Managing uncertainty: A case for using real options with option pricing model (OPM) to evaluate capital investment

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ABSTRACT: The pulp and paper industry relies heavily on the traditional discounted cash flow-based net present value (DCF-NPV) for making capital investment decisions. The deficiency of the DCF-NPV model is that it is static; once a pattern of cash flow is established, management does not have the option to change the direction when new information is available. However, flexibility to alter the investment decision is a powerful strategic and capital investment tool. Abundant research has established strong precedence for applications of "real options" in operational and strategic settings to provide useful insights in the evaluation of irreversible investments under uncertainty. The binomial or Black-Scholes option pricing model (OPM) for strategic planning and capital investment has been used in many other industries but not in the pulp and paper industry. The pulp and paper industry, though very capital intensive, has provided poor to moderate return on investment or return on capital and has never used the OPM and the flexibility it offers for capital investment decisions. This paper makes a case for using OPM for capital investment decisions by using the example of a hypothetical North American mill considering investments to modernize its papermaking operation.

Application: Mills can use OPM-real options value to decide when to harvest timber, when to rebuild paper machines and new facilities, when to close mills, when to enter into new markets, and when to invest in new technologies.

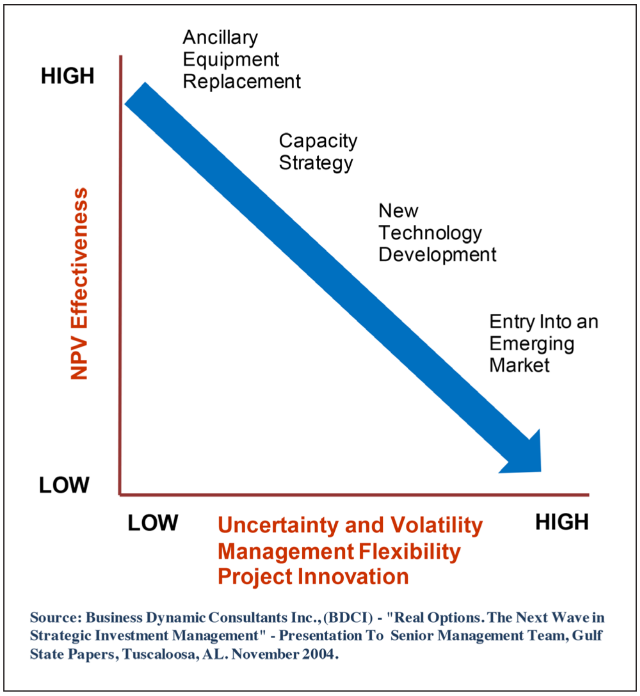
TAPPI PEERS 2012 Honorable Mention ★★★

Uncertainty is the randomness of the external environment. Managers cannot change the level of uncertainty, but they can mitigate asset exposure to uncertainty through flexible investment decisions. Unlike uncertainty, an exposure of a firm's asset to adverse economic consequence is risk; uncertainty is manageable, whereas risk exposure is not.

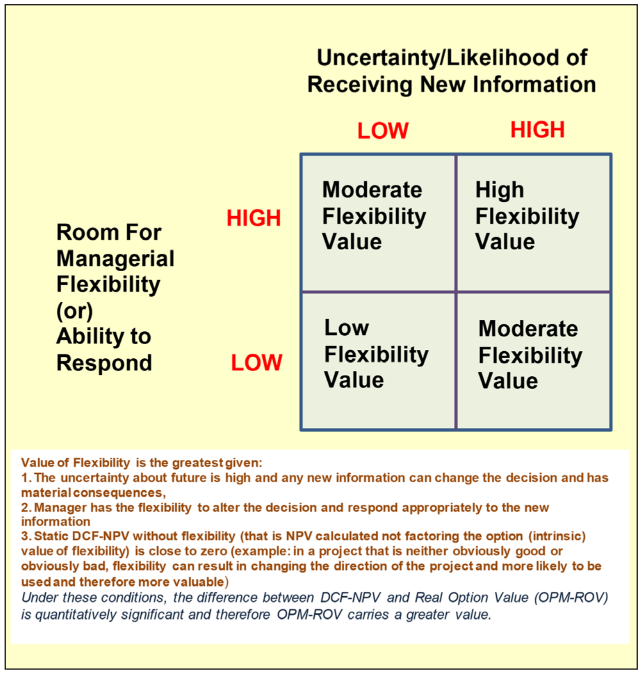
An option is the right – though not the obligation – to take an action in the future. Options are valuable only when there is uncertainty. The flexibility to take action in the future has certain intrinsic value. The option pricing model (OPM) captures that intrinsic value as an "option" premium and then adds that to net present value (NPV), calculated as present value of the assets minus required capital expended to acquire the assets. Therefore, NPV and real options value (ROV) calculated using OPM are similar in their functionality but the latter is more flexible and dynamic than the discounted cash flow-based net present value (DCF-NPV) and has the flexibility for mid-course correction, as and when new information is available.

