Techniques for expanding the sphere of statistical process control

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When product changes affect process variability, the use of actual minus target values, percent of target value, and Z-scores can allow breakthroughs in quality improvements.

As more and more North American paper companies make firm commitments to the continual pursuit of quality improvement, formal, corporate-wide systems for quality control are being instituted. The most successful of these use statistical process control (SPC) to monitor and control processes and to help attain improvements in quality and productivity.

If a mill produces only a few grades, SPC may be relatively easy to use. However, when many grades are produced and these grades are changed often, the implementation of SPC can be difficult and confusing. There are workable alternatives to the standard SPC charts for mills in which the production mix makes implementation difficult.

Overview of SPC

In the forest products industries, the most commonly used control charts are the Shewhart control charts, which include the $\bar{X}$ (X-bar) and R (range) charts. The $\bar{X}$ control chart plots the average of subgroup readings over time, while the R chart plots their range, which is the maximum value minus the minimum value. These charts include center and control limit lines, which often enable us to detect problems or "out-of-control situations" long before they become apparent.

Variations on these charts are the individuals charts—the X chart and the moving range chart. The X chart plots the actual process readings over time, and the moving range chart plots the ranges of consecutive readings. These charts also contain center and control limit lines. The calculations necessary to construct these control charts are simple and well documented (*1–3*).

These control charts are interpreted by looking for patterns or occurrences that are unlikely to happen by chance when the process is running without problems. Commonly used patterns that indicate problems or changes in the process are:

- Any points outside the upper and lower control limits
- Seven or more consecutive points on the same side of the center line
- Six or more consecutive increases or decreases
- