Some observations towards improved predictive models for box compression strength

DOUGLAS W. COFFIN

ABSTRACT: A McKee-Urbanik type equation for box compression testing (BCT) strength prediction was evaluated using the box data of McKee, Gander, and Wachuta and the tube strength data of Batelka and Smith. Instead of separating BCT into separate panel loads, an effective width was deemed adequate for the data set. Although not much can be done to improve the fit of the McKee data over the McKee formula, the fit of the tube data was slightly better than previously reported in the literature. Unlike previous literature, the fitting parameters used in the model directly target the uncertainty we have in predicting a critical buckling load for a panel (K = edge restraint, κ = twisting resistance, α = relative scaling). The lack of ability to improve the McKee data set is attributed to the variety of boxes tested and the dependence of BCT on unmeasured properties. The century-old take-home message reiterated in this paper is that focused research is needed to replace estimated or fitted parameters with independently measured quantities, which should result in a decrease in the number of boxes with unsatisfactory predictions of BCT.

Application: This paper provides analysis demonstrating that additional improvement in box compression test strength (BCT) prediction should result from the inclusion of additional characterizations of corrugated board and keen observations of box failures.

The 2015 Centennial Celebration of TAPPI provides the opportunity to reflect on our knowledge of corrugated box strength and establish a prudent path for future advancements. Although much progress has occurred, the basic foundations that guide future developments have remained the same. On April 6, 1914, the Interstate Commerce Commission ruled that "there are no transportation differences between the movements of suitable commodities from California terminals eastbound in wood and in fiber" [1]. This ruling (Pridham decision) helped level the playing field and allowed corrugated paperboard, with its great advantage of weight and low cost, to dominate the market.

An article from August 14, 1915, appearing in *The American Stationer* [2], touted the advantages of corrugated as being lightweight yet strong, sufficiently flexible, capable of flat-empty storage/shipping, and low cost. The advantage of corrugated for obtaining high bending stiffness for low weight was illustrated via a photo comparison of three-point bending specimens of corrugated versus pasteboard. The article suggested that corrugated was ideal for the express shipping of small articles, which is important today—especially with e-commerce. Although this article was promotional, it pointed out the engineering and economic advantages of corrugated board, which hold just as well in 2015 as in 1915.

The importance that the corrugated industry had placed on experimental observation and research and development is gleaned from the 1915 article [2]. The Corrugated Fibre Company News reported one such example of investment in research in January 1917 [3]. The company had formed a committee and established four aims or questions:

1. What is the function of each component of the corrugated board?
2. What is the relative importance of the various components?
3. What constitutes a thoroughly satisfactory box for freight shipments?
4. What tests show most clearly the ability of the board to withstand the stresses of transportation?

Even though we have come far, these are still open questions for which answers could lead to innovation and maintained competitiveness for the corrugated industry. The Corrugated Fibre Company [3] summarized that at corners and edges, shearing, tensile, stretch and folding were important, while on sides and ends compression, flexure, puncture and cushion were important. They concluded:

> "The members of the Corrugated Fibre Company organization as well as the committee members felt that the ultimate working out of these problems was of sufficient importance to the industry to warrant a painstaking thorough research and study by a qualified engineer

> and chemist who could give undivided attention and thought to the question and accordingly the company has recently established a Fellowship at the Mellon Institute of Industrial Research and School of Specific Industries University of Pittsburgh, where it is thought that the problems presented as well as others affecting the corrugated paper industry can and will be given the study and scientific research that they deserve.
>
> *"It seems remarkable that an industry which has grown to such magnitude as this one should have so far progressed without some such research work being undertaken."*

This last emphasized statement is quite striking and resonates in the present time. In the long run, investing in development is a better approach compared to manipulating the free market, such as was attempted by the wooden boxmakers in the early 1900s. A great contemporary example from the corrugated industry is the development of the corrugated common footprint [4] in response to poor performance of the existing products and increased competition from plastic crates. In contrast, paper grocery bags may see some renewed interest because of an environmental push against the use of plastic bags, but the push is unlikely to be successful unless accompanied by innovation that meets consumer demands.

The research noted in [3] was carried out Malcolmson [5], who developed a rotating drum box tester. About 20 years after these initial studies, Malcolmson [6] discussed the importance of compression tests for corrugated boxes. This denotes a shift from measuring performance with tests such as Mullen/burst and drum rotation, which are important for handling, to a focus on compression, which is indicative of stacking performance and lifetime.