Approximately 85% to 90% of all global companies use traditional DCF-NPV calculations in one form or another to fund capital projects, and the pulp and paper industry is no exception [1]. Although the application of ROV using the Black-Sholes OPM has been used in areas such as oil exploration, research and development for new drugs, airplane leasing, etc. [2-4], it is unheard of in the pulp and paper industry. The promise of ROV is that it provides an empiric technique for calculating future flexibilities in valuing real-world projects using OPM. It also provides flexibility in making mid-course corrections if and when new information about markets, pricing, and competition is available [4].

The pulp and paper industry is capital-intensive. Over the last three decades (1979–2009), on average, the industry has spent about 8.2% of its sales revenue on capital projects. However, the return on capital was less than 7.5%, and the weighted average cost of capital for this period stands at 7.2%. That means that the industry on average earned only US$ 0.54 on each capital dollar spent; that statistic translates to a negative economic value added of about US$ 47 billion (in 2009 dollars)[5]. (Economic value added measures the actual value addition that a project will bring to the company and unlike simple internal rate of return calculation, includes the weighted average cost of capital that is being used to finance the project.) Ironically, 50% of the capital expenditure was spent on repair or replacement projects that provide little or no return on investment. This in itself provides impetus and a powerful argument for using a better



1. Net present value (NPV) effectiveness.



2. How much is managerial flexibility worth?

tool than DCF-NPV to screen projects for cost improvement and growth.

Many times the NPV does not capture changes in project risk and modifications to the environment created by competitive investment [6]. Also, the NPV model is inadequate for handling the required changes needed in discount rate as a project unfolds. One other flaw of the NPV rule is its unrealistic assumption that, all else being equal, investment decisions are fully reversible and therefore should be implemented via a "now or never" model. The NPV rule works better in some situations than others (**Fig. 1**). However, in situations that involve greater uncertainty and a managerial need for a flexible response, the real options model has more value (**Fig. 2**) [7].

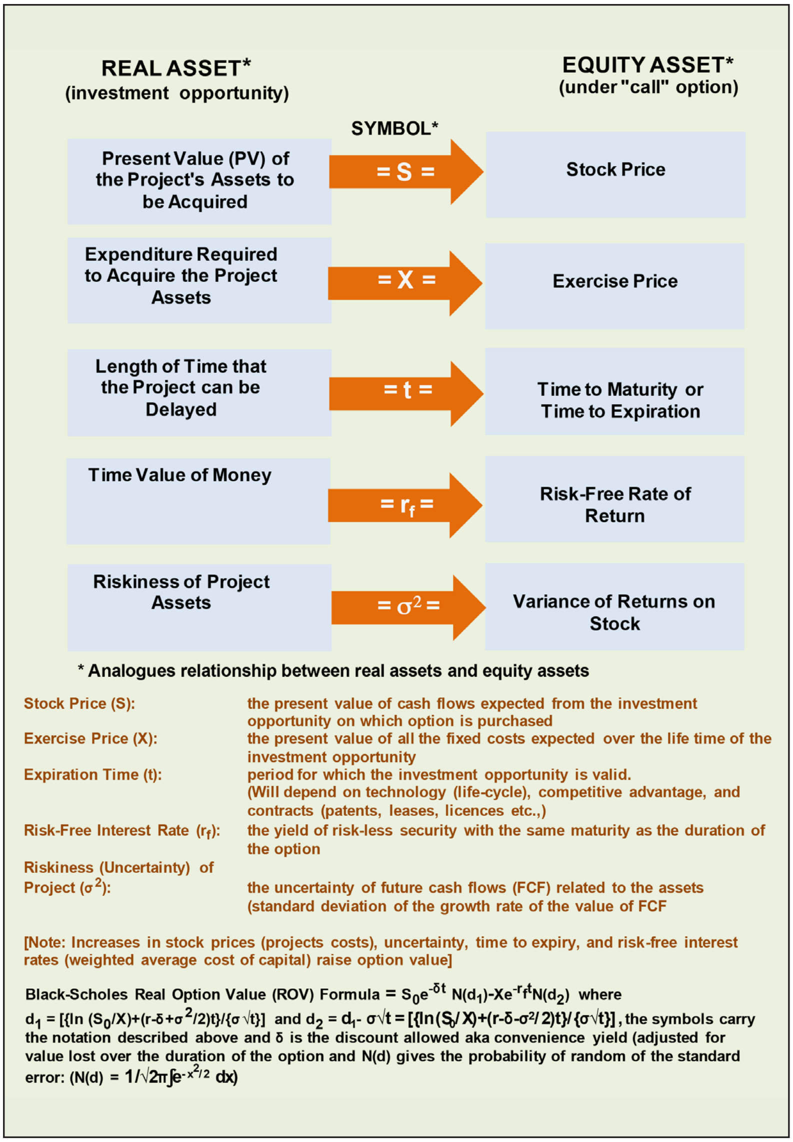
Recognizing real options can help decision makers assess the profitability of new projects and understand whether to proceed with the later phases of projects that have already been initiated. Real options are especially valuable for projects that involve both a high level of uncertainty and opportunities. **Table I** lists possible areas for application of real options in the pulp and paper industry.

