Psychophysical tests for visual-numerical correlation of whiteness formulas

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ABSTRACT: Measures of whiteness are important to the paper industry. Such measures must have good correlations with visual observations. In this study, psychophysical tests were conducted to determine if a visual correlation could be found between the International Commission on Illumination (CIE) whiteness formula and a formula previously derived by our group. The lightness value of the substrate has a significant effect on the perception of whiteness, so observer trials were carried out at approximately the same lightness levels. Perceptions of whiteness and their changes with different light sources were studied. Psychophysical tests were conducted using D_{50} and D_{65} light sources for visual correlations, under controlled viewing conditions at a 2° field angle. The psychophysical results were then compared with the two formulas. Considering available light sources for observer experiments and the standards used by the graphic arts industry, along with the effect of optical brightening agents on whiteness perception, recommendations for use of a different light source by the paper industry are provided.

Application: The Fleming-Aksoy whiteness formula overcomes limitations of the current CIE whiteness formula by showing better visual-numerical correlation and helps in determining optimum whiteness on bluer tone.

Paper whiteness is a key concern in the paper and printing industries. The perception of color depends on the type of light source used for viewing. In the printing industry, D_{50} is used as the standard illuminant. D_{50} approximately represents horizon daylight [1-3]. In the paper industry, illuminant C [4] is used for the calculation of color values. Practically, illuminant C is not a readily available light source for viewing conditions. As a result, the International Commission on Illumination (CIE) discontinued illuminant C and replaced it with D_{65} [1,2,5]. The difference between these two illuminants is that D_{65} is richer in the ultraviolet (UV) region than illuminant C [6]. Ideally, an illuminant used for calculation and its corresponding light source should be used for the visual inspection of paper to obtain a better correlation between measurements and perception of color. In our experiments, we used a D_{50} (CRI 95) [7] because most paper made for the printing industry and printing is viewed under a D_{50} illuminant. We also used D_{65} (CRI 90) because it is normally used as a daylight illuminant [1].

The lightness value of a coated paper affects the perception of paper whiteness. In the CIELAB color space, theoretically, a paper with color values of $a^* = 0$ and $b^* = 0$ with a lightness value (L^*) of 100 are accounted as standard white, although paper white always has some tint. If the amount of tint increases, the "white" is no longer white. So it is important to define a boundary where white is no longer white, but is a light shade. Numbers should match what the eyes perceive.

ILLUMINANTS

Over time, many attempts have been made to derive formulas to calculate whiteness as a one dimensional scale. The CIE whiteness formula [8] was accepted by the paper industry for calculating the whiteness of paper. The CIE whiteness formula works on the assumption that "bluer is whiter." This assumption is correct, but it has limits. The CIE formula is limited by inequalities. The Fleming-Aksoy formula [9] uses the assumption that after a certain limit, increasing the blue cast results in "white remaining no more white," but instead becoming a light shade of blue. This assumption is very important for deciding the maximum amount of optical brightening agents or blue colorant added to papers.

Definitions

Definitions for various illuminants include the following:

- **Illuminants** — An illuminant is a set of numbers defined by a relative spectral power distribution that might or might not be physically realizable as a source [10,11]. All illuminant data are published by CIE. So, some of CIE standard illuminants are mentioned in the following.
- **CIE standard illuminant C** [4,12] — This is derived from illuminant A. It was an early daylight, or overcast sky equivalent. Illuminant C has a correlated color temperature of 6774 K. According to CIE [1,2,5,8] recommendations, illuminant C should be replaced with illuminant D_{65}, but the paper industry still uses C because TAPPI standards have not been updated.