For the last 50 years, box compression testing (BCT) has dominated development. Little [7] identified edgewise compressive testing ([ECT], used to denote intrinsic compressive strength), bending stiffness, and box perimeter as three important factors that can be used to predict the compressive strength of a box (denoted as BCT). Significant research in corrugated board was conducted throughout the 1950s, and one major outcome was the McKee Equation, which was published in 1963 by McKee, Gander, and Wachuta [8]. The successful use of this equation to predict package performance ultimately lead to the 1991 changes to the Uniform Freight Classification (UFC) Rule 41 and the National Motor Freight Classification (NMFC) Item 222 to allow qualification based on minimum ECT. This change resulted in a major shift in the development of various grades of high performance liners. Today, the industry could benefit from research that answers the previously posed questions [3].

Frank [9] recently provided a very thorough review of progress in corrugated board with a focus on compressive strength. His purpose was to capture the vast amount of research conducted in the past and identify areas for fruitful new research. He concluded that much work still needs to be done to develop robust models that can be applied more broadly. He pointed out that both natural variability inherent in paper and the vast array of corrugated structures that exist in the market place leave us with unacceptable predictive capability. He suggested that the industry still requires refinement in methods to measure key parameters. He also finds that significant work needs to be done to understand the role of scores and flaps on performance. It is not clear from the previous literature which new parameters should be measured to improve predictive capability.

The fundamental needs of the corrugated industry have not changed in the last hundred years, but to progress further we will need to delve deeper into the problems. The existing literature does not put forth a systematic evaluation of how new parameters could improve our understanding of box compression strength. In addition, perhaps too much emphasis has been placed on the meaning of the fitting parameters in the McKee-Urbanik type formulas. The purpose of the following sections of this paper is to further examine the McKee-Urbanik BCT predictions, clarify some issues, establish specific additional parameters to be measured and propose a level of acceptability for any improved model.The final objective of this paper is to put forth the notion that such improvement will only result when new key properties are introduced into the characterization of corrugated board.

THEORY: PREDICTIVE EQUATION FOR BOX COMPRESSIVE STRENGTH

The short-form McKee formula for BCT, as presented by McKee, Gander and Wachuta [8], is shown in Eq. (1):

$$BCT = 5.874 ECT\sqrt{hZ} \quad \text{with} \quad d/Z \geq 1/7 \tag{1}$$

where *ECT* is the edge-loaded compressive strength, *h* is combined board caliper, *Z* is the perimeter of the box, and *d* is the depth of the box. For a specific combined board, one measures only *ECT* and *h*. The fact that Eq. (1) is still being utilized after fifty years is a testament of its efficacy. Equation (1) suggests that to design a corrugated board, one should focus primarily on maximizing ECT and secondarily on maximizing *h*. McKee, Gander, and Wachuta [1] warned that Eq. (1) should be used with caution and presented their simplified formula as:

$$BCT = aECT^{b}\sqrt{D_{MD}D_{CD}}^{\,1-b} Z^{2b-1} \tag{2}$$

where D_{MD} and D_{CD} are bending stiffnesses in the machine direction (MD) and cross direction (CD), respectively. It was reported that for a = 2.028, and b = 0.746, the prediction was accurate to about 6% on average for boxes with a perimeter in the range of 30-135 in. Equation (2) requires three measured properties of the corrugated board (*ECT*, D_{MD}, and D_{CD}) — one additional property compared to Eq. (1), and moreover, bending stiffnesses are more cumbersome to measure than caliper. Even the accuracy and general applicability of Eq. (2) has been called into question [10]. The literature has many examples of extensions of the McKee Equation to broaden applicability, as in [9]. Urbanik [11] devoted much time to understanding and predicting BCT, and the reader again is referred to Frank's review [9] for a more complete listing of his contributions. The most recent application of his models to existing data sets was completed by Urbanik and Frank [10]. They developed a single model that was accurate to an 8.5% error on all single-wall regular slotted carton (RSC) boxes available in the literature over a 46 year span.

The basis for the McKee Equation comes from studies of compressive strength of panels for aerospace applications [12]. The base equation is empirical and can be stated [10] as:

$$\frac{P_{\max}}{P_f} = \begin{cases} \alpha\left(\dfrac{P_{cr}}{P_f}\right)^{\mu} & P_{cr} < P_f \\ \alpha & P_{cr} > P_f \end{cases} \tag{3}$$

where P_{max} is the maximum panel load, P_f is the intrinsic compressive strength (= *ECT*), and P_{cr} is the critical buckling load in units of load per unit length for the panel with width w and depth d. The concept behind Eq. (3) is that failure is partially indicated by pre-failure characteristics of the board and thus the standard practice is to use theoretical equations for the linear elastic buckling load of the panel, rendering Eq. (3) semi-empirical. If this approach works, there is no need to incorporate inelastic behavior or more complex expressions for the buckling load. For a panel with width w, the elastic critical buckling load is given as:

$$P_{cr} = \frac{4\pi^2}{w^2}\sqrt{D_{MD}D_{CD}}\left(\lambda + \frac{\kappa+1}{2}\right) \tag{4}$$

