A new generation of paper machine trim programs

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ABSTRACT *There are two basic approaches to solving trim problems in the paper industry: linear programming and sequential pattern generation. A new and more powerful approach combines the two methods, taking advantage of the most useful capabilities of each while eliminating the weaknesses of both. In most cases, the solution will contain patterns generated by both approaches. In the rare situation in which the problem is best solved exclusively by one method or the other, the procedure is capable of giving that type of solution as well.*

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All procedures based on linear programming can be traced back to the seminal work of Gilmore and Gomory (*1*, *2*) in the early 1960s. They described how the next pattern to enter the basis could be found by solving an associated "knapsack" problem. Problems of minimizing trim losses could be solved by linear programming techniques without first generating every feasible slitting pattern.

A large number of slitter patterns may exist where narrow widths are to be trimmed from a wide paper machine. Pierce (*3*) showed that in such situations the number of slitting patterns can easily run into the millions. However, only a small fraction of all possible slitting patterns need to be considered in finding the minimum trim loss solution. The delayed pattern generation technique developed by Gilmore and Gomory made it possible to solve trim loss minimization problems in much less time than would be required to generate the slitting patterns to prepare the input for a general-purpose linear programming algorithm such as MPSX from IBM.

The second approach to solving trim problems involves generating slitting patterns sequentially in much the same way as it is done manually. Each slitting pattern and its associated usage value, or the number of times the pattern is used, satisfies a portion of the total set of production requirements. As each pattern is selected, the remaining production requirements are reduced by the output of this pattern. The procedure terminates when all the production requirements have been satisfied. The first "industrial grade" sequential procedure capable of routinely outperforming human trimmers was developed at Mead Papers in 1967 (*4*).

With two fundamentally different approaches available to solve trim problems, focus naturally turned to identifying the advantages and disadvantages of each approach and the conditions under which each might be applied. In earlier articles, I described these issues in the context of single-machine and multiple-machine roll trim problems (*5*, *6*).

Linear programming vs. sequential pattern generation

There are two major disadvantages with using linear programming for solving roll trim problems. The first is that the values of the pattern usage variables generally will not be integers, which are required in trimming rolls of fixed diameters. Therefore, the linear programming solution will have to be "rounded" to integer values.

The second disadvantages of linear programming is that it is not possible to control explicitly anything other than trim loss. In most situations, trim loss is not the only consideration. In some situations, it may be important to limit slitter changes or to process some minimum number of production rolls before changing the slitters. In others, it may be important to keep the rolls for each order localized in the production run to avoid staging of materials and to facilitate shipment.

For solving problems by sequential pattern generation, the advantages and disadvantages are the reverse of those for linear programming. Using sequential procedures, it is possible to limit slitter changes by selecting high usage patterns or to keep orders localized by choosing patterns that complete orders that have been started. The only major drawback is the inability to guarantee low trim loss.

It is possible that the last widths to be trimmed will not trim well and the last patterns generated will have excessive amounts of trim loss. This would be the case if a 200-in. paper machine is being trimmed, and the only remaining requirement is 100 rolls that are 41 in. wide. The only situation in which this is not a real possibility is when the ordered widths are narrow compared to the paper machine width. If an average of at least nine ordered widths are slit from each production roll, finding low trim loss is usually quite easy using sequential procedures (*5*).

If trim loss is the only consideration, then linear programming will undoubtedly be the best way to solve the problem. If factors other than trim loss are important and low trim loss is easy to obtain because the ordered widths are narrow, then a sequential procedure will work best. Most actual trim problems do not fall in either of these two narrow categories. This reality has led to trim systems in which a problem might be solved by both methods. The scheduler is then asked to select the answer he likes best.

Although this dual solution approach will generally provide better results, it still does not take advantage of the unique benefits of the linear programming and sequential approaches.

Integrated solution procedure

The integrated solution procedure is easiest to understand by viewing it from the standpoint of using linear programming to gain better control over the trim loss in a procedure using sequential pattern generation. There are three major issues here. First, a decision must be made as to what will be considered acceptable trim. Second, a pattern that appears to be acceptable may lead to unacceptably high trim loss. Third, there are fewer ways to combine the remaining widths near the end of a sequential solution procedure.

A decision on acceptable trim

The first issue is that as patterns are generated to achieve some objectives such as minimizing slitter changes, a decision must be made on accepting that pattern based on trim loss. In some cases, two inches of side trim may be acceptable, but in others it may not be. This decision must depend on the minimum trim loss attainable, which can be determined beforehand only by first solving the problem by linear programming.

Deceptive pattern solutions

The second issue is that a pattern under consideration in the sequential procedure may have low trim loss and look quite acceptable in terms of solution objectives such as minimizing slitter changes. However, if that pattern is selected, it may lead to a solution with unacceptably high trim loss.

Consider the following example with two orders to be trimmed on a 100-in. paper machine. The first order is for 120 rolls that are 36 in. wide, and the second is for 100 rolls that are 10 in. wide.

Let (A_1, A_2, TL) represent a slitting pattern for orders one and two and its associated trim loss. The pattern (0, 10, 0) with a usage of 10 has no trim loss and completes the order for 10-in.-wide rolls. This means that the pattern (2, 0, 28) with a usage of 60 will be needed to complete the solution. This solution has the same number of patterns but much more trim loss than a solution of (2, 2, 8) with a usage of 50 and (2, 0, 28) with a usage of 10.

Because the 10-in. width is needed to provide trim for the 36-in. width, the pattern (0, 10, 0) should not be selected in this situation. It is possible to construct another sample problem where this pattern should be selected. The difficulty is to know when a given pattern should or should not be selected.

An imperfect but effective procedure is to use the dual prices from the optimal linear programming solution to calculate the reduced cost of the pattern. If the reduced cost is not zero or close to zero, entering this pattern into the solution will lead to an increase in trim loss. Linear programming theory dictates that the reduced cost of each pattern in the solution is zero. The reduced cost of a nonsolution pattern indicates how much the solution will worsen if this pattern is forced into the solution.

Limitations near the end of a sequential solution

The third issue is that near the end of a sequential solution procedure, there will be fewer ways to combine the remaining widths to achieve low trim loss. This limitation makes it more imperative that linear programming be used to fit the sizes together for minimum trim loss. Therefore, it may be useful to use linear programming to solve the residual problem once most of the benefits from using a sequential procedure have been achieved.

If the residual problem has no satisfactory linear programming solution, then the last pattern generated by the sequential procedure is discarded. The remaining requirements are enlarged to reflect this, and the new residual problem is solved by linear programming. This procedure is repeated until a satisfactory linear programming solution to the residual problem is found. The worst possible case is that all the patterns found by the sequential procedure are discarded, and the whole problem is solved by linear programming.

Summary of improving the solution using linear programming

The use of linear programming to improve any sequential procedure can be summarized as follows. First, use

I. Required sizes for the example

No. of rolls required	*Required size, in.*
12	35.000
2	34.000
22	33.000
11	32.500
5	32.000
5	31.250
9	30.750
4	29.625
10	29.000
24	28.500
4	28.000
12	26.250
6	26.000
18	25.000
20	23.000
1	22.250
3	21.750

Max. usable width = 214.00 in.
Max. slitter capacity = 10 rolls.
Max. overrun = 3 rolls/size.
Minimum of 3 rolls for a slitter change.

II. Solution sizes for the example: four patterns

No. of rolls for each usage	Size, in.
Pattern 1	
1	35.000
2	33.000
1	32.500
1	29.000
1	26.250
1	25.000
Usage value = 12. Loss/roll = 0.25 in.	
Pattern 2	
1	32.000
1	31.250
1	29.625
1	26.000
1	25.000
3	23.000
Usage value = 7. Loss/roll = 1.125 in.	
Pattern 3	
4	28.500
2	28.000
1	22.250
1	21.750
Usage value = 3. Loss/roll = 0 in.	
Pattern 4	
1	34.000
3	30.750
3	28.500
Usage value = 4. Loss/roll = 2.25 in.	

III. Overruns and trim loss

Width size, in.	No. of rolls overrun
34.000	2
33.000	2
32.500	1
32.000	2
31.250	2
30.750	3
29.625	3
29.000	2
28.000	2
26.000	1
25.000	1
23.000	1
22.250	2
Total trim loss = 19.875 in.	

the results from an initial linear programming solution to determine the minimum possible loss of trim. Use this minimum to set pattern acceptance criteria relating to trim loss.

Second, use the dual prices from the initial linear programming solution to identify patterns whose reduced costs are not near zero. The objective is to reject those patterns that may lead to large increases in trim loss.

Third, the pattern in the final solution may come from either the sequential procedure or from the solution to the residual linear programming. In most cases, there will be a mixture of both types of patterns. However, a real strength of the procedure is that the patterns can be exclusively from one source or the other. Therefore this procedure is capable of solving any kind of problem.

If there is only one way to trim the orders, then linear programming may be needed to get a good answer. In this case, the whole set of order requirements becomes the residual problem to be solved. In other situations, the sequential procedure may proceed to completion without running into trim difficulties, and the linear programming procedure will never be called on to solve a residual problem.

Applying the procedure: an example

Let us look at the use of the procedure in a hypothetical situation, in which minimizing slitter changes is extremely important.

Problem and solution

Ordered widths ranging from 20 to 40 in. must be trimmed from production rolls with a maximum usable width of 214 in. At least three production rolls must be processed before making a slitter change. Overruns of up to three rolls are permitted on each width. This amount is unusually high, but it is permitted to limit slitter changes and to fill out the last pattern, which also must have a usage of at least three. The requirements are listed in **Table I.**

There are two primary objectives that the integrated solution procedure must be able to deal with here. The first objective is to avoid combining all the easy-to-trim widths (with low dual prices) and leaving the hard-to-trim widths (with high dual price) to the end.

The second objective is to finish the requirements for as many widths as possible in each pattern. As shown in **Table II**, the requirements for 17 ordered widths are completed with only four slitter changes. On average, more than four orders are completed by each slitter change.

Linear programming theory tells us that a linear programming solution to this problem will have 17 slitter changes. Some of these will not be used because the usage values will be rounded down to zero. Nonetheless, the number will be closer to 17 than to 4. The only way to guarantee that a small number of slitter changes will finish the requirements is to specifically search for patterns that will complete as many orders as possible. This search requires that the quantity of each width still to be produced must also be considered when generating patterns.

For example, as the tables show, the initial requirement is 12 rolls for both the 35-in. and 26.25-in. widths. If the requirements for the 32.5-in. width is overrun by one roll, the quantity for that width is also 12. If the 33-in. width is included in a pattern twice while these sizes are included once, such a pattern used 12 times will satisfy all four requirements without violating the overrun limit for any width.

Careful inspection of Pattern 1 in Table II indicates that every width in the pattern except for the 25-in. width has its requirements met from that first pattern. The overruns for each width size are listed in **Table III.** The total trim loss is 19.875 in.

Procedure outline

The procedure is outlined as follows.

Step 1. Solve the problem initially as a linear programming problem. In the unlikely event that the linear programming solution meets the slitter change restrictions, this answer is used. If not, save the dual prices and the minimum percentage trim loss and go to Step 2.

Step 2. Sort the untrimmed order requirements by quantity from low to high. Look for the order quantity that has the largest number of occurrences. Observe that two orders require 12 rolls in Table I.

Step 3. Set up to search for a pattern that can be used 12 times that contains these two sizes, and that has a total trim loss consistent with the amount in the linear programming solution. If there are any widths requiring quantities of 13 or 14 rolls, ignore them in this search. If such a width is included in the next pattern, the residual quantity will cause a low usage pattern to be generated later.

Sort the remainder of the sizes into a list after the two order widths requiring 12 rolls as follows. First, use overruns to get to 12. Note that 11 rolls are needed for the 32.5-in. width. Making 12 of this size will not violate the overrun tolerance.

Second, use multiples of 12. A size requiring 24 rolls may be able to be included twice in the pattern.

Step 4. Search exhaustively for a pattern that will meet the goal for pattern trim loss based on the minimum trim loss possible from the linear programming solution at a usage of 12. If a pattern is found, use the linear programming dual prices to reject any pattern that has a reduced cost in excess of 0.03.

If a pattern is accepted, save this pattern, reduce the requirements, and return to 12. If no pattern is found, select the next most promising pattern quantity and go to 3. Feasibility is assured by eventually taking the pattern with the least trim loss that can be used at least three times.

Step 5. When the number of widths with unscheduled requirements is reduced to 4 or less, the residual problem is solved using linear programming. This step is taken to avoid large amounts of trim cost at the end of the sequential procedure. For the solution presented in the Table II, the first 3 patterns were generated by the sequential procedure, and the fourth pattern was generated by first finding a linear programming solution and then rounding the usage values to zero or integer values of 3 or more.

Conclusions

A new approach for solving paper machine trim problems integrates the most powerful features of both the linear programming and sequential pattern generation procedures. Because this type of procedure is capable of generating solutions that are either strictly linear programming or strictly sequential in nature, it can generate answers that are as good as either of those two approaches used individually.

By combining these two approaches, it is possible to develop more effective sequential procedures to deal with those concerns other than trim loss, while at the same time using linear programming to gain better control of the trim loss incurred, especially in the last few patterns generated.

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