A corporate investment opportunity is like a call option because the corporation has the right but not the obligation to acquire (invest in) the operating assets of a new business or invest in the existing business [7-11]. In 1973, Black and Scholes [7] devised an "arbitrage-free" solution to value options. Later, Merton provided rigorous proof for the Black-Scholes OPM [8]. Simple option pricing for exchange-traded calls (buying) and puts (selling) is fairly straightforward, but the real challenge is to apply the OPM to non-traded assets.

Many projects involve spending money to buy or build a

Application	Example
Flexible technology	• Multi-fuel versus single-fuel boiler • Fiber and grade switching
Expansion, reversibility, and optimum capacity choice	• Paper machine rebuilds • New lines • New facilities • Mill closures • New market-entry • Exiting from the market
Multi-stage investments	• Research and development • Natural resources development
Optimal timing and product market competition	• Optimal time to build • Implement multi-stage investments • Implement emission limits • Harvest timber
Valuation	• Mergers and acquisitions • Joint ventures • Contract rewards and damages for acceleration and delay

I. Possible areas for application of real options in the pulp and paper industry.



3. Mapping of capital investment decisions to option pricing — Black-Scholes real options value (ROV)-based NPV.

productive asset [9]. Spending money to exploit such an opportunity is analogous to exercising an option on a share of stock (here the share of stock is the asset). To apply real options, it is important to establish a correspondence between the project's characteristics and option on a share of stock (**Fig. 3**) [9-11]. This is the basis for the calculation of real options, either through the Black-Scholes OPM or through any other OPM, such as the binomial model, etc. [4,12-14]

Most of the real options evaluations are developed for European call options because of their simplicity. Unlike the American call option, the European call option is exercisable only on one day, its expiration date. The option that is being synthesized this way, though not a perfect substitute for the real opportunity, nevertheless provides a quantifiable relationship between NPV (or Tobin's Q or marginal Q) and real options. Such a quantifiable relationship is important when applying corrections to the NPV model to account for the value of deferring investment decisions, and for the changing value of investment under delayed conditions. The model developed using the European call option should be corrected to the American call option, which is what real world investments are like, that is, options exercisable anytime during the course of an expiration period. Benaroch and Kauffman [15], Berrada [16], Berrada and Pilavachi [17], Brealey and Myers [18], and Mason and Merton [19] discuss in detail the application of Black-Scholes OPM to the American option.

A project embeds a real option [13] when it offers management the opportunity to take some future action in response to events occurring within the firm and its business environment. One of the fundamental strengths of the OPM is that, although option valuation is more complex, it is generally less subjective than DCF approaches. The Black-Scholes OPM is especially interesting in this regard and can be applied to real world assets such as building a new paper machine or modifying an older paper machine. However, a correction must be made to the Black-Scholes OPM with regards to exercise time (t). Three approaches are available for dealing with that. The first is to continue using the unadjusted Black-Scholes OPM and regard the resulting value as a "floor." The second is to value the option to each potential date through sensitivity analysis. The third is to use a modified binomial model to consider the possibility of early exercise. Because the binomial OPM is the first approximation of the Black-Scholes OPM, the second approach, sensitivity analysis of t, is the most comprehensive approach.

OPTIONS OR BETS – LEVERAGING FLEXIBILITY

With an understanding of what real options are, it is easy to realize that they are embedded in a whole range of management decisions. Options are everywhere, but there is an important difference between what an option is and what a bet is. Options involve uncertainty about the future and the ability of management to respond to new knowledge as and when it is available. That means that there is a value to managerial flexibility. If management cannot respond in a material manner to new developments, the situation represents more of a gamble than an option. When talking about the reactive flexibility of a real option, we are talking about its advantage as a valuation tool, and the payoff comes from the proactive flexibility with which we can increase the value of an option once it is acquired.

Six real-option levers that can be pulled to manipulate the value of the real options (**Fig. 4**) [20] are:

• **Lever 1** represents the value of the underlying asset (S). By pulling Lever 1, the manager can improve the present value of the expected operating cash flow if a price increase or production increase is possible. Another important issue is the non-lognormal distribution of S (**Appendix**). Hull [21] distinguishes between several such situations and characterizes the resulting Black-Scholes OPM quantitatively. The non-lognormal distribution of S is possible only when the calculated option price is small in absolute terms, something that would normally suggest the deficiency of the underlying investment.

• **Lever 2** represents the present value of the cost associated with the acquisition of asset (X). The cost is reduced in two specific ways. One is by leveraging the economy of scale, the other is by leveraging the economy of scope.

• **Lever 3** represents the uncertainty of expected cash flows (cash flow volatility), denoted by the variance. The value of the option is high when there is a greater chance of flexibility. This is perhaps the crucial difference between OPM and static NPV analysis.