If all four sides of the panel are simply supported, λ is the value that minimizes Eq. (4) and satisfies:

$$\text{Simple Supports: } \sin^2\left(\pi\chi\sqrt{\lambda}\right) = \sin^2\left(\pi\chi\sqrt{1+\lambda}\right) \tag{5}$$

and if the top and bottom are fixed, λ is the value that minimizes Eq. (4) and satisfies

Fixed-top and bottom:

$$(1+\lambda)\sin^2\left(\pi\chi\sqrt{\lambda}\right) = (\lambda)\sin^2\left(\pi\chi\sqrt{1+\lambda}\right) \tag{6}$$

In Eqs. (3) and (4):

$$\chi = \sqrt[4]{\frac{D_{MD}}{D_{CD}}}\frac{d}{w}, \quad \kappa = \nu_m + 2\frac{D_{MD-CD}}{D_{MD}D_{CD}}$$

ν_m is an in-plane Poisson ratio and $D_{MD\text{-}CD}$ is the twisting stiffness. Equation (5) is equivalent to forms used in other references, for example [10], but this form was chosen simply to give a direct comparison to Eq. (6), which was included to assess strength data for tubes. Urbanik and Frank [10] use an "empirical improvement" factor, τ, in the form, χ^{τ}. This allows for a change in the relationship between box depth and critical buckling load. Unfortunately, their function does not change the critical buckling load at the value at $\chi = 1$ and thus cannot accurately represent the differences between, say Eqs. (5) and (6), without another scaling factor. Thus, the parameter α in Eq. (3) (suggested to be related to inelastic buckling load in [9,10]) must play a double role and limits any physical interpretation of the parameters.

To implement Eqs. (3) and (4), you must measure four combined board properties (*ECT*, D_{MD}, D_{CD}, κ)). In practice, κ is not measured but set to a value such as κ=1, which corresponds to the Baum approximation for shear modulus of paper [13].

When considering Eqs. (1)-(3), the tradeoff is for improved representation of the critical buckling load at the expense of measuring more combined board properties. One could consider including transverse shear stiffness [14] or nonlinear material behavior [11], which requires additional measured properties. Even without changing the form of Eqs. (3)-(6), we have two quantities (κ and resistance proffered by edges) not typically measured for corrugated board and likely contributing to a loss of predictive capability.

Equation (4) is valid for a panel, and an extension to a box is required. Inherent in McKee's Eqs. (1) and (2) is an assumption that the width is 1/4 the perimeter. Thus, there is no distinction between square and rectangular boxes. The approach of Urbanik, as illustrated in [10], was to separate BCT to individual panel loads. The drawback of using panel loads is that it introduces another degree of freedom in fitting, and even if the separated loads seem reasonable, it may not reflect the actual distribution of load. Thus, I question the interpretation of α assigned in reference [10] as an inelastic buckling load, because it is based on empirically determined maximum panel loads not measured loads. To avoid this issue, I specify an effective width as:

$$w = \sqrt{\frac{l_1^2 + l_2^2}{2}} \tag{7}$$

where l_1 and l_2 are the lengths of the sides of the box. Equation (7) is equivalent to saying that the effective panels form an equivalent square box with the same diagonal length as the rectangular box. This author believes Eq. (7) is sufficient for the data being considered.

The final model utilized was obtained to compare to a select set of published compressive test data by combining the first part of Eq. (3) with Eq. (4) and an approximation that covers both Eqs. (5) and (6). This procedure yields:

$$\frac{BCT}{ECT \cdot Z} = \alpha \left[\frac{P_{cr\infty}}{ECT} \left(\lambda_{\chi} + \frac{\kappa + 1}{2} \right) \right]^{\mu} \tag{8}$$

where

$$P_{cr\infty} = \frac{4\pi^2}{w^2} \sqrt{D_{MD} D_{CD}} \quad \text{and} \quad \lambda_{\chi} = \frac{K}{\chi^2} \tag{9}$$

The value of K allows one to account for more or less restraint of buckling at the top and bottom edges (creases). A larger value of $K = 0.67$ would yield a curve similar to that prescribed by Eq. (6) (fixed), and a small value of $K = 0.1$ would yield a curve closer to Eq. (5) (simple). If K were zero, the sensitivity of buckling to depth (χ) would disappear. The value κ accounts for the effect of twisting shear, and the terms α and μ are empirical factors, which are likely specific for various corrugated boards and box/tube geometries. I believe the current approach is more reasonable than that reported in the previous literature. If the resulting model is not satisfactory, the logical next step would be to develop a more rigorous form of Eqs. (4)-(6).

By eliminating the second condition of Eq. (3), the criteria that α represents a maximum panel load is not enforced. Thus, α represents a scaling factor accounting for the fact that the actual magnitude of the buckling load is not known. Unless one has data for very small perimeter boxes or tubes where the compressive load reaches a plateau, the value of α is only estimated. In addition, in the limit as the panel size decreases (to dimensions of *ECT* specimens), the strength should equal if not exceed the *ECT* value. The separation of *BCT* to panel loads requires both *ECT* and P_{cr}; therefore, α cannot be fully determined because neither the edge restraint nor the value of κ is known *a priori*. The present analysis using Eqs. (7)-(9) is fundamentally different to that reported in the literature, and using it with data from the literature can illustrate the importance of the parameters (α, μ, κ and K.)

RESULTS: COMPARISON OF BCT PREDICTION TO SELECT DATA FROM LITERATURE

To utilize Eqs. (8)-(9), the values of *BCT*, *ECT*, D_{MD} and D_{CD} reported in [8,15] were used. The fitting parameters are α, μ, κ and K. Three cases are presented: case 1, $\kappa = 1$ and α is allowed to vary; case 2, $\alpha = 1$ and κ is allowed to vary; and case 3, all parameters are allowed to vary. The cases were chosen to illustrate that our ignorance of the true values of these parameters has little influence on the ability to fit the data. The takeaway message is that to improve the accuracy of our BCT prediction, we need to measure K and κ rather than use them as fitting parameters.

Eq. (8) can be re-arranged to give:

$$\lambda_{\chi} + 1 = \frac{\kappa - 1}{2} + \frac{\left(\frac{BCT}{Z \cdot \alpha} \right)^{1/\mu} ECT^{(1-1/\mu)}}{P_{cr\infty}} \tag{10}$$

which provides for a direct comparison to the curves given by Eqs. (5) and (6) describing the dependence of box depth (χ) on the critical buckling load.

The results of McKee, Gander and Wachuta [8] were chosen because they are the basis for the parameters of the McKee Equation. The tube strength data of Batelka and Smith [15] were chosen because they represent a large set of data, 108 single-wall (SW) tubes, produced from only three liner and medium sources for three distinct board types. The best fit for the McKee data obtained by Urbanik and Frank [10] gave an average error of about 6%, not very different than the original McKee Equation. Thus, a two parameter model is sufficient and K is set to the limiting value of zero, reducing the number of parameters for the McKee data to two for cases 1 and 2, and three parameters for case 3. Urbanik and Frank [10] obtained a best fit of the data of Batelka and Smith [15] of about 4.5% absolute error using a four parameter model with BCT split into panel loads. The four parameter model offered a vast improvement over the McKee Equation and even a two-parameter equation applied to the panel loads (average absolute error 12.5%). Thus, for the Batelka data, cases 1 and 2 use three parameters and case 3 uses four parameters.

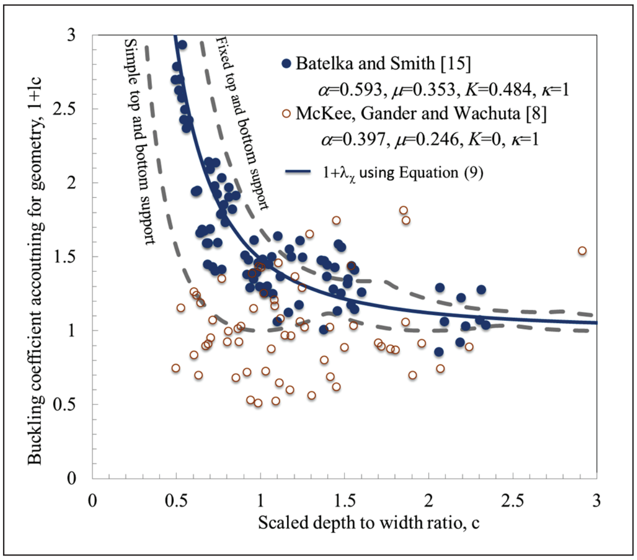
The analysis utilized the effective width defined in Eq. (7). The parameters (**Table I**) were determined using Microsoft Excel to minimize the sum of the squared error for each fit and to present the results using Eq. (10) (**Figs. 1-3**).

After extending some effort, it was decided that the McKee data could not be improved beyond an average error of 6.0 %. Using the assumption that $\kappa = 1$ and $\lambda_z = 0$, Eq. (8) can be used to visualize the McKee data (**Fig. 4**). For clarification, the data was separated into three distinct regions based on the error being under or over 6.0%. After extensive analysis, no significant correlation of this split to any other parameters reported by McKee, Gander, and Wachuta [8] could be found, including flute size and manufacturing joint. I did find a significantly improved empirical fit for the Batelka data just using two variables, $P_{cr\infty}$ and d. The basic form was the same as Eq. (8), but the parameters α and μ were taken as function of d with a total of five fitting parameters required to optimize the fit. This model gave an average error of 2.8% compared to 4.2% given in **Table I**.