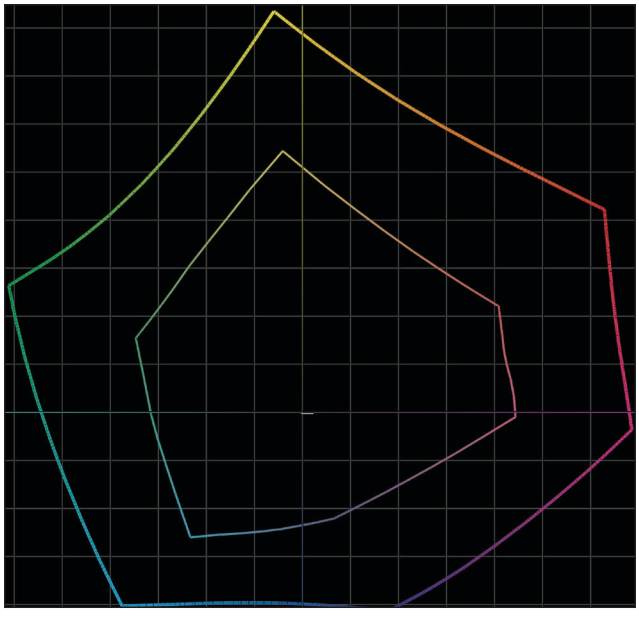
- **CIE D series illuminants** [1,13] — The method of calculation for this series is based on a correlated color temperature. D_{65} is recommended [5] by CIE as a standard illuminant. It represents daylight at noon, with a color temperature of 6504 K. D_{50} has been adopted by the graphic arts industry in the United States as a standard illuminant that represents horizon sunlight with a color temperature of 5000 K.

Illuminant and light source are distinct concepts. An illuminant is defined by mathematical data, which are used in colorimetric calculations. A light source is a physical source that emits electromagnetic radiation in the visible region. Not all illuminants have physical light sources. Light sources are made by targeting illuminant data. The difference between target (illuminant) and light source is characterized by the color-rendering index [7].

PAPER WHITENESS AND COLOR REPRODUCTION

Process color inks are transparent, and light is reflected from paper. The color gamut [14] is based on the ink and substrate combination. For the same ink, whiter and smoother paper gives a wider gamut. In **Fig. 1**, the same process color pigments are used for printing on coated and news stock. Coated stock gives a wider gamut than news stock. Because the whiteness of paper plays an important role in color reproduction, its measurement and accruement assessment is necessary.

Indeed, "paper white" is a well-known contributor to the color gamut [14] of color printing on a substrate by a given process. However, other properties, such as permeability [15] of the paper, are stronger contributors. The color gamut volume in CIELAB space for a printing device can be interpreted as the number of colors that can be printed within a ΔE tolerance of √3 [14]. In fact, the loss of optical brightening effects from accelerated lightfastness effects leads to a reduction of color gamut volume by 6%-8% [16]. A brighter paper white also improves contrast and increases color gamut [16].



1. Color gamut of newspaper (inner boundary) and coated stock (outer boundary).

CIE whiteness formula

The CIE whiteness formula is based on the idea that bluer is whiter. CIE recommended an equation for whiteness in 1981 [8,17]. This formula was inspired by Ganz [18-20]. This equation uses the CIE chromaticity coordinates of the sample and illuminant along with the Y tristimulus value (Eq. [1]) [21].

$$W_{CIE} = Y + 800(x_n\text{-}x) + 1700(y_n\text{-}y) \quad (1)$$

where x and y are the CIE chromaticity [21] co-ordinates and x_n and y_n are the corresponding co-ordinates for the perfectly reflecting diffuser in the reference illumination.

Whiteness is based on a one-dimension scale, on the blue-yellow axis with a dominant wavelength of 466 nm. This equation is complemented by the tint value equations, which are on the red-green axis of the opposing color theory [8]:

$$T = 1000(x_n\text{-}x) - 650(y_n\text{-}y), \text{ for a } 2° \text{ observer} \quad (2a)$$

or

$$T = 900(x_n\text{-}x) - 650(y_n\text{-}y), \text{ for a } 10° \text{ observer} \quad (2b)$$

A positive value of T indicates greenishness and a negative value indicates reddishness.

These equations are to be used only in a limited region. The criteria for whiteness, W, must fall within the limits given by [8]:

$$5Y\text{-}280 > W_{CIE} > 40 \quad (3a)$$

and the tint value T shall fall within the limits given by:

$$3 > T > -3 \quad (3b)$$

These inequalities define a region of applicability of W_{CIE} in chromaticity space (Yxy). According to this definition, the perfectly reflecting diffuser has a whiteness of 100 and a tint value of zero.

The CIE whiteness formula is used for the relative evaluation of whiteness by the same instrument and without reference to a white scale [8]. This system of equations does not clarify whether the whiteness has any component of blueness or yellowness. Evaluations with the formula are significantly improved if the sample illumination is stabilized and fitted as closely as possible to a desired illuminant. This also improves the matching of different measurement instruments for whiteness, but the tint deviation or hue value still cannot be adequately matched.

