

# Computational model for water drainage in fourdrinier paper machines

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**ABSTRACT** *A computational model based on Taylor's equation and the filtration equation was developed to evaluate the location of the wet line (5% consistency) on fourdrinier paper machines. Drainage at table rolls and low-vacuum elements was evaluated by the model. Comparison of the results obtained using the drainage model and by measuring the consistency profile in paper machines presented an agreement within 10%. The value of the specific filtration resistance used in the model was determined by means of experimental runs in a modified laboratory drainage and vacuum retention tester. This device measures dynamic specific filtration resistance, which is different from the constant pressure or constant flow approaches used in the past.*

## KEYWORDS

Analysis  
Consistency  
Drainage  
Paper machines  
Vacuum boxes  
Water removal  
Wet ends

The simulation of paper machine drainage is a useful tool that can be used not only during the design of new paper machines or the rebuilding of existing ones but also in the preparation and evaluation of stocks for new paper grades in the mill. For the design engineer who establishes paper machine parameters like positioning of drainage devices and vacuum pressure to be applied in low-vacuum boxes, a simulation of the consistency profile over the wire can save time and raw material during the adjustment period.

Unfortunately, from the viewpoint of fluid dynamics, the drainage of water during the formation of the fiber web is a complex phenomenon, in which the flow is subject to such forces as pressure pulses, vibrations, induced turbulence, wire deformation, and instabilities. The combination of these occurrences with the rheological behavior of the water and fiber mixture and the influence exerted by chemical additives on drainage characteristics of the stock makes the mathematical treatment of the drain-

age phenomenon practically impossible without major simplifications.

Applying different hypotheses and simplifications, several researchers have studied the problem of drainage from both theoretical and experimental approaches. Mathematical models for table roll drainage can be found in the works of Wrist (1), Taylor (2, 3), and Bergstrom (4). For a web of cellulose fiber, Ingmanson (5) developed filtration equations for the constant flow rate and constant pressure differential across the fiber web. However, there is a lack of reports about the use of these models to predict drainage and consistency profiles along the forming wire.

To build a computational model that simulates the drainage at a fourdrinier paper machine, we decided to use the equation developed by Taylor (2) for drainage at a table roll and the filtration equation (5) for drainage at low-vacuum boxes. The computational model performs a sequential mass balance along the wire using the results obtained from the evaluation of

the drainage in one device to evaluate the drainage in the subsequent one.

There are equations for drainage at a table roll that were developed using a less simplified model—the equation proposed by Bergstrom (4), for instance. The use of one of them, however, would only increase the complexity of the computational model with little increase in the precision of the results. This we have concluded because one of the constraints of all models is the laboratory procedure to evaluate the drainage resistance of the fiber mat.

In conjunction with the construction of the computational program, a new method for determining the filtration resistance of the pulp was developed. For this purpose, a laboratory drainage and vacuum retention tester was modified to measure the resistance under dynamic conditions.

## Theoretical background

### Filtration equation

The filtration equation was used to

evaluate the drainage flow rate because of the gravitational and pressure forces in low-vacuum boxes. It was also used as the basis for determining the drainage resistance of the pulp. This equation was derived from Darcy's Law for flow in porous media and the Kozeny-Carman equation for flow through a packed bed. Ingmanson (5) showed that these equations can be used for a compressible mat of cellulose fibers and derived a form of Eq. 1 for filtration:

$$Q = \Delta P A / \mu R_t \quad (1)$$

where

$Q$  = flow rate,  $m^3/s$

$\Delta P$  = pressure across the fiber mat, Pa

$\mu$  = fluid viscosity,  $N \cdot s/m^2$

$A$  = cross section of the fiber mat,  $m^2$

$R_t$  = total drainage resistance,  $m^{-1}$ .

#### Taylor's equation for drainage at a table roll

For his analysis of the drainage at a table roll, Taylor assumed that the flow under the wire is nonviscous and that Bernoulli's equation therefore applies along a stream line in the nip. He assumed that the wire is tangent to the roll and does not deflect. Gravitational forces are negligible compared to inertia forces. The length of the drainage zone is small relative to the radius and is therefore large relative to the greatest depth of the zone (2). His solution for uniform resistance along the drainage length without mixing of flow inside the nip is given by Eq. 2:

$$Q/L = 0.295 \rho^2 R U^3 / (\mu^2 R_t^2) \quad (2)$$

where

$L$  = machine width, m

$R$  = table roll radius, m

$U$  = machine speed, m/s

$\rho$  = white water specific mass,  $kg/m^3$ .

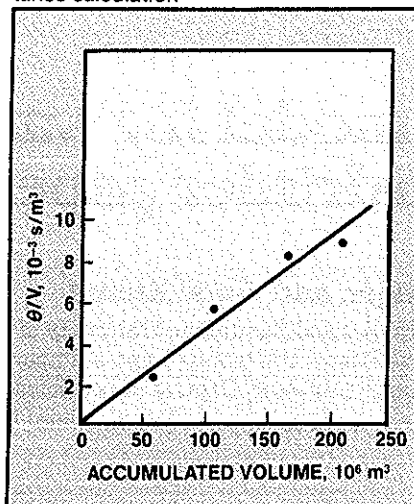
When total mixing occurs inside the nip, the factor 0.295 increases to 0.405 (6).

