdot \dot{\boldsymbol{\omega}}_i \quad (15)$$

where $\mathbf{I}$ is the tensor of inertia, and $\boldsymbol{\omega}$ is the angular velocity.

The hydrodynamic force and hydrodynamic torque of a prolate spheroid in an unbounded creeping flow can be expressed as:

$$\mathbf{F}_i^{(h)} = A_{ij} (\mathbf{U}_j^\infty - \mathbf{U}_j) \quad (16)$$

$$\mathbf{T}_i^{(h)} = C_{ij} (\boldsymbol{\Omega}_j^\infty - \boldsymbol{\omega}_j) - H_{ijk} E_{jk}^\infty \quad (17)$$

In Eq. (17), i and j are the index of direction, and $\mathbf{U}^\infty$, $\boldsymbol{\Omega}^\infty$, $\mathbf{E}^\infty$ are the velocity, vorticity, and rate of strain of the ambient flow, respectively. The resistance tensors are defined by:

$$A_{ij} = 6\pi\mu a [X_A d_i d_j + Y_A (\delta_{ij} - d_i d_j)] \quad (18)$$

$$C_{ij} = 8\pi\mu a [X_C d_i d_j + Y_C (\delta_{ij} - d_i d_j)] \quad (19)$$

$$H_{ijk} = 8\pi\mu a^3 Y_H \varepsilon_{ijl} d_l d_k$$

where X_A , Y_A , X_C , Y_C , and Y_H are only functions of eccentricity e , and d is the unit directional vector along major axis of the spheroid.

$$X_A = \frac{8}{3} e^3 [-2e + (1 + e^2)L]^{-1} \quad (20)$$

$$Y_A = \frac{16}{3} e^3 [2e + (3e^2 - 1)L]^{-1} \quad (21)$$

$$X_C = \frac{4}{3} e^3 (1 - e^2) [2e - (1 - e^2)L]^{-1} \quad (22)$$

$$Y_C = \frac{4}{3} e^3 (2 - e^2) [-2e + (1 + e^2)L]^{-1} \quad (23)$$

$$Y_H = \frac{4}{3} e^5 [-2e + (1 + e^2)L]^{-1} \quad (24)$$

$$L = \ln\left(\frac{1+e}{1-e}\right) \quad (25)$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2} \quad (26)$$

Here, a and b are semi-major axis and semi-minor axis of the spheroid.

Y represents the internal resistance torque at joints, which includes torsional torque Y_t and bending moment Y_b . According to Bernoulli-Euler Law, the bending moments can be given by:

$$Y_b = \frac{EI}{2a}(\theta - \theta_e)n_b \quad (27)$$

where θ is the angle of bending in radiant, and θ_e is the equilibrium bending angle. EI is the flexural rigidity, and n_b is the unit vector normal to the plane of bending.

For a uniform fiber under torsion, the torsional torque Y_t can be written as:

$$Y_t = k_t \frac{GJ}{2a}(\phi - \phi_e)n_t \quad (28)$$

where ϕ is the angle of torsion in radiant, and ϕ_e is the equilibrium torsion angle. GJ is the torsional rigidity, and n_t is the unit vector normal to the plane of torsion.

Eqs. (2) and (3) can be discretized in time as

$$\frac{m_i}{\Delta t}(\dot{r}_i^{n+1} - \dot{r}_i^n) = F_i^{(h)} + F_i^{(c)} + \sum_{a=1}^N S_{ia} X_a \quad (29)$$

$$\dot{I}_i \cdot \omega_i + \frac{I_i}{\Delta t}(\omega_i^{n+1} - \omega_i^n) = T_i^{(h)} + T_i^{(c)} + \sum_{a=1}^N S_{ia} [(C_{ia} \times X_a) + Y_a] \quad (30)$$

where Δt is the time step of discretization and n is the number of time steps.

By some manipulation of the governing equations, the translational and rotational equations of motion may be cast in the form of Eqs. (2) and (3), where in those equations, N^1 , N^2 , N^3 , N^4 , M^1 , M^2 , and M^3 are matrices associated with position and properties of fiber, velocity and properties of flow field. The matrices are as follows:

$$\begin{aligned} N_{il}^1 &= \delta_{il} \left(C_i + I + \frac{I}{\Delta t} \right) - \sum_{j=1}^N \tilde{d}_{ij} \cdot A'_{j \cdot} \tilde{b}_{lj} + \sum_{j=1}^N \sum_{k=1}^N \tilde{d}_{ij} \cdot A'_{j \cdot} (A')^{-1} \cdot A_k \tilde{b}_{lk} \\ N_{il}^2 &= \tilde{d}_{il} - \sum_{l=1}^N \tilde{d}_{il} \cdot A'_{l \cdot} (A')^{-1} \\ N^3 &= 1 \\ N_i^4 &= C_i \cdot \Omega_i^\infty - H_i \cdot E_i^\infty + \sum_{j=1}^N \tilde{d}_{ij} \cdot A_j \cdot U_j^\infty - \sum_{j=1}^N \sum_{l=1}^N \tilde{d}_{ij} \cdot A'_{j \cdot} (A')^{-1} \cdot A_l \cdot U_l^\infty \\ &\quad + \sum_{a=1}^N S_{ia} Y_a + \frac{I \cdot \omega_i^n}{\Delta t} - \sum_{j=1}^N \sum_{l=1}^N \tilde{d}_{il} \cdot A'_{l \cdot} (A')^{-1} \cdot \frac{m_j}{\Delta t} \dot{r}_j^n + \sum_{j=1}^N \tilde{d}_{ij} \frac{m_j}{\Delta t} \dot{r}_j^n \\ M_{ij}^1 &= \tilde{b}_{ji} - \sum_{k=1}^N (A)^{-1} \cdot A_k \cdot \tilde{b}_{ik} \\ M_{ij}^2 &= (A)^{-1} \\ M_i^3 &= \sum_{j=1}^N (A)^{-1} \cdot A_j \cdot U_j^\infty + \sum_{j=1}^N (A)^{-1} \cdot \frac{m_j \dot{r}_j^n}{\Delta t} \end{aligned}$$

where $A'_{i \cdot} = A_i + m_i I_{3 \times 3} \Delta t$ and $A' = \sum_{i=1}^{N_a} A'_{i \cdot}$. $\tilde{d}_i$ and $\tilde{b}_i$ are anti-symmetrical matrices of vector $\tilde{d}_i$ and $\tilde{b}_i$. The details are given by Ross and Klingenberg [14].

ABOUT THE AUTHORS

This study resulted from trying to understand the influence of forming fabric geometry on the topographic wire mark in paper. In our previous research, we studied how simulated wood fibers interact with a single cylinder, which is representative of one filament in a forming fabric. We extended that earlier research to consider interaction with a realistic fabric geometry.

Fiber interaction with a forming fabric is an example of a fluid-structure interaction. The physics of such interactions is complex. We addressed it by decoupling the fluid mechanics from the fiber structure response to the fluid mechanical forcing.

The fiber deformations we predict here (and that we confirm in a follow-up paper) are much larger than the topographic wire mark observed in pressed and dried paper. It therefore is apparent that pulp mat pressing and drying not only dewater but also

substantially smooths paper.

Paper machine fabric manufacturers can use this information to design and recommend better fabrics for mills. Experimental confirmation will be the next step in this research.



Li



Green

Li is a doctoral student and Green is professor, University of British Columbia, Department of Mechanical Engineering, Vancouver, BC, Canada. Email Green at green@mech.ubc.ca.