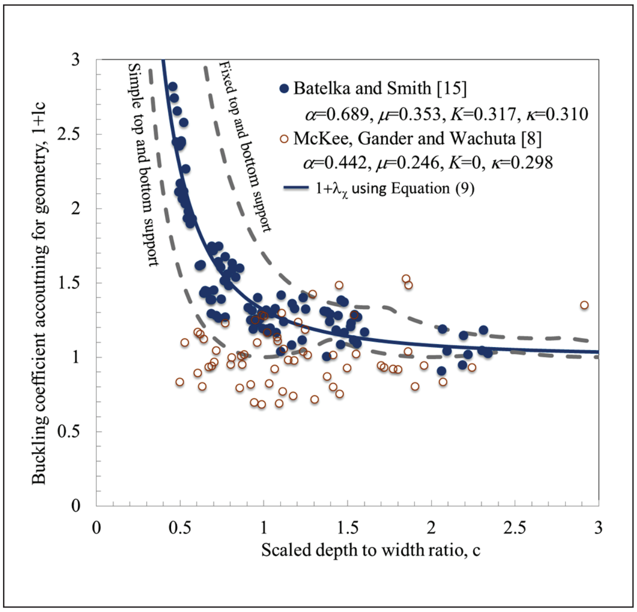
To visualize the distribution of the absolute errors caused by the model, experimental cumulative distribution curves (**Fig. 5**) were created. The cumulative probability of a given absolute error was determined as number of data points with

Data Set	Case	α	μ	K	κ	Average Absolute Error, %
McKee [8]	1	0.397	0.246	0	1	6.1
	2	1	0.246	0	-0.953	6.1
	3	.442	0.246	0	0.298	6.1
Batelka [15]	1	0.593	0.353	0.484	1	4.2
	2	1	0.353	0.110	-0.544	4.2
	3	0.689	0.353	0.317	0.310	4.2

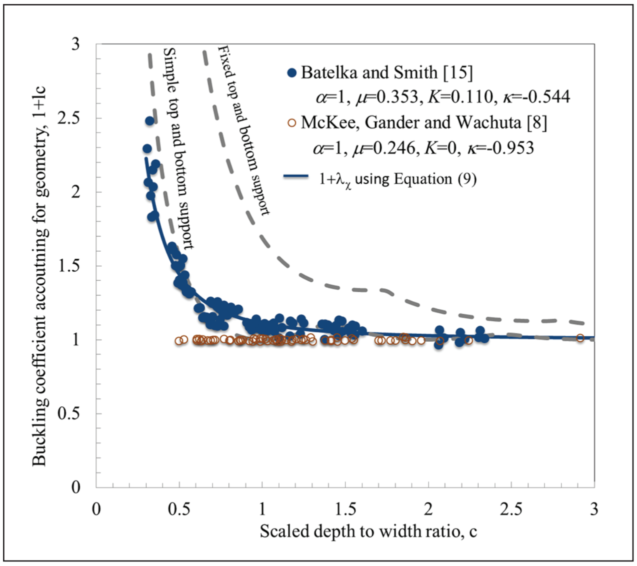
I. Parameters from fitting of Eq. (8) for data from references [8] and [15] (integer values indicate prescribed parameters).



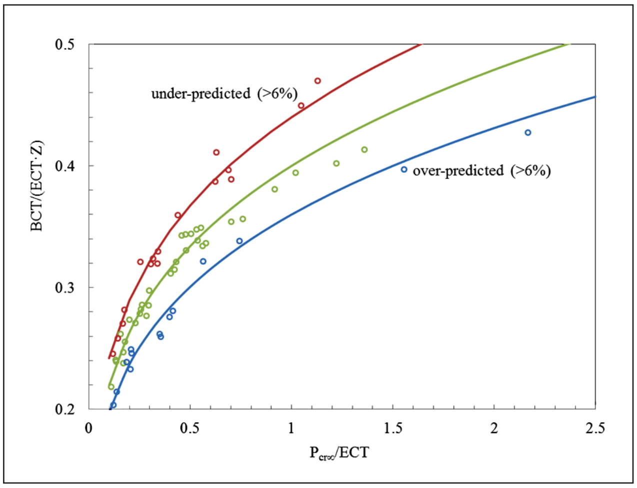
1. Comparison of dependence of critical buckling load on box depth to case of simple supports and fixed-top and bottom for case 1 from Table I.



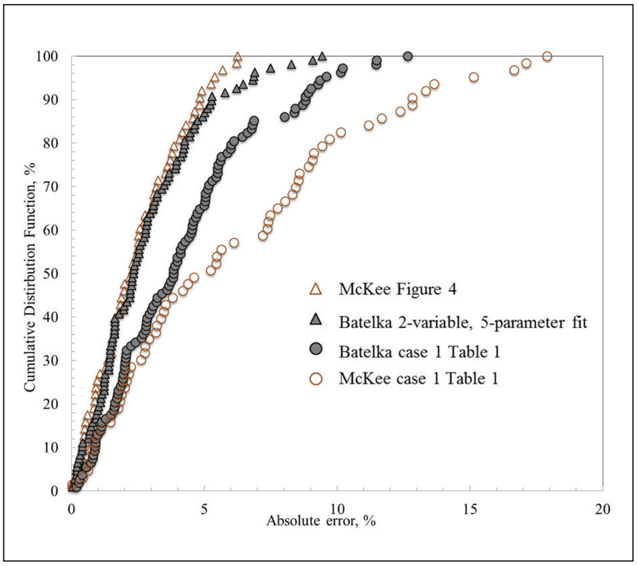
3. Comparison of dependence of critical buckling load on box depth to case of simple supports and fixed-top and bottom for case 3 from Table I.



2. Comparison of dependence of critical buckling load on box depth to case of simple supports and fixed-top and bottom for case 2 from Table I.



4. McKee, Gander, and Wachuta [8] data plotted as a function of limiting critical buckling load and separated to three curves based on error being over- or under-predicted by 6%.



5. Cumulative distribution function of absolute error for the various fits of box compression testing (BCT) strength data.

absolute error less than or equal to the point of interest divided by the total count of all data. Figure 5 summarizes the errors produced by the fitted-models. For example, the best empirical fits for both data sets yield that approximately 90% of the data had an error of less than 5%, while the predictions corresponding to the results of Table I are not nearly as good. Only case 1 is shown in Fig. 5 because cases 2 and 3 were indistinguishable from case 1.

DISCUSSION

The McKee data shown in Fig. 4 is very appealing in the sense that it suggests there is some property that could be measured with three distinct levels to explain the differences. The data reported by McKee, Gander, and Wachuta [8] does not provide any insight into this. For the large assortment of box types tested by McKee (meant to represent the variety observed in industry at that time), a significant amount were not well predicted by the McKee Equation. Any vertical line drawn in Fig. 4, would cross McKee Equivalent boxes, meaning boxes where the McKee Equation predicts the same BCT relative to ECT and perimeter. A box whose data point falls on the upper curve has an actual strength greater than McKee predicts, and a box on the lower curve has an actual strength that is lower than predicted by McKee. If you produced the box that is on the lower curve, it means a competitor whose box is on the upper curve has an 18% improved strength advantage over you. It would be interesting to find out why one seemingly equivalent box is significantly stronger than another. If you produce the box on the upper curve, you also would be interested in knowing why, because you can either save grammage or place the box in more severe service. In either case, this 18% difference from high to low is significant enough to warrant further investigation. The cumulative distribution curves in Fig. 5 suggest that if one were able to improve the predictive capability for BCT, the average error could be reduced from 6.1% to 2.7%. In addition, 90% of boxes would have an error less than 4.8%. Compare this to the current McKee model yielding 50% of boxes would have an error greater than 5%. The only way to achieve this improvement in prediction is to introduce new measurements for the boxes beyond the typical properties (ECT, D_{MD}, and D_{CD}). The first step suggested here is to measure values for K and κ for each box or board type. The next step, if required, would be to improve the robustness of Eqs. (4)-(6).

The results suggest that a good estimate of the critical buckling load is lacking. The value of κ is not typically measured, and this parameter adds directly to the magnitude of the critical buckling load. In addition, the resistance to buckling offered by the folded edges is not known. Flaps could add eccentricity and increase panel bulging. This implies that we do not know the factor K, and even the factor α cannot be independently determined for most BCT data sets. The three cases of fits given in Table I show that a wide range of α, κ, and K could be obtained with equally good predictions of BCT. The results shown in Fig. 1, where $\kappa = 1$, the typical assumption for paper and corrugated, are not appealing because the data is spread and some boxes seem to be stiffer than proffered from fixed supports. Case 2, where $\alpha = 1$, looks very appealing as observed in Fig. 2, but the negative value of κ is unsatisfying, without further elucidation from theory. Case 3 is an intermediate case where a minimum squared error was found for all four parameters. The data for Batelka is mostly between the limits of fixed support and simple support curves. The data of McKee are mostly at or below the simple support curve. This suggests that all three parameters (α, κ, and K) and their relative magnitude are important for proper BCT prediction. For a given data set, one can get a best fit, but if the actual values of say κ and K vary for the different boards included in the data set, predictive capability suffers.

A predictive equation for BCT is typically fit to the mean value for a number of box tests for each specimen type. Batelka and Smith [15] completed 12 BCT tests for each data point, and the average 95% confidence limit for the mean BCT was 3%. Carlson [16] suggested that the coefficient of variation for box testing is on the order of 7.3%, yielding a 95% confidence limit of the mean for 12 BCT tests on the order of 4.2%. The cumulative distribution curve for the 5-paramter fit of the Batelka data gave an average error of 2.7%, which is comparable to the 3%. Unfortunately, the nature of this "best" fit involves the depth directly, instead of a dimensionless depth parameter. This finding could indicate that our knowledge of the mechanics of the failure is lacking and not captured by current models (more robust forms of Eqs. [4]-[6] required). The fit of McKee based on Fig. (4) also has an average error less than 3%, but we have no basis to justify this fit, and the indication is that more knowledge of those boxes is required. Urbanik and Frank [10] suggested that the best a model could do to fit data would be at a level of about 4% and that 7% to 8% would be adequate. Their numbers are based on estimates of standard deviations. When fitting a specific set of data, the expectation should be higher and one should use some confidence limit for the mean, not the standard deviation of the measurements. For example, the 95% confidence limit of the mean could be chosen to judge quality of the fit. The 4.2% fit of the Batelka data was good, indicating that there are few unknown large differences between the various tested boards, but as shown in Fig. 5, only 85% of the data had an error less than 6.9%. If additional properties of each board had been reported, the robustness of the fit could be improved.

If we compare the best fit error of the Batelka data from the current fit, 4.2%, with a 3-parameter model, and that of Urbanik and Frank [10], 4.5% (three parameters and split into panel loads), the fit is marginally better but the ease of the fit is much improved. This suggests that the entire box acts together and that using the effective width with the allowance for undetermined depth dependence on buckling (K and κ) provides a reasonable model for BCT, even for rectangular and shallow boxes. In the long run, the approach of Urbanik and Frank [10] may be more robust, but the separation to panel loads is premature for the given measured properties and the use of the empirical factor τ is questionable. The simplified model shown here was sufficient. More importantly, the three cases of fits show the importance for the need for additional measured quantities and an improved expression for the critical buckling load. Urbanik [17] discussed similar matter, which led to the correction parameter (τ used in [10]). Therefore, it is proposed that the use of K is more appropriate. Peterson and Schimmelpfenning [18] experimentally showed that the top and bottom edges of tubes were restricted from rotation, implying that the critical buckling curve for a tube is closer to the fixed conditions curve, as shown in Fig. 3. They also showed that when a box was restrained from rotation, it could achieve a higher BCT. The main difference between tubes and boxes is in the top and bottom edge conditions (K). Renman [19] presented a method to test the effects of the top and bottom folded creases on compressive behavior. He found that the compressive strength of a section of board with the flaps was about 50% lower than the ECT strength. In addition, the flap and crease caused additional moment.

The results and analysis given previously suggest that current data-sets cannot be used to distinguish the role of important parameters (α, κ and K). If we are to develop a model for compressive strength that can distinguish between boxes that current models under and over predict, additional measurements to determine these as independent parameters (α, κ and K) are required. These measurements might be for the combined board, such as the twisting stiffness (κ), or if Eq. (4) is modified, even transverse shear stiffness [14,20]. To determine K, the resistance of the folds to rotation [19] is needed. Today, it is likely that high performance boards will require modified test methods to yield meaningful measurements [21, 22].

CONCLUSIONS

A critical evaluation of a simple model for BCT was conducted to determine some of the technology gaps in predictive capability. The proposed model identified two key parameters (K and κ) that should be obtained from independent measurements. The comparison of three fits to existing tube data and the McKee box data suggest that the range of parameters giving equally good fits is wide, and one needs independent measures of these properties to remove the uncertainty. Independent quantification of the effects of geometry and edge restraint (K) and material properties (κ) on panel buckling is warranted.

The conclusions in the following sections are made with respect to the four questions posed in 1917 [3], although limited to the scope of this article.

1. What is the function of each component of the corrugated board? [3]

The results and discussion of the predictive model for *BCT* indicated that the magnitude of the shear twisting term (κ) is essential to align the critical buckling load to an appropriate depth dependence. It was shown that the standard practice of choosing a value for κ will influence the values of other fitting parameters, even if the overall fit is adequate. This means that the BCT for boards that possess a low value of κ will be over-predicted. The results also show that the role of the top and bottom flaps need to be assessed with measurements to determine the appropriate value of K for a given board. Again, if one board/box combination offers less resistance (lower K), the *BCT* will be over-predicted. In order to obtain the α term as a measure of maximum panel strength, square boxes/ tubes or panels of sufficiently small size need to be tested. It is not sufficient to calculate a maximum panel load using panel loads extrapolated from BCT for a rectangular box. In other words, α needs to be split into a product of maximum load and a scaling factor for the critical buckling load of a very deep box.

2. What is the relative importance of the various components? [3]

In the current context, the large range of α, κ, and K values that provides equally good fits of the data indicates that further study of corrugated board panels under edge-wise compression is required. Measurements of additional properties such as flap resistance (K), twisting shear (κ), and possibly transverse shear and nonlinear effects (changing Eqs. [4]-[6]) are needed to allow for a more proper ordering of the relative importance of each component of a box.

3. What constitutes a thoroughly satisfactory box for freight shipments?

If a box maker produces a box that consistently underperforms compared to the McKee formula, the present analysis (Fig. 4) suggests that another seemingly equivalent box exists that surpasses expectations based on McKee. Based on the data of McKee, this difference could be as much as 18%. The potential for an 18% increase in strength or a similar decrease in weight should be sufficient motivation to undertake research that will uncover the missing information alluded to in the answers to the previous two questions. The fact that no significant correlation to other reported information could be found for under- and over-predicted McKee data suggests that a better assessment of important differences in commercial boxes is required. The current analysis shows that measured values of K and κ should not be neglected.

The other issue of importance here is to specify what constitutes a satisfactory prediction of performance. For BCT, I suggest that one needs to establish a level based on the cumulative distribution function of the error (as shown in Fig. 5). For example, one might establish that the calibration of a model should fit that data such that 90% of the predictions are within 5% of the mean values. Any validation, verification, or application of the model will likely result in an increase in error, so the calibration requirement should be strict.

4. What tests show most clearly the ability of the board to withstand the stresses of transportation?

All the findings of this paper point to the fact that additional characterization of the board is required if one wants a more reliable *BCT* prediction. Not only are more measurements of the corrugated board necessary, but more observations of failure mechanisms during the testing need to be recorded. Our capability to move beyond McKee and Urbanik is not hindered by our lack of knowledge in mechanics or our inability to model complex problems; rather, it is a lack of measured characterization of corrugated board and box compression. With the right measured properties and recorded observations, we can develop a predictive model that meets needs identified by Frank in 2014 [9] and may serve the industry for the next 100 years. I suggest the first priority is to develop methods to determine K and κ.

The last century was fruitful for knowledge of box compression, yet as indicated in this paper there is still a lack of full fundamental understanding and a lack of measured distinctive properties. In 1911, The American Paper Products Company of Saint Louis, MO [23], advertised a corrugated shipping container for eggs with "Unlimited Stacking Strength" at low weight. If a true claim, perhaps we have lost some knowledge of corrugated board compared to that of our predecessors. It is more likely that we have gained the knowledge that success is obtained when strength is balanced against weight and cost. I have tried to show in this article that predicting onset of panel bulging should be sufficient to predict box strength, but that additional parameters that affect panel bulging, and possibly more robust equations for the critical buckling load, are required to achieve an adequate prediction. **TJ**

ABOUT THE AUTHOR

Box performance has been of interest to me for 20 years, and the simplicity and usefulness of the McKee equation has always fascinated me. In 2013, I was asked to prepare a poster and I noticed that the presentation day was the 50th anniversary of the McKee, Gander, and Wachuta paper that presented the McKee formula. It just seemed like a good idea to look at it again.

There is much good literature in this area, and of course some of the ideas I present here can be found elsewhere. I think it is worth stating that one cannot much improve upon the McKee-Urbanik approach to BCT without adding some new measurement to characterize the board. In this paper, I have provided a more systematic evaluation of how one should go about developing an improved predictive equation and pointed out two measurements (K and κ) that should be made independently.

Because 2015 is TAPPI's Centennial Anniversary, I thought it would be interesting to look into what was happening in the corrugated box industry a century ago. The activity I found reported in literature surprised me. The most interesting part of this research for me was when I convinced myself that the McKee data could be split into three groups: under-performing, adequately performing, and over-performing. I went on to find that I could develop multiple fits to a robust data set (Batelka and Smith), each with the same over all accuracy. It emphasizes that to differentiate two seemingly equivalent boxes will require the measurement of new properties.

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Knowledge is power, and it is important to know that the McKee equation only gives us an estimate of BCT. Once we have optimized ECT and bending stiffness, further improvement in performance will come from optimizing some property we currently do not even measure.

I think now is the time for the industry to undertake a major study on corrugated board performance, but with more intent focus on assessing the role of twisting shear and edge-restraint on panel bulging.

Coffin is professor, Department of Chemical and Paper Engineering, Miami University, Oxford, OH, USA. Email Coffin at coffindw@miamioh.edu.

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