A restriction on the CIE formula is that samples must not dif-

fer greatly in tint and fluorescence [8]. The formula produces relative, not absolute, whiteness assessments. This is seemingly adequate for commercial use in many cases. However, it considers bluer to be whiter, which is not acceptable to the eyes if the blueness becomes so high that the sample is no longer whiter, but instead a light shade of blue. Again, the measuring instruments must have illumination resembling daylight.

If the sample illumination is stabilized, assessments with the CIE formulas are significantly improved, and samples to be compared do not have to be measured at the same time. This also improves the matching of different measuring instruments for whiteness. However, the tint deviation or hue value still cannot be adequately matched.

In 2009, CIE formed TC1-77, "Improvement of the CIE-Whiteness and Tint Equations," a technical committee recommended by Joanne Zwinkles [22]. Among the issues raised by Zwinkles were:

1. Measurement geometry
2. Illumination
3. Limited range of validity
4. CIE whiteness limits

Zwinkles serves on this committee, as does one of the authors (Fleming), with Robert Hirschler as chair. Recent progress has been reported [23], including work by other members of the committee. Our report presents data and interpretations cited by TC1-77 in the 2012 report.

Fleming-Aksoy whiteness formula (W_{FA})

Recently, Aksoy proposed two new whiteness formulas [9], based on a combination of theoretical and empirical considerations. One formula, N_{FA}, corresponds to a maximum whiteness at the perfectly reflecting diffuser, i.e., zero chroma in the CIE L*a*b* space (or HunterLab space) [24]. The other formula, W_{FA}, is based on the assumption that observers prefer a more "blue" white, but only if it is not too blue. Thus, a local maximum in whiteness is expected. The coefficients in W_{FA} were determined from the CIE formula by requiring that the two formulas have the same value and derivatives with respect to a* and b* at zero chroma (a* = b* = 0) for a given Y (or L* value).

These formulae are given by Eqs. (4) and (5):

$$N_{FA} = L\left(1/2\right)^{(C/C_0)^2} \quad (4)$$

and

$$W_{FA} = Y\left(1/2\right)^{[a^*(a^*-2a_1^*)+b^*(b^*-2b_1^*)]/C_2^2} \quad (5)$$

where $C = (a^{*2} + b^{*2})^{1/2}$ is the chroma and C_0 and C_2 are characteristic chroma values determined from the boundary region of applicability defined by inequalities 3. The coordinates for the local maximum whiteness at constant lightness Y (or L*) are a_1^* and b_1^*. They are determined by equating the corresponding derivatives of W_{FA} and W_{CIE} with respect to a_1^* and b_1^* to one another (Eqs. [6] and [7]) [9]:

$$a_1^* = \alpha\left(Y_n/Y\right)^{4/3}\left[C_2^2/200\ln 2\right] \quad (6)$$

$$b_1^* = -\beta\left(Y_n/Y\right)^{4/3}\left[C_2^2/200\ln 2\right] \quad (7)$$

where $\alpha = 3x_n(900y_n - 800z_n)/500$ and $\beta = 3z_n(800x_n + 1700y_n)/200$, with Y_n, x_n, and y_n being the illuminant luminance and chromaticity values.

EXPERIMENTAL

In the experiment, the Fleming-Aksoy whiteness formula, Eq. (5) (W_{FA}), was tested against the CIE whiteness formula by conducting psychophysical tests. C_2 was taken as the largest Chroma value consistent with inequalities 3. Tests were conducted under the following conditions:

- **Field angle**: A 2° field angle was used because practically, when many patches are compared, it is confined to distances corresponding to a 2° observer window. So, the printing industry also uses the 2° observer in colorimetric calculations.
- **Sample size**: 25 × 25 mm was chosen to accommodate 2° observers, considering size of the standard viewing booth available for the experiment.
- **Sample to observer distance**: To accommodate a 2° field angle in the available standard viewing condition booth, we used a distance of 72 cm.
- **Light sources**: D_{50} and D_{65} were used for illuminating samples.
- **Background and surround**: A standard viewing booth has walls that are painted gray, but to get good contrast, we used a black background and surrounding.
- **Sample making**: Only one kind of coated paper was used to eliminate the effect of the other variables. Small patches were printed on paper by using an International Color Consortium (ICC) profile for an HP Color LaserJet CP3505 printer (Hewlett-Packard; Palo Alto, CA, USA) with Adobe Photoshop CS5 software (Adobe Systems Inc.; San Jose, CA, USA). The patches were designed to have L*a*b* values of approximately 92,0,b*, where b* is between 0 and -10. These printed patches were used for testing.

Psychophysical tests

We used the pair comparison method as the basis for this psychophysical test. Because of the large number of samples, an elimination method was used for selection of the whitest patch. Twenty-three observers with proper training participated in conducting the psychophysical sampling. The following procedure was used for conducting these tests:

1) Conduct Ishihara test to confirm observers are not colorblind.
2) Allow observer eyes to adapt to illuminating condition.
3) Observer has to select the whitest patch among all randomly placed samples for three times.

4) Patches are organized in sequential manner b* = 0 to b* = -10 for selection.
5) Considering a patch selected by an observer, patches around it are shown for pair comparison, until the observer selects one patch constantly for five times.
6) This procedure is repeated for the other light source.

Elimination method

1) Show observer b* = 0 and b* = -1 and ask are they different or not?
2) If observer can point to the difference, then select observer for the test.
3) Show all 11 patches randomly placed to observer under D_{50}.
4) Observer has to select the whitest patch.
5) Repeat steps 1 and 2 three times.
6) Place patch in sequential order from b* = 0 to b* = -10.
7) Observer has to select whitest patch.
8) Based on the above selected whitest patch, show two patches before and after selected patch for selecting whitest patch randomly and sequentially.
9) To check consistency show ±1 patches around it randomly and sequentially.
10) Repeat step 7 three times to check consistency.

This process was repeated after changing to the D_{65} bulb.

Measuring conditions

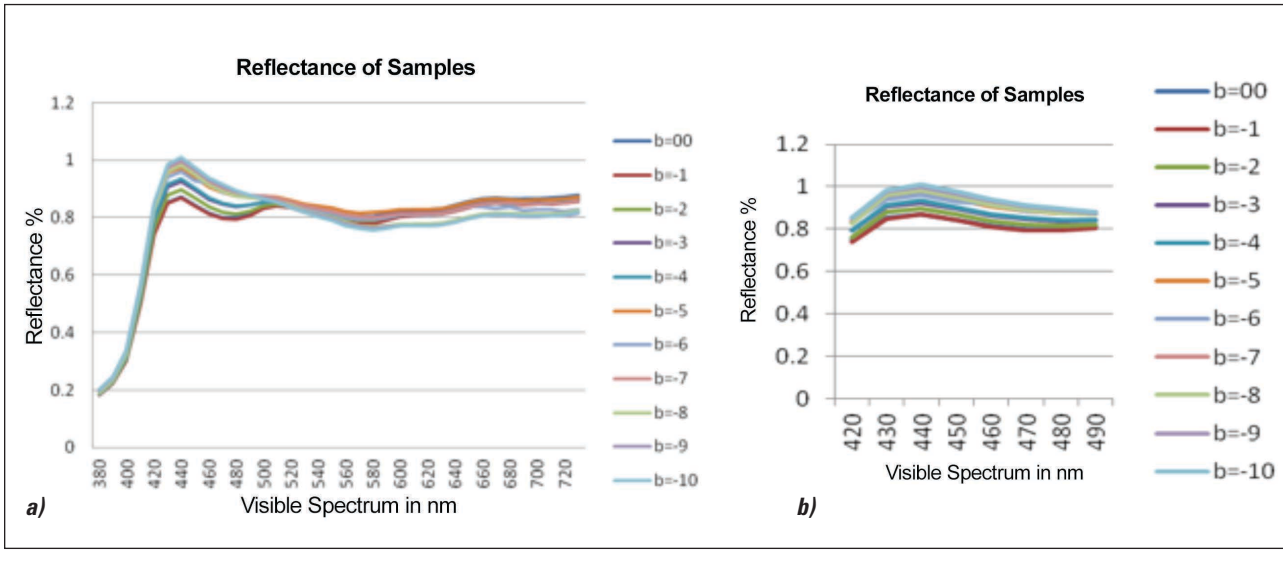
An i1io chart reading system (X-Rite; Grand Rapids, MI, USA), without a UV filter, was used for obtaining reflectance data. Averaging of three measurements was used. Repeatability of instrument is within ±0.1 ΔE_{76}. The same color patch can show different geometry of instrument, light source, range, number of detectors, types of light splitting system, aperture size, etc.

RESULTS AND DISCUSSION

The reflectance of the samples was measured to calculate the CIE colorimetric values. The reflectance graphs of all the samples are plotted in **Figs. 2a** and **2b**. Analysis of the reflectance data shows that the paper sample used in the experiment contains optical brightening agents. Almost all patches have a similar trend in reflectance. In the region of 400 nm to 470 nm, as the bluer content of the paper increases, the reflectance increases. The patches having b* = -5 and b* = -4 show a higher reflectance in the green and red regions, but not so high in the blue region. But the patch of b* = -7 shows a higher reflection in the blue region than the b* = -5 and b* = -4 patch, but slightly less in green and red regions. This could be a reason that the maximum number of observers selected b* = -7 as the whitest patch. A summary of the observations is given in **Table I**.

When the white patches are plotted in CIELAB color space, they lie near the b* axis. Every white patch has a slight color cast of red, green, blue, or yellow. As per CIE, bluer is whiter, so whiter patches continue to the blue region on the –b* axis. As per W_{FA} formula, the whitest patch is shown by the arrow in **Fig. 3**. After that, the calculated whiteness decreases. Also, whiteness largely depends on illuminant used. In Fig. 3, illuminant D_{50} was used to calculate CIELAB color coordinates for plotting.

As shown in Table I, color difference values (ΔE_{76}) for the tested patches from the target color values were all below 1.0, except for two patches under D_{50} illuminant, namely, b* = -5 and b* = -7. ΔE_{76} values were 1.47 and 1.08 for b* = -5 and b* = -7, respectively. These results show that, as –b* increases, the CIE whiteness increases, but in the Fleming-Aksoy formula, the whiteness diminishes after a certain level. In the CIE formula until the patch with b* = -3 for the 2° observer and b* = -5 for the 10° observer, there is little effect of the illuminant. After that, for the same patch, the whiteness



2. (a) Reflectance of samples used, and (b) reflectance data in the 420-500 nm region.

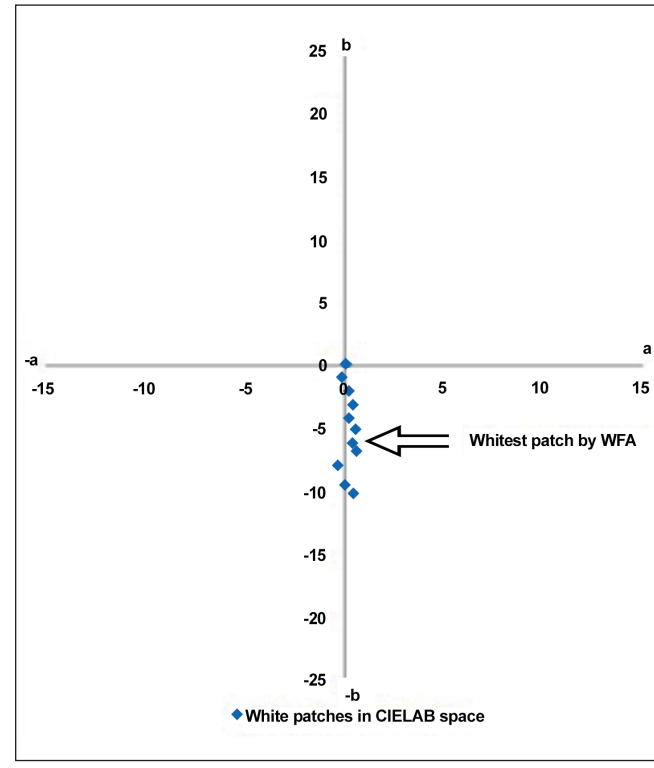
Ideal Patch	Measurements	Color Diff.	Measurements	Whiteness Values							
CIELAB	CIELAB	ΔE_{76}	CIELAB	CIE	W_{FA}	CIE	W_{FA}	CIE	W_{FA}	CIE	W_{FA}
	$D_{50/2°}$	D_{50}	$D_{65/2°}$	$D_{50/2°}$	$D_{50/2°}$	$D_{50/10°}$	$D_{50/10°}$	$D_{65/2°}$	$D_{65/2°}$	$D_{65/10°}$	$D_{65/10°}$
92,0,-0	92.71, 0.08, 0.14	0.72	92.71, -0.12, 0.23	81.72	81.72	80.52	80.47	81.21	81.18	79.69	79.69
92,0,-1	91.9, -0.16, -0.88	0.22	91.92, -0.27,-0.76	84.18	83.85	82.74	82.48	84.05	83.77	82.25	82.05
92,0,-2	92.27, 0.18, -2.05	0.33	92.3, 0.23,-1.94	90.00	88.51	88.56	87.54	90.40	89.01	88.58	87.77
92,0,-3	92.77, 0.56, -3.02	0.95	92.8, 0.56, -3.01	95.61	92.70	94.18	92.02	96.47	93.52	94.65	92.69
92,0,-4	92.47, 0.24, -4.03	0.52	92.52,0.54, -3.9	98.75	93.07	97.15	92.75	99.94	94.26	97.91	93.77
92,0,-5	93.37, 0.52, -5.00	1.47	93.43,0.98,-4.89	104.78	98.64	103.24	98.41	106.38	99.78	104.39	99.69
92,0,-6	92.62, 0.38, -6.12	0.73	92.7,0.89,-5.94	107.92	94.79	105.94	95.75	109.73	96.44	107.27	97.48
92,0,-7	92.9, 0.56, -6.79	1.08	92.98,1.22,-6.65	111.33	96.67	109.53	97.89	113.54	98.08	111.23	99.69
92,0,-8	91.82, -0.3, -7.93	0.36	91.94,0.49,-7.72	113.91	82.21	111.78	86.06	116.46	85.93	113.77	89.15
92,0,-9	91.66,-0.03,-9.42	0.54	91.79,0.95,-9.22	119.99	72.51	117.81	79.21	123.06	76.73	120.30	82.76
92,0, -10	91.52,0.36,-10.04	0.60	91.65,1.39,-9.84	122.41	66.81	120.12	74.68	125.65	70.29	122.76	78.13

I. Whiteness by CIE and Fleming-Aksoy (WFA) formula.

number is greater for D_{65} than D_{50}. In the Fleming-Aksoy formula, whiteness is always more for D_{65}. This shows that the calculated whiteness depends on the kind of illuminant/light source used. The CIE formula shows as the blue contents of a patch increases, the calculated whiteness increases. Field angle has a very nominal effect on whiteness, but both field angles follow the same direction in the CIE and Fleming-Aksoy formulas. So a change in field angle does not show much of a difference in the predicted whiteness of paper, with either formula.

As shown in Table I, the Fleming-Aksoy whiteness increases with decreasing (negative) b* values (blue) and reaches to a maximum at b* = -5 under D_{50} for observers at 2° and 10°. Then as the b* value decreases further, the Fleming-Aksoy whiteness decreases slightly for b* = -6 and then increases slightly for b* = -7. This decrease in whiteness continues at lower b* values but far more rapidly at b = -8 and lower. This apparent double maximum is because the L* value was lower than the target for b* = -6. Similar results are obtained with the Fleming-Aksoy formula under the D_{65} illuminant in regards to change in whiteness values with varying b* values in the blue region. **Table II** shows results of the psychophysical tests.

Under the $D_{50/2°}$ light source, the maximum number of observers selected the patch corresponding to b* = -7 as the whitest patch. Under $D_{65/2°}$, the greatest number of observers selected the patch corresponding to b* = -5. However, the expected value of b* is -6.1 for $D_{50/2°}$ and -5.2 for $D_{65/2°}$, based on all the observer results. When psychophysical results are compared with the CIE formula, then the patch corresponding to b* = -10 under $D_{50/2°}$ is the whitest patch under both light sources, but for observers it is not the whitest. Under light source $D_{50/2°}$, after the patch corresponding to b* = -8 under $D_{50/2°}$, observers feel that it is no more white, but a light blue shade. Under $D_{65/2°}$, the observer patch corresponding to b* = -7 under $D_{50/2°}$, observers feels that it is no longer white.



3. White patches in CIELAB color space

This indicates that the CIE whiteness formula fails to follow what observers see. Therefore, if a paper manufacturer has two papers with CIE b* = -7 and b* = -10 in $D_{50/2°}$, then the CIE whiteness formula says the patch with b* = -10 is whiter, when visually it is not. This result could mislead manufacturers seeking to make the whitest paper and lead them to add more expensive optical brightening agents, when it actually hurts the perceived whiteness.

However, the Fleming-Aksoy formula is consistent with the observer results for both light sources. For the D_{50} illuminant,

$D_{50/2°}$	Selection	$D_{65/2°}$	Selection
Sample	Frequency	Sample	Frequency
b = 0	1	b = -2	1
b = -4	1	b = -5	17
b = -5	2	b = -6	2
b = -6	4	b = -7	2
b = -7	15	b = -9	1
Mean b	-6.1	Mean b	-5.2
STD	1.6	STD	1.2

II. Results of psychophysical tests: selection of whitest patch.

the largest value of W_{FA} (98.6) occurs for nominal b* = -5 (-5.0 actual). A secondary maximum (96.7) occurs for nominal b* = -7 (-6.8 actual). The calculated value at nominal b* = -6 (-6.1 actual) is 94.8. Thus, these are consistent with the expected value of b* = -6.1. Furthermore, the calculated W_{FA} values indicate nearby local maximum values (100.3, 94.9, 96.8) for the corresponding Y values (83.8, 82.1, and 82.7) at b* = b_1* = (-7.2, -5.7, and -6.2) corresponding to measured b* = (-5.0, -6.1, and -6.8), respectively. Similarly, for the D_{65} illuminant, the largest value for W_{FA} (99.8), occurs for nominal b* = -5 (-4.9 actual). A secondary maximum (98.1) occurs at nominal b* = -7 (-6.6 actual). In between, at nominal b* = -6 (-5.9 actual), the calculated value is 96.4. Again, these are consistent with the expected value of b* = -5.2. Furthermore, the calculated W_{FA} values indicate nearby local maximum values (102.1, 96.8, and 98.8) for the corresponding Y values (84.0, 82.3, and 82.9) at b* = b_1* = (-7.1, -5.8, and -6.3) corresponding to measured b* = (-4.9, -5.9, and -6.6), respectively.

CONCLUSION

The CIE whiteness formula has poor visual-numerical correlation. This causes confusion between accepting visual or numerical results for selection of whitest papers among those available. The CIE whiteness formula has limitations for comparing whiteness with higher blue tone under any D_{50} and D_{65} light sources. The Fleming-Aksoy formula shows much better visual-numerical correlation than the current CIE whiteness formula. Visual and numerical results are very close. Observer/Field angle appears to have little impact on whiteness scale. The Fleming-Aksoy (W_{FA}) formula is suitable for the paper industry under light sources D_{50} and D_{65} for both field angles.

Recommendations

Based on the results of this study, we make the following recommendations:

1) Additional tuning of Fleming-Aksoy formula to fit other psychophysical tests.
2) Replacing of illuminant C by D_{50} or D_{65} for paper optical properties tests.
3) Replacing the CIE whiteness formula with the Fleming-Aksoy formula. **TJ**

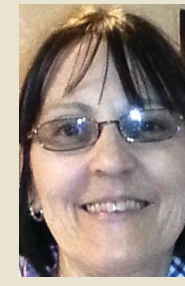
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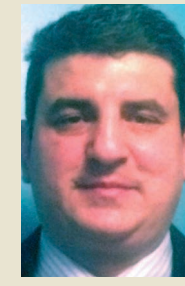
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ABOUT THE AUTHORS


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Fleming


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Whiteness and its perception have been topics of interest, and dispute, for many years. We had done some theoretical and experimental work in this area, but it had failed to gain traction because of "noisy" observer data. We decided to conduct more controlled observer trials. These results are presented in this paper.

The most difficult part of this study was verification of the consistency of observer responses. However, the observer trials were in even better agreement with our formula than expected. The formula was based on simple assumptions about rate of whiteness variation and range of "near white" region in the original CIE whiteness formula.

Use of our formula can enable mills to cut back on optical brightening agent additions to achieve near optimal white rather than the bluer than optimal favored by the current CIE whiteness formula.

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