### Specific filtration resistance

#### Theoretical principles

The total drainage resistance, which is a function of the flow pattern and composition of the stock, is the sum of the effects of resistance imposed by the wire and by the fiber mat. In the initial phases of the mat formation

1. A typical plot for the specific filtration resistance calculation



the resistance imposed by the wire has the same order of magnitude as the resistance imposed by the mat, but as the thickness of the mat increases, the importance of the wire decreases.

One of the constraints on applying equations derived from theoretical analysis of the flow is the determination of the filtration resistance imposed on the water by a compressible mat of cellulose fibers. Using the measurement of drainage flow rate in the paper machine and the drainage equations, the filtration resistance can be calculated. However, this process is expensive and time consuming, it needs to be repeated when modifications are introduced, and it restricts the use of mathematical models for predicting drainage length. To avoid these restrictions, a laboratory procedure for evaluating the drainage resistance was necessary.

In this work, we used the concept of specific filtration resistance defined in terms of the basis weight of the fiber mat. We measured the drainage rate and pressure during the deposition of a mat in conditions equivalent to those encountered under the actual filtration process. Because the applied vacuum pressure remained quasi-constant during the filtration process, Eq. 3 can be used to calculate the specific filtration resistance (5, 7):

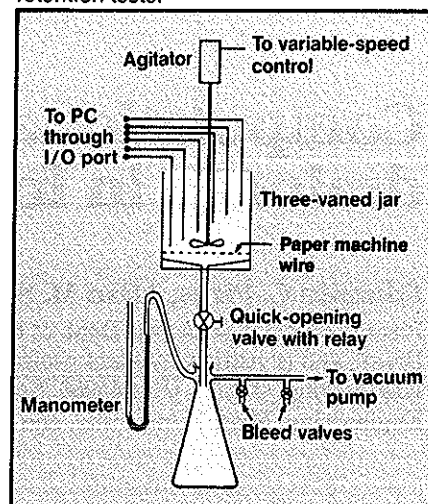
$$\Delta\theta / \Delta V = (\mu C SFR / A^2 \Delta P) V_1 + R_w \mu / A \Delta P \quad (3)$$

where

$SFR$  = specific filtration resistance,  $m/kg$

$\Delta\theta$  = time interval to filter the volume,  $V$ , s

2. Diagram of the drainage, vacuum, and retention tester



$\Delta V$  = filtrate volume,  $m^3$

$C$  = fiber concentration,  $kg/m^3$

$V_1$  = accumulated volume of filtrate,  $m^3$

$R_w$  = wire resistance,  $m^{-1}$ .

If this relationship applies to the experimental process used for the measurement, a graph of the ratio  $\Delta\theta / \Delta V$  as a function of the accumulated volume  $V_1$  should result in a straight line. The slope and the intercept of the line are used to evaluate the specific filtration resistance and the wire resistance, respectively. Figure 1 illustrates the process for one value of  $SFR$ .

Assuming that the total drainage resistance increases linearly with the increase of the fiber mat basis weight (8), then the total drainage resistance is given in Eq. 4:

$$R_t = SFR BW + R_w \quad (4)$$

where

$R_t$  = total drainage resistance,  $m^{-1}$

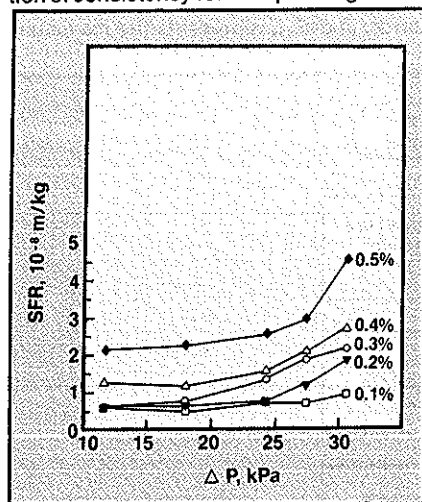
$BW$  = basis weight of the fiber mat,  $kg/m^2$

$SFR$  = specific filtration resistance,  $m/kg$ .

#### Experimental apparatus and procedures for the measurement of the specific filtration resistance

A drainage and vacuum retention tester (DVRT), based on a modified Britt jar, was fitted with six electrodes to monitor the drainage time of a standard volume of slurry (Fig. 2). The time was measured by a micro-computer connected to the set of elec-

3. Specific filtration resistance as function of consistency for 750 rpm of agitation



trodes through an input/output port. The volume between consecutive electrodes was kept constant and was established by the height of the electrodes. The vacuum pressure, induced by a portable vacuum pump, was set and controlled in the range of 2.4–33.0 kPa. To increase the turbulence, a three-vane jar was used for all measurements. To measure  $R_w$ , a sample of the wire used in the paper machine was also used in the DVRT.

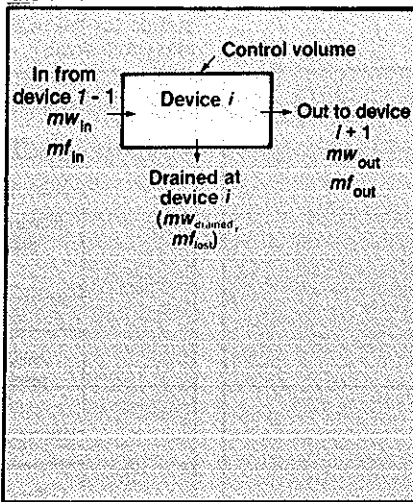
For a complete characterization of a pulp sample, five levels of consistency (0.1, 0.2, 0.3, 0.4, and 0.5%) and vacuum pressure (11.9, 18.6, 25.4, 28.8, and 32.2 kPa) were chosen. Three different shear rates (500, 750, and 1000 rpm) were also chosen for the tests. As an example, Fig. 3 illustrates the results of the variation of  $SFR$  with consistency for a pulp sample taken from a paper machine.

To simulate drainage in a paper machine, a complete characterization of the pulp is not always necessary. The measurement of  $SFR$ , using only the range of consistencies and pressures that are expected in the paper machine, is sufficient to provide the information needed to perform the simulation.

### Mass balance over the wire

As water is drained by any kind of drainage device, the average consistency of the slurry over the wire increases and a mat of fibers is deposited at the same time, developing a web with increasing basis weight. Measurements performed by Tellvik and Brauns (9) demonstrated that the activity that occurs over the wire is

4. Control volume for mass balance over the wire



not sufficient to dislodge the fiber already deposited, and for this reason two layers are formed. In their experiment, for a speed of 6.25 m/s, just above the wire was one layer with a high consistency of fixed fibers. Above it, the slurry continued to present a consistency that was only 0.01% higher than the headbox consistency, even after the drainage of 40% of the water.

For any section of the slurry over the wire, the mass balance can be represented as in Fig. 4. In particular, after the drainage at a drainage device  $i$ , a mass balance can be used to evaluate the average consistency and the basis weight of the fiber deposited on the wire.

### Consistency evaluation

By definition, the consistency inside the control volume is given by Eq. 5:

$$CW = mf_{vc} / (mf_{vc} + mw_{vc}) \quad (5)$$

where

$CW$  = consistency inside the control volume, dimensionless

$mf_{vc}$  = mass of fiber inside the control volume, kg

$mw_{vc}$  = mass of water inside the control volume, kg.

A simple mass balance on the control volume leads to the values for the mass of fiber and water inside it. However, since the paper machine works continuously, the balance must be treated in terms of mass flow rate. The mass balance is executed after each major drainage calculation—that is, after the calculation of drainage between the headbox slice and first drainage de-

vice—and after the final value for the drainage at each one of the devices is calculated. The initial values for this calculation are the slurry concentration at the headbox and the mass flow rate at the headbox slice. Based on the amount of water drained at each one of the drainage devices, one can apply a first-pass retention coefficient, taking into consideration the fact that part of the fiber is not retained over the wire.

### Basis weight evaluation

At each drainage device, the basis weight is increased by the amount of fibers that are drained in that particular device and are thus deposited on the web. According to the observations of Tellvik and Brauns (9), this amount is given by

$$mf_{dep} = C_{hb} mw_{drain} f_{ret} \quad (6)$$

where

$mf_{dep}$  = mass rate of fiber deposited on the web, kg/s

$C_{hb}$  = concentration at the headbox, kg/m<sup>3</sup>

$mw_{drain}$  = flow rate of water drained at device  $i$ , m<sup>3</sup>

$f_{ret}$  = retention coefficient.

Thus the increment in basis weight is

$$BW_{inc} = mf_{dep} / L U \quad (7)$$

where

$BW_{inc}$  = basis weight increment, kg/m<sup>2</sup>

$L$  = machine width, m

$U$  = machine speed, m/s.

The retention used in this expression is the first-pass retention familiar to most papermakers.

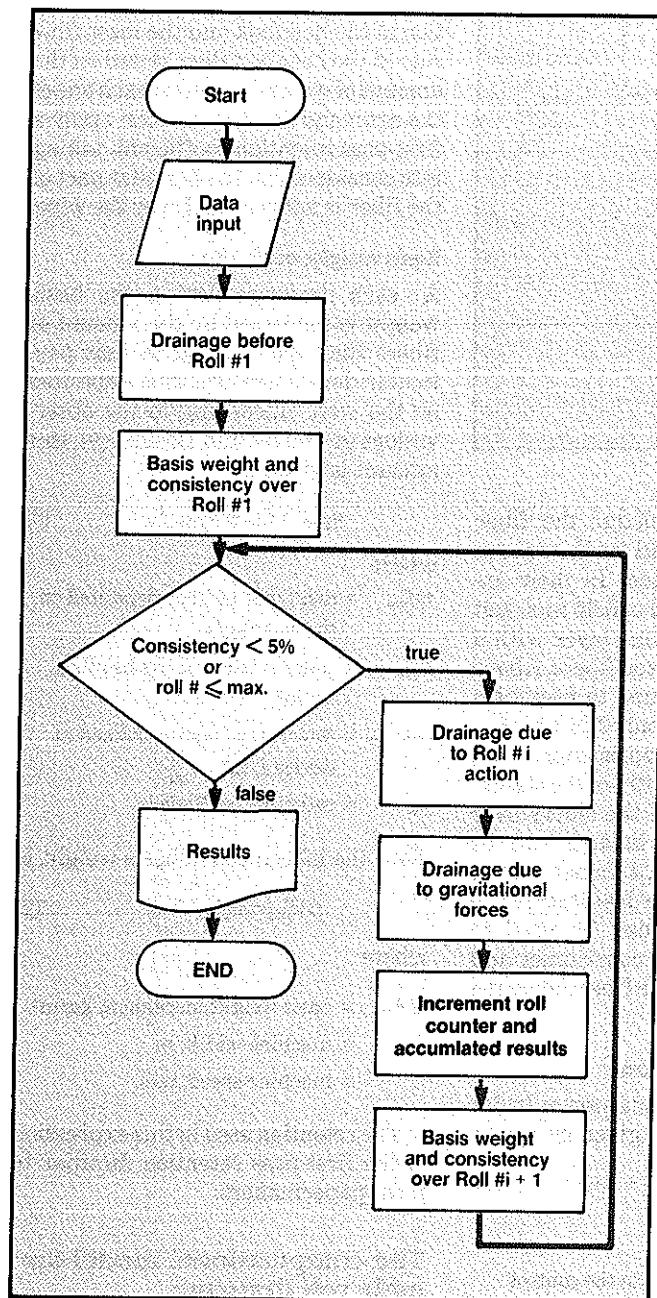
## The computational model for table roll drainage

### General development of the model

The computational model we developed evaluates the consistency and drainage rate along the wire, starting from the headbox slice up to a point where the consistency reaches a value equal to or greater than 5%, which is considered the wet line (10). For values of consistency greater than 5%, the validity of the mathematical models for drainage is not certain.

The general flow of the program is depicted in Fig. 5. Before the calculation of drainage at the table rolls, the drainage that occurs between the slice and the first roll is evaluated. In this

5. General flow of the computational model

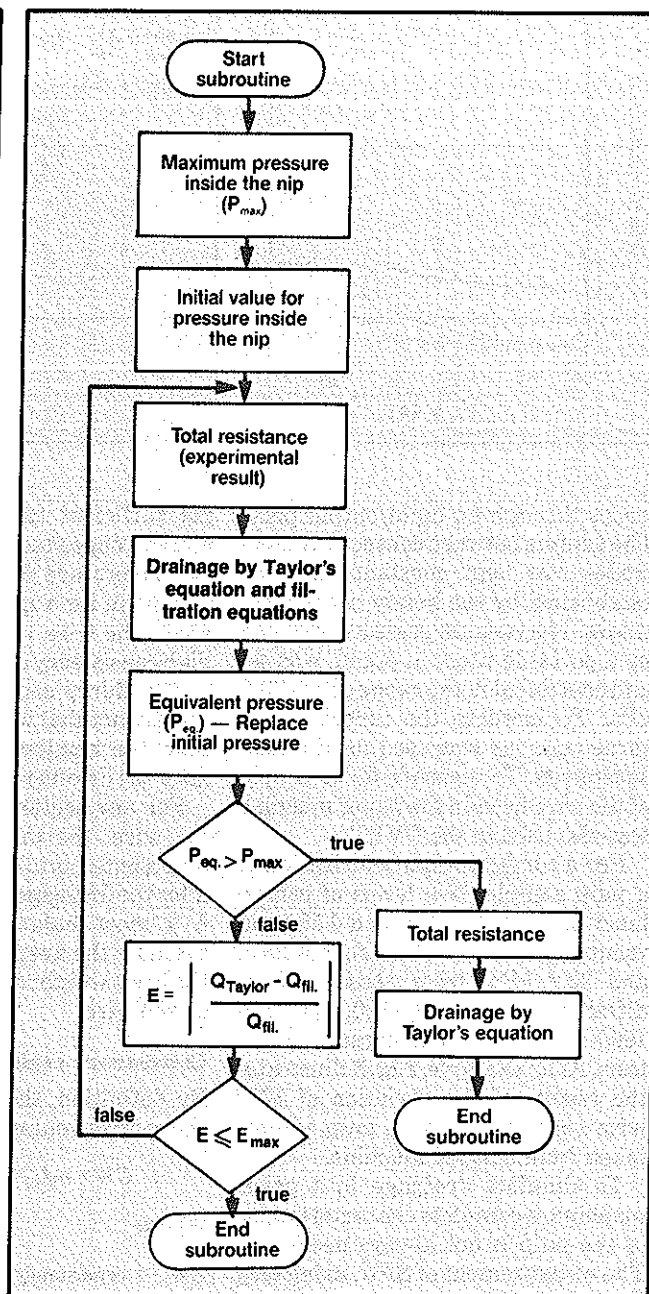


zone, one can consider that only the wire presents a resistance against the flow, which is caused by gravitational forces, and therefore the filtration equation applies. After the drainage is determined, a mass balance is used to evaluate the basis weight of the fiber already deposited on the wire and the average consistency of the slurry.

Inside the roll section, using a loop structure, the program performs the following:

- Calculates the drainage due to the action of Roll  $i$
- Calculates drainage due to filtration between Roll  $i-1$  and Roll  $i$
- Accumulates these values
- Updates the roll counter
- Calculates the basis weight
- Calculates the average consistency with which the slurry will reach the next roll.

This process is repeated while the consistency is smaller than 5% or while there are rolls needed to account for drainage.

6. Calculation of the induced drainage ( $Q$  indicates flow rates, as calculated from Taylor's equation and the filtration equation.  $E$  stands for the relative error, and  $E_{max}$  represents the maximum allowable error.)

#### Computation of drainage due to table roll action

The drainage resulting from table roll action is a function of the pressure that acts on the fiber mat. The interaction between the roll movement and the mat resistance creates a zone of low pressure under the wire, causing drainage. This zone is called the "roll nip."

Two methods to evaluate the pressure inside the nip can be used. The first is based on theoretical (2) and experimental evidence (11) to the effect that the maximum vacuum is

## I. General information for the table roll and the low-vacuum box simulations

|                                                      | Table<br>roll | Low-<br>vacuum<br>box |
|------------------------------------------------------|---------------|-----------------------|
| Wire speed, m/s                                      | 2.99          | 2.15                  |
| Dry basis weight, g/m <sup>2</sup>                   | 65.2          | 174.0                 |
| Flow rate at slice, (m <sup>3</sup> /s)/m            | 0.0503        | 0.056                 |
| Headbox consistency, %                               | 0.36          | 0.75                  |
| Stock temperature, °C                                | 31.6          | 36.0                  |
| Wire resistance, $\times 10^6$ L/m                   | 4.300         | 4.300                 |
| Water viscosity, $\times 10^{-4}$ N·s/m <sup>2</sup> | 7.740         | 7.084                 |
| Fiber retention, %                                   | 80            | 80                    |
| Avg. head pressure over wire, Pa                     | 24.5          | 24.5                  |

## II. The equation and coefficients for specific filtration resistance

|                                                                         | Turbulence               |                          |                          |
|-------------------------------------------------------------------------|--------------------------|--------------------------|--------------------------|
|                                                                         | High                     | Medium                   | Low                      |
| Equation                                                                |                          |                          |                          |
| SFR = A*DP + B*DP <sup>2</sup> + C*CW + D*CW <sup>2</sup> + E*DP*CW + F |                          |                          |                          |
| where DP = pressure drop, kPa<br>and CW = consistency, %                |                          |                          |                          |
| Coefficients                                                            |                          |                          |                          |
| A                                                                       | -0.17117                 | -0.25193                 | 0.20671                  |
| B                                                                       | $47.3396 \times 10^{-4}$ | $60.0595 \times 10^{-4}$ | $49.9947 \times 10^{-4}$ |
| C                                                                       | -354.15                  | -618.54                  | -158.27                  |
| D                                                                       | $10.7243 \times 10^4$    | $11.9814 \times 10^4$    | $54.1429 \times 10^4$    |
| E                                                                       | 10.5408                  | 17.8761                  | 15.458                   |
| F                                                                       | 1.78143                  | 3.13043                  | 2.28746                  |

## III. Summary of results for the table roll and the low-vacuum box simulations

|                                               | Table<br>roll | Low-<br>vacuum<br>box |
|-----------------------------------------------|---------------|-----------------------|
| Total length, m                               | 6.3           | 0                     |
| Number of rolls to drain                      | 22            | ...                   |
| Number of boxes to drain                      | ...           | 6                     |
| Final consistency, %                          | 1.77          | 5.93                  |
| Final basis weight, g/m <sup>2</sup>          | 41.5          | 143.8                 |
| Total induced drainage, (m <sup>3</sup> /s)/m | 0.03584       | 0.05154               |
| Total gravity drainage, (m <sup>3</sup> /s)/m | 0.07255       | ...                   |
| Total drainage, (m <sup>3</sup> /s)/m         | 0.04309       | ...                   |

equal to  $\frac{1}{2} \rho U^2$ . The second is the comparison with an equivalent vacuum pressure derived from the filtration equation. The equivalent pressure method is based on the assumption that the flow rate evaluated using Taylor's equation should be equal to the flow rate evaluated using the filtration equation. This method can only be used if the condition of maximum vacuum ( $\frac{1}{2} \rho U^2$ ) is satisfied; otherwise the first method is used.

Figure 6 charts the calculation of the drainage flow rate induced by the action of the table rolls. Both maximum vacuum pressure and equivalent

flow criteria are used in this process. Initially the maximum vacuum pressure that can be achieved inside the nip is calculated, and an initial value is given to the equivalent pressure. Using the equivalent pressure, the drainage resistance is evaluated based on experimental results. Next, the drainage flow rate is calculated from Taylor's equation and from the filtration equation.

Using the filtration equation, a new equivalent pressure is calculated from the result predicted by Taylor's equation. This pressure is compared against the value of the maximum

pressure that can exist inside the nip. If the equivalent pressure is smaller than the maximum, then the relative difference of the two flow rates is compared against a standard, and the cycle is repeated until the required precision is reached. Based on the value of the equivalent pressure, the cycle can be terminated whether or not the required precision is reached. At any cycle, if the equivalent pressure is greater than the maximum, a new filtration resistance is calculated for the maximum pressure, and the flow rate is evaluated from Taylor's equation.

### The computational model for drainage of low-vacuum boxes

This model has the same structure used in the table roll drainage model. Since each box can be treated as a roll for the mass balance evaluation, only the procedure to calculate the flow rate induced by the drainage device was changed. In this case, all pressures are previously determined, and the calculation of the specific filtration resistance is immediate. Also, the flow rate can be evaluated by the filtration equation alone, since the angle of the blades of the boxes was zero for this particular project.

A more rigorous approach to the problem would be to consider each slot of the vacuum boxes separately, but the uncertainties that accompany the calculations and the lack of experimental data for this case would prevent any improvement in the final result.

## Results

Table I presents general information for both the table roll and the low-vacuum box simulations, Table II gives the equation and coefficients, and Table III summarizes the results. Complete simulation results are listed in Tables IV and V.

### Results for the table roll model

For model validation, the simulation of a table roll paper machine was performed. The stock consisted mainly of hardwood pulp. A complete characterization of the pulp in terms of specific filtration resistance as a function of consistency and vacuum pressure was performed to establish a relationship among these variables. A regression analysis was used to obtain the correlation equation of SFR as a function of consistency and vacuum.



The high-turbulence values in Table II were used to obtain the values in the simulation for consistency less than 0.8%. A high turbulence level corresponds to 1000 rpm in the modified Britt jar. The medium turbulence level is 750 rpm, and it was used to compute the value of *SFR* in the consistency range 0.8% consistency <1.6. The low turbulence level is 500 rpm and was used for the *SFR* value when consistency was greater than 1.6%. The consistency profile along the wire was measured and compared against the predicted value (Fig. 7).

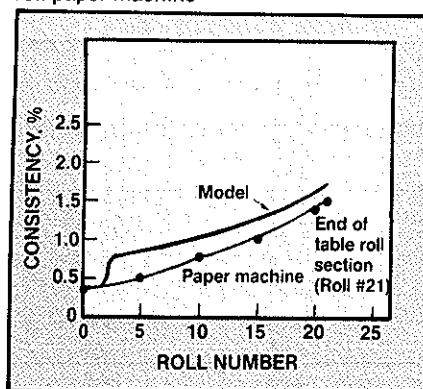
Initially the model predicts a steep increase in the consistency, but subsequently the predicted and the measured curve converge. The steep initial increase may be due to two factors. The first is the assumption, used for the derivation of Taylor's equation, that the drainage resistance has a uniform distribution along the nip. The second factor is the use of only one value of drainage resistance in each nip for the calculation of the drainage flow rate.

The computational model uses only the value for drainage resistance with which the mat enters the nip zone. In the first and second roll, the mat is being formed rapidly, and this phenomenon causes a nonuniform distribution of the drainage resistance in the nip zone. This nonuniformity persists along the table but with decreasing intensity, and it tends rapidly to a quasi-uniform distribution as the increment in the basis weight decreases. These factors cause the prediction of a higher drainage flow rate than the one that occurs in the paper machine. The correction of this effect depends upon measurement of drainage flow rate and the determination of drainage resistance profiles specifically for these rolls. The use of a correction factor will lead to a better overall agreement between the predicted and measured consistency profiles.

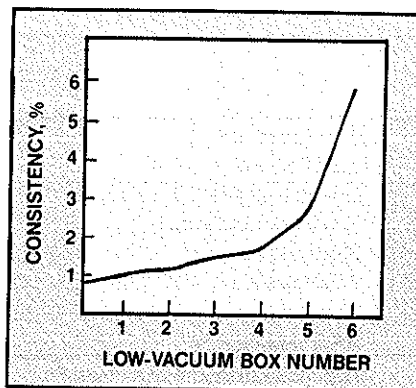
#### Sensitivity analysis—table roll model

A sensitivity analysis with  $\pm 10\%$  variation of input parameters to the drainage simulation model indicated that with the exception of the interaxis distance and the average hydraulic pressure exerted by the slurry over the wire, all other parameters cause measurable variations in the output. In Table VI, we have selected for purposes of comparison (a) the number of rolls to drain to 1.5% consistency,

7. Comparison between the predicted and measured consistency profiles for a table roll paper machine



8. Consistency profile for a low-vacuum paper machine



#### IV. Results for the table roll simulation

| Roll | Diameter, mm | Position, m | Consistency, % | Basis weight, g/m <sup>2</sup> | Mat resistance, $\times 10^7$ L/m | Induced drainage, $\times 10^{-4}$ (M <sup>3</sup> /s)/m | Gravity drainage, $\times 10^{-4}$ (m <sup>3</sup> /s)/m |
|------|--------------|-------------|----------------|--------------------------------|-----------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| 1    | 108          | 0.20        | 0.37           | 1.4                            | 0.089                             | 264                                                      | 14.1                                                     |
| 2    | 108          | 0.45        | 0.75           | 28.2                           | 2.196                             | 10.3                                                     | 2.91                                                     |
| 3    | 108          | 0.71        | 0.79           | 29.5                           | 2.303                             | 9.52                                                     | 2.68                                                     |
| 4    | 108          | 0.96        | 0.84           | 30.7                           | 3.148                             | 5.55                                                     | 2.13                                                     |
| 5    | 108          | 1.22        | 0.87           | 31.4                           | 3.229                             | 5.31                                                     | 2.00                                                     |
| 6    | 108          | 1.47        | 0.89           | 32.1                           | 3.306                             | 5.09                                                     | 1.96                                                     |
| 7    | 108          | 1.72        | 0.93           | 32.8                           | 3.381                             | 4.89                                                     | 3.66                                                     |
| 8    | 108          | 2.18        | 0.97           | 33.6                           | 3.472                             | 4.67                                                     | 2.12                                                     |
| 9    | 108          | 2.46        | 1.00           | 34.3                           | 3.543                             | 4.50                                                     | 3.59                                                     |
| 10   | 108          | 2.93        | 1.05           | 35.1                           | 3.629                             | 4.31                                                     | 2.27                                                     |
| 11   | 108          | 3.24        | 1.09           | 35.7                           | 3.699                             | 4.17                                                     | 2.84                                                     |
| 12   | 108          | 3.63        | 1.14           | 36.4                           | 3.773                             | 4.02                                                     | 2.72                                                     |
| 13   | 108          | 4.01        | 1.19           | 37.0                           | 3.845                             | 3.89                                                     | 1.56                                                     |
| 14   | 108          | 4.24        | 1.23           | 37.5                           | 3.903                             | 3.79                                                     | 1.69                                                     |
| 15   | 108          | 4.49        | 1.28           | 38.1                           | 3.961                             | 3.69                                                     | 1.74                                                     |
| 16   | 108          | 4.75        | 1.34           | 38.6                           | 4.018                             | 3.59                                                     | 1.64                                                     |
| 17   | 108          | 5.00        | 1.40           | 39.1                           | 4.074                             | 3.50                                                     | 1.62                                                     |
| 18   | 108          | 5.25        | 1.46           | 39.6                           | 4.128                             | 3.42                                                     | 1.67                                                     |
| 19   | 108          | 5.51        | 1.52           | 40.1                           | 4.182                             | 3.34                                                     | 1.59                                                     |
| 20   | 108          | 5.76        | 1.60           | 40.6                           | 4.234                             | 3.27                                                     | 1.64                                                     |
| 21   | 108          | 6.02        | 1.68           | 41.0                           | 4.266                             | 3.22                                                     | 1.76                                                     |
| 22   | 108          | 6.30        | 1.77           | 41.5                           | ...                               | ...                                                      | ...                                                      |

which was the final consistency in the table roll section of the actual paper machine, and (b) the consistency after the tenth roll, which was chosen arbitrarily. The range of variation in input parameters was chosen based on experimental errors that are expected on some of these values.

Based on the range of the response to the input parameters, the following observations were drawn. The greatest sensitivity of the model, with a range of 230%, is its sensitivity to machine speed, while the next greatest sensitivity is to the headbox flow rate with a variation range of 46%. Temperature and wire resistance cause the same range of variation, 30–32%.

The same occurs for consistency and specific filtration resistance, but in a lower range of 20–22%.

Care should be taken in the interpretation of results presented in Table III. There are interactions in the machine speed, stock flow rate, and consistency at the headbox. In actual operation, if the basis weight is to be kept constant these three variables are changed at the same time.

#### Results for the low-vacuum box model

For this model, only the final result, the position of the 5% consistency line, was compared to the measured value in a paper machine, which resulted in

## V. Results of the low-vacuum box simulation

| Box | Length, mm | Vacuum, Pa | Consistency, % | Basis weight, g/m <sup>2</sup> | Mat resistance, $\times 10^7$ L/m | Induced drainage, $\times 10^{-3}$ (m <sup>3</sup> /s)/m |
|-----|------------|------------|----------------|--------------------------------|-----------------------------------|----------------------------------------------------------|
| 1   | 100        | 2119       | .81            | 14.6                           | 1.636                             | 14.5                                                     |
| 2   | 100        | 3738       | 1.06           | 55.0                           | 9.532                             | 5.30                                                     |
| 3   | 160        | 2992       | 1.20           | 69.8                           | 10.13                             | 6.40                                                     |
| 4   | 160        | 3983       | 1.45           | 87.7                           | 16.00                             | 5.48                                                     |
| 5   | 270        | 3492       | 1.78           | 102.9                          | 16.88                             | 7.69                                                     |
| 6   | 270        | 5729       | 2.72           | 124.4                          | 30.92                             | 6.97                                                     |
| ... | 0          | ...        | 5.93           | 143.8                          | ...                               | ...                                                      |

## VI. Sensitivity analysis for the table roll model

| Parameter       | Variation in number of rolls (1.5% cons.) | Variation in Roll #10 |
|-----------------|-------------------------------------------|-----------------------|
| Diameter        | 3 - (-3)                                  | 9.1 - (-15)           |
| Speed           | 13 - (-12)                                | -26 - 200             |
| SFR             | -4 - 5                                    | 15 - (-7.3)           |
| Flow rate       | -7 - 8                                    | 33 - (-13)            |
| Interaxis       | 0 - 0                                     | 1.8 - 0.9             |
| Stock head      | 0 - 0                                     | 2.7 - 0.9             |
| Retention       | -3 - 3                                    | 6.3 - 1.8             |
| Temperature     | 5 - (-4)                                  | -11 - 21              |
| Wire resistance | -4 - 2                                    | 21 - (-9.1)           |
| Consistency     | -5 - 6                                    | 14 - (-6.4)           |

## VII. Sensitivity analyses for the low-vacuum box model

| Parameter             | Variation in consistency over Vacuum Box #5, % |
|-----------------------|------------------------------------------------|
| Open area             | -6.0 - 6.6                                     |
| Speed                 | 5.4 - 6.0                                      |
| SFR                   | 4.8 - 4.2                                      |
| Flow rate             | 14 - 9.0                                       |
| Position of first box | 1.2 - (-0.6)                                   |
| Stock head            | 1.2 - (-0.6)                                   |
| Retention             | 0.6 - 0.6                                      |
| Temperature           | 3.6 - 4.2                                      |
| Wire resistance       | 0.6 - (-0.6)                                   |
| Consistency           | 3.6 - 4.2                                      |

an agreement of within 10% (Fig. 8). The low-vacuum boxes on the machine had slots and not drilled covers. The complete set of data input and the results of the simulation are listed in Table V. In this case, the specific filtration resistance of the pulp was measured only for the headbox consistency and for the range of pressures encountered in the paper machine.

### Sensitivity analysis—low-vacuum boxes model

Since the number of low-vacuum boxes is small, a large step in consistency occurs from one box to the next. The sensitivity analysis was therefore restricted to the comparison of the variation in consistency after the fifth box. For only six low-vacuum boxes (as in this work), variations in the number of boxes to reach a fixed consistency are not as meaningful for comparison purposes as the variation of consistency in a fixed position. A variation of  $\pm 10\%$  was again applied to the input data, as shown in Table VII.

Based on the range of variation in

the output, the flow rate is the most important factor in determining the consistency profile, with a variation of 23%. In second place are machine speed and the open area at the vacuum boxes, with a range of 11–13%. Temperature, consistency, and specific filtration resistance cause a variation between 8% and 9%. The position of the first vacuum box, the wire resistance, and the retention factor are not as important in determining the consistency profile.

## Conclusions

A computational model was developed for predicting the average consistency profile of the stock along the wire in fourdrinier paper machines. The model is based on Taylor's equations and filtration equations for drainage in paper machines. To validate the model, the results were compared against measurements performed in two paper machines, one that has only table rolls and another that has only vacuum boxes. For the table roll model,

the position of the 1.5% consistency line, which was the end of the table roll section, was compared with the predicted result, and an agreement within  $\pm 20\%$  was reached. The comparison with the consistency profile showed greater differences, particularly for the initial part of the table. Further studies can provide correction factors, and better agreement is expected. For the low-vacuum box model, the position of the wet line, at 5% consistency, was compared with the predicted result. The prediction is within  $\pm 10\%$  of the measured value.

A laboratory procedure for determining the specific filtration resistance under dynamic conditions was developed. The apparatus is based on a modified version of the Britt jar, applying pressure and shear rates that are comparable to the values encountered in paper machines. Part of the success of the simulation model arises from the use of this new procedure. More tests will increase the confidence in the simulation process and in the method for measuring the specific filtration resistance.

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