• **Lever 4** will extend the opportunity duration and raises the value of the option because it increases total uncertainty. Although this is a simple approach to employ, there are advanced Black-Scholes models, including the one developed by McDonald and Siegel [22], which enable an analyst to determine optimal investment timing (when the model's underlying assumptions are met).

• **Lever 5** will reduce the value lost by waiting to exercise the option. In the traded quality situation, this is the cost of waiting until after the payment of a dividend (which lowers the stock value and therefore option payoff). In a real option situation, the cost of waiting would be higher if an early entrant seized the initiative.

• **Lever 6** represents the risk-free interest rate. This is not an issue in a proactive flexibility (risk-neutral) environment, because no one can influence the risk-free rate, which is decided by central monetary authorities. However, any expected increase in the interest rate raises the value of an option, despite the negative effect on NPV, because it reduces the present value of the exercise price.

The question of which levers to pull is one of the internal and external constraints on the operations of a company. These constraints may be technical or market-related. A company would also be concerned about factors such as the delay between investment and payoff, as well as constraints on incremental investment.

HOW REAL OPTIONS CAPTURE THE VALUE OF FLEXIBILITY

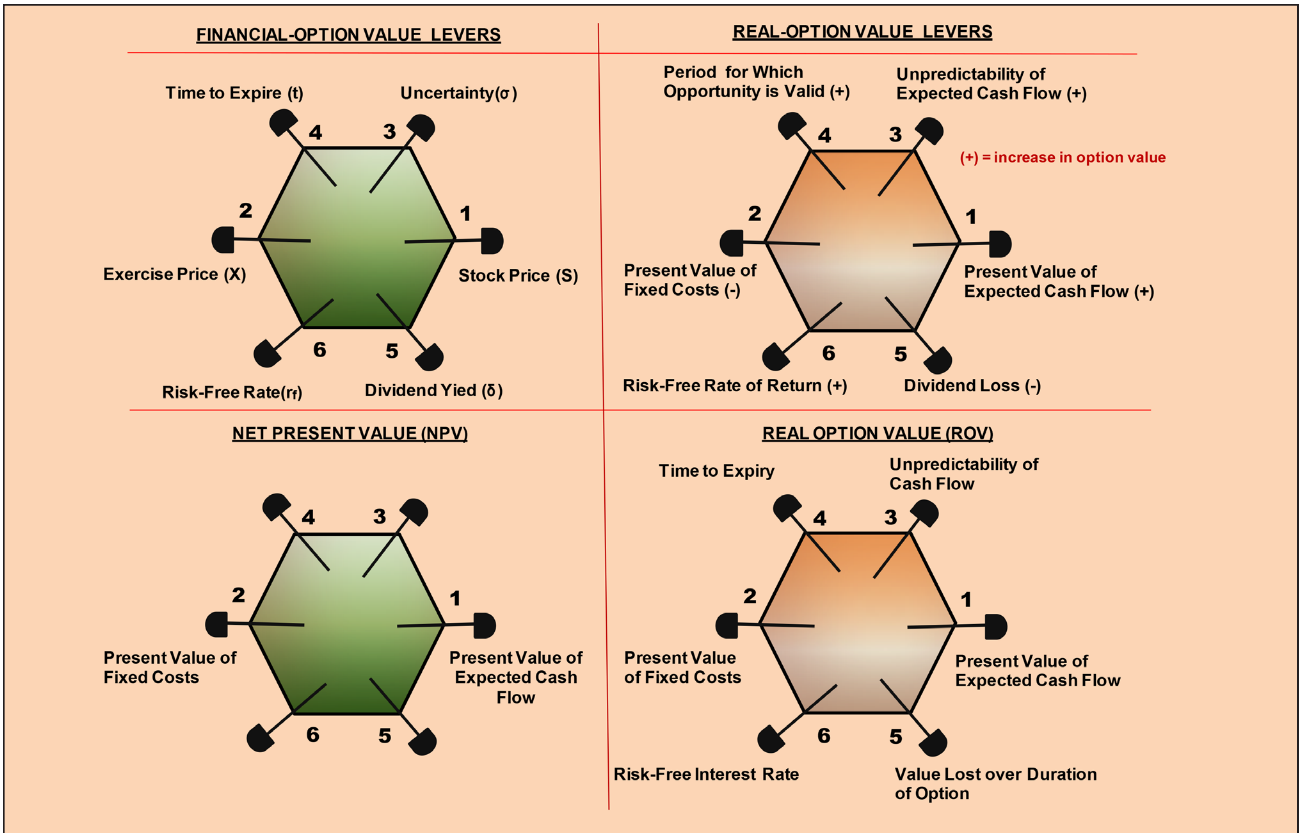
Imagine that a hypothetical North American mill has one paper machine producing 86000 finished metric tons per year (FMTPY) of writing and printing paper. Management is considering three mutually exclusive alternatives for the machine:

- **Alternative 1:** Do nothing.
- **Alternative 2:** Rebuild the paper machine for US$ 51 million, increasing production to 125000 FMTPY.
- **Alternative 3:** Replace the existing machine with a new 262000 FMTPY paper machine for US$ 175 million. The new paper machine will be built up with an option to increase production speed from 6560 ft/min to 7870 ft/min to make 315000 FMTPY at a cost of US$ 33 million. A percentage of the option premium (US$ 6.6 million, equal to 20% of the US$ 33 million) will be paid upfront to the machine vendor with contractual understanding that exercising the option of expanding or not will be at the sole discretion of the management. This paid-up premium will be forfeited if the management at the end of the current project's perpetuity (in this case, 7 years) decides that the market conditions do not dictate moving forward with the speeding up of the machine to gain the additional 53000 FMTPY. In the event that it is decided at the end of the 7-year period that the market demand and price support are conducive and warrant expansion, the paper machine vendor will complete the expansion with the rest of the premium of US$ 26.4 million being paid.

NPV is calculated using a 10.0% weighted average cost of capital (with 1.8% risk premium or risk-free rate of 8.2%). The risk beta (β) coefficient used for the equity analysis is 1.5 (capital intensive matured industry with uncertain cash flow). The following assumptions are also used in the NPV calculation: the perpetuity valuation is for 7 years with 16% of the project remaining value (residual book value) at the end of the project expiration. Double decline depreciation is used in present value factor calculation to obtain the present value of cash flow. The equity analysis was completed using Damodaran's perpetual valuation model, and NPV for all the three alternatives are calculated [23]. Consider the paper machine example. Rebuilding the paper machine at a cost of US$ 51 million to gain an additional 39000 FMTPY would provide a NPV of US$ 68.86 million at a weighted average product price of US$ 990/metric ton (cost, insurance, freight [CIF]) the entire 7-year period. (Note: Market data is collected quarterly for the 1979-2011 period. The woodfree sheet price for 60 g/m^2 writing and printing paper is based on RISI index pricing CIF Rotterdam and used for forecasting price for years 2013-2020. Price movements follow geometric Brownian motion and are mean reverting. Instead of the discrete Monte-Carlo technique, the auto regressive integrated moving average [ARIMA] technique is used to forecast the price.) On the other hand, with the complete paper machine replacement (approximately 262000 FMTPY) option, the calculated NV is US$ 160.43 million. **Table II** shows a summary of the results.

Based on the static NPV calculation, the company would have abandoned the new paper machine project and passed up any opportunity that arose from the price movement of the product.

The NPV plus options premium (real options) method



Source: Leslie, K.J., and Michaels, M.P., "The Real Power of Real Options". The Mckinsey Quarterly 3: 97-106(1997).

4. Six levers of real options: comparison of NPV and ROV methodologies.

would look at this investment from a different perspective:

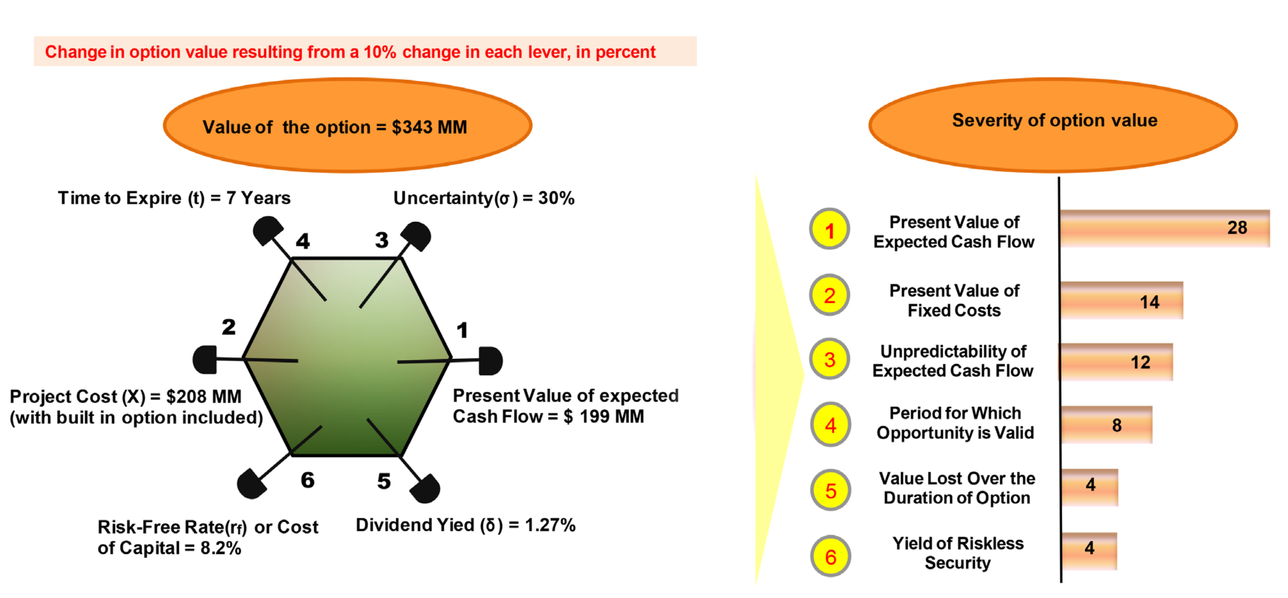
- Future writing and printing paper demand is uncertain, and price for the product is cyclical.
- The rebuilt paper machine would be at its structural production limit if the demand for writing and printing paper strengthened and therefore, if there is an upward movement in the pricing of the product.
- In the future, the new paper machine could be sped up from 6560 ft/min to 7870 ft/min at a cost of US$ 33 million.

In this case, option valuation would recognize the importance of uncertainty. The two major sources of uncertainties affecting the project are: 1) the tonnage increase, and 2) the price that the tonnage will command. It is fair to say that where the capacity (tonnage) is concerned, there is no uncertainty and the new machine project would result in increasing production from 86000 metric tons to 262000 metric tons, an increase of 176000 metric tons from an investment of US$ 175 million. Additionally, with the option built into the project to increase the machine speed from 6560 ft/min to 7870 ft/min at an investment of US$ 33 million, the mill gains an additional 53000 metric tons in the future, and with that, increased cash flow.

Similarly, based on historic price trends, price forecasting is fairly straightforward. Deviation to prices can be calculated using a discrete simulation model (e.g., the Monte-Carlo simulation) or using moving averages of price from quarter to quarter by a model such as the ARIMA model. However, any de-

Alternative	NPV	Project Cost	NPV of Opportunity	Decision
1. Do nothing (ca. 86000 finished metric tons per year [FMTPY])	US$ 0	US$ 0	US$ 0	Reject
2. Rebuild to 125000 FMTPY	US$ 68.86 million	US$ 51 million	US$ 17.86 million	Invest
3. New paper machine (ca. 262000 FMTPY)	US$ 160.43 million	US$ 175 million	US$ -15.43 million	Reject

II. NPV calculation for rebuild and new machine alternatives.



5. Pulling the real options lever.

viation in the price from the forecast would change the weighted average used in calculating the cash flow. For the sake of argument, let us say that the price fluctuation would result in a 30% deviation (σ) around the growth rate of the value of operating cash inflow. Holding the option to increase machine speed from 6560 ft/min to 7870 ft/min at US$ 33 million obligates the company to incur an annual fixed cost of approximately US$ 4.75 million (US$ 33 million/7 years, rounded up to the second decimal), representing a dividend payout of 2.71% ([US$ 4.75 million/US$ 175 million]*100 = 2.71% value of the asset) (Note: For calculating the dividend payment the full option value of US$ 33 million is used). The expiration time (t) for the option is 7 years and the risk-free interest (r_f) is 8.2%.

Suppose there is a 50% chance that after investing US$ 175 million + US$ 33 million (for a total of US$ 208 million) in option for future expansion with the new paper machine, paper prices remain high and demand is strong. Management is rewarded with strong sales for the next 7 years, allowing the company to exercise its option of increasing the machine speed from 6560 ft/min to 7870 ft/min for an incremental production increase of 53000 metric tons (from 262000 finished metric tons to 315000 finished metric tons) at a US$ 33 million investment (US$ 6.6 million as paid-up option premium + US$ 26.4 million for the balance of option payment), thus resulting in an operating income stream with a present value of US$ 882.82 million. The estimation of the value of real options is empirically depicted as follows:

$$\text{ROV} = [(882.82 * e^{-0.0271*7}) * (0.52) - (208 * e^{-0.082*7}) * (0.32)] =$$
$$(\text{US\$ } 379.74 \text{ million} - \text{US\$ } 37.41 \text{ million}) = \text{US\$ } 342.73 \text{ million}$$

On the other hand, suppose there is a 50% probability that demand turns poor, and hypothetically, the operating rate has to be dropped from 90.1% to 72% (= % uptime * % prime metric tons) and price is dropped from 25% to US$ 742.50/metric ton. In that situation, the NPV of operating income stream is only US$ 55.32 million. The traditional NPV analysis of this bet would put the present value of future operations of the paper machine at US$ 199 million (the average of US$ 343 million and US$ 55 million). Definitely, that is not enough to offset the investment of US$ 208 million (US$ 175 million + an option premium to speed up the machine at US$ 33 million) in the new paper machine.

Alternative	NPV + Real Options	Decision
1. Do nothing	US$ 0	Reject
2. Rebuild	US$ 68.86 million (US$ 68.86 million NPV + US$ 0 ROV)	Reject
3. New paper machine	US$ 503.1 million (US$ 160.4 million NPV + US$ 342.7 million ROV)	Invest

III. NPV calculation for rebuild and new machine alternatives.

The option approach to this problem is depicted in **Fig. 5** and is based on the six-lever approach advocated by Leslie and Michaels [22].

If writing and printing paper demand in 7 years exceeds the capacity of the new paper machine, then the new machine will be sped up. If the demand does not exceed capacity, the new machine will not be modified. The value of the option to expand the new paper machine in the 7th year is US$ 503 million (US$ 160 million in NPV + US$ 343 million in ROV) (**Table III**). The best decision is to build a new paper machine with an option exercisable at the end of the 7th year

	Cash Flow Based	Risk Adjusted	Multi-Period	Captures Flexibility
Real options value	Yes	Yes	Yes	Yes
Discounted cash flow-based net present value	Yes	Yes	Yes	No
Data tree analysis	Yes	No	Yes	Yes
Economic profit	Yes	Yes	No	No
Earnings growth	No	No	No	No

IV. Key criteria for decision making tools.

for expansion because of the real options premium. Here we have a classic decision tree analysis [24]. How does ROV differ from decision tree analysis? Option valuation differs from decision tree analysis in calculating the values in accordance with the "no arbitrage" principle. There is no price difference between two equities, whether a risk-neutral or risk-averse investor invests in them (see Appendix). On the other hand, in the decision tree analysis, one calculates the expected cash flow for a probable situation and then discounts it at a chosen rate, usually the weighted average cost of capital. A risk-averse investor would apply a huge discount rate, would adjust the project for risk, or would leverage in a way that would result in such low internal rate of return that the project would never be implemented. Are there differences between ROV and other investment decision-making tools, such as DCF-NPV, economic profit, and earnings growth? **Table IV** shows key differences between various decision-making tools. Both ROV and data tree analysis capture the mechanics of flexibility. However, only ROV adjusts for risk.

Options that depend on multiple sources of uncertainty are called nested options. The NPV calculation cannot capture the value of the nested option because NPV calculations depend on single-point forecasting and decision making. Dixit and Pindyck [9] explain that "by not accounting properly for the options that research and development investments may yield, naïve NPV analysis leads companies to invest too little in their future." This statement is universal, applying to everything from entering a new market to new product development.

A real options strategy emphasizes the logic of strategic opportunism, forcing managers to compare and exploit every incremental opportunity arising from their investments. Such an approach is critically important in mature industries like the pulp and paper industry. Real options strategies promote strategic leverage, encouraging managers to exploit situations in which incremental investments can keep their companies "in the game" and ensure that they come out ahead.

CONCLUSIONS

This paper illustrates the concepts of real options using the OPM as a strategic and investment tool. ROV allows the manager to move from taking risks to managing uncertainties. Use of real options significantly affects the structure of projects and the management of risk. Making irreversible investment decisions in the face of uncertainty is risky. Being able to change a decision as new information becomes available helps reduce risks. Real options management is a disciplined quantitative and qualitative framework that helps enterprises build superiority in all of the three value levers: organization (a better way of making decisions), strategy (a new way of thinking), and execution (a different way of doing business). Why real options now? Companies using the real options method manage their risk and earn extraordinary returns in unfavorable environments because they follow a strategy of making incremental investments. **TJ**

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This paper is dedicated to the memory of Frederick Christiansen, my mentor, guide, and friend. He was director (Process, Technology, and Engineering) at Jacobs Engineering Company, a director at Jacobs Consultants, and a passionate advocate of the real options model for capital investment decisions. The author wishes to acknowledge the senior management team of Cellulosic Fiber Division, Weyerhaeuser NR Company, for permission to publish this paper. Appreciations are also extended to Bob Williams, general manager and vice-president, Weyerhaeuser (Port Wentworth) operations, and Todd Howard, operations manager, for their support and encouragement. Appreciations are also extended to Niti Parthasarathy for her help in the preparation of this manuscript.

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APPENDIX

Non-lognormal distribution of asset valuation: solution to Black-Scholes equation

The Black-Scholes equation is a partial differential equation used to track the price of the option (in this case the asset price) over time. It assumes that one can perfectly hedge the option (arbitrage-free) by buying and selling the underlying asset in just the right way and consequently "eliminate risk." This hedge, in turn, implies that there is only one right price for the option, as given by the following equation:

ABOUT THE AUTHORS

Capital decisions made in the pulp and paper industry cost millions of dollars; this industry has a poor track record on return on investment and needs a more effective financial instrument to realize better returns on capital dollars invested. When new information is available, companies should have the flexibility to respond to it by moving forward, abandoning, or postponing decisions about investments. Real options valuation (ROV) provides that flexibility for senior management.

At the 2008 TAPPI EPE [reference 6 in Literature Cited) conference, the theoretical concept of project flexibility and how any competitive environment might change the investment decision-making process was presented. A theoretical approach to real options also was introduced. In this paper, however, for the first time, the practical application of ROV through the option pricing model (OPM-ROV) for capital investment decision-making in the pulp and paper industry is outlined.

OPMs are commonly used as quantitative instruments in the financial equity markets. However, simplifying such complex models for use in large capital projects was the most difficult aspect of this research. Additionally, the pulp and paper industry traditionally uses the discounted cash flow-net present value (DCF-NPV) model, and that this would be the first application of OPM for the industry made my research more complicated. To address this, investment in a paper machine project was used as a beta case to test this complex concept.

Parthasarathy

Capital spending is not static; avenues are available for senior management to be flexible in deciding to move forward or postpone or abandon a capital project, even when capital has already been committed. Such flexibility does have an intrinsic value to it and can be quantified through OPM, which is commonly used in the equity markets to exercise options (call or put) on stock holdings.

Mills can use OPM-ROV to decide when to harvest timber, when to rebuild paper machines and new facilities, when to close mills, when to enter into new markets, and when to invest in new technologies, such as an integrated biorefinery, etc.

At Weyerhaeuser, we have successfully used OPM-ROV on small capital projects. However, refining the OPM methodologies for application in large turnkey projects will be a challenge and is the next step in this evolution.

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$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0.$$

A derivation of the Black-Scholes differential equation is provided by Hull [21]. According to Hull, the price of the underlying asset follows a geometric Brownian motion, which means that while the price fluctuates it is always mean reverting to a central line (the so-called "trend price"):

$$\frac{dS}{S}=\mu\,dt+\sigma\,dW$$

where W is Brownian motion analogues to simple random walk. Note that W, and consequently its infinitesimal increment dW, represents the only source of uncertainty in the price history of the stock. Intuitively, $W(t)$ is a process that "wiggles up and down" in such a random way that its expected change over any time interval is 0. Thus, the previous equation states that the infinitesimal rate of return has an expected value of $\mu\, dt$ and a variance of $\sigma^2\, dt$.

The payoff of an option $V(S,\ T)$ at maturity is known. To find its value at an earlier time, we need to know how V evolves as a function of s and t. By Itō's lemma for two variables we have:

$$dV=\left(\mu S\frac{\partial V}{\partial S}+\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)dt+\sigma S\frac{\partial V}{\partial S}\,dW.$$

In a delta-hedge portfolio, consisting of short option and long $\frac{\partial V}{\partial S}$ shares at any given time t, the value of these holdings is:

$$\Pi=-V+\frac{\partial V}{\partial S}S.$$

Over the $[t, t+\Delta t]$, the total profit or loss from changes in the values of the holdings is obtained by the derivatization of the above equation, as follows:

$$\Delta\Pi=-\Delta V+\frac{\partial V}{\partial S}\Delta S.$$

Discretization of the equations for dS/S and dV leads to replacing differentials with deltas, as follows:

$$\Delta S=\mu S\,\Delta t+\sigma S\,\Delta W$$
$$\Delta V=\left(\mu S\frac{\partial V}{\partial S}+\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta t+\sigma S\frac{\partial V}{\partial S}\Delta W.$$

and appropriately substituting them into the expression for $\Delta\Pi$:

$$\Delta\Pi=\left(-\frac{\partial V}{\partial t}-\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta t.$$

Note that the ΔW term has vanished. Thus, the uncertainty has been eliminated and the portfolio is effectively riskless. The rate of return on this portfolio must be equal to the rate of return on any other riskless instrument; otherwise, there would be opportunities for arbitrage. Now assuming the risk-free rate of return is r, we must have over the period $[t, t+\Delta t]$ $r\Pi\,\Delta t=\Delta\Pi$.

If we now equate our two formulas for $\Delta\Pi$, we obtain the following equation:

$$\left(-\frac{\partial V}{\partial t}-\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}\right)\Delta t=r\left(-V+S\frac{\partial V}{\partial S}\right)\Delta t.$$

Simplifying, we arrive at Black-Scholes partial differential equation:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0.$$

Solving for V and S over an exercise time of t, the equation has two solutions, with one having non-lognormal distribution. With the assumptions of the Black-Scholes model, this second order partial differential equation holds for any type of option as long as its price function V is twice differentiable with respect to s and once with respect to t. Different pricing formulae for various options will arise from the choice of payoff function at expiry and appropriate boundary conditions.

Congruence between present value and real options calculations

When the price of writing and printing paper (P) develops according to Itō's lemma, in the Feynman-Kac equation $dP=\alpha P dt+\sigma(t,P)dw$, the change in price is partly deterministic and partly stochastic and specified by α(t,P)dt. In the differential equation, α is called Feynman-Kac constant and obtained by solving for the variables t and P by assuming a fixed σ. The Feynman-Kac formula will give the same answer as a present value calculation for all payments that are symmetric in P. The key to this congruence is the fact that the expected value of the above stochastic process is $E[P(t)] = P_0 e^{\alpha t}$. Then, the stochastic term disappears because the expected value of deterministic Itō integral becomes zero. In that case, the present value approaches the real option value and both become one and the same.

Writing and printing paper is a commodity; however, the paper machine that produces the writing and printing paper is an investment. In calculating the NPV and again the ROV, the PV factor and cumulative cash flow comes from the total metric tons made and the price that the product (in this case writing and printing paper) commands (adjusting for transportation, delivered price, or in case of offshore delivery, the CIF price) per unit volume (finished metric tons). Therefore, in the Black-Scholes OPM to calculate the ROV, the stock price (S) is replaced by the present value of expected cash flow and the strike (exercise) price (X) is the present value of the fixed costs (in this case, the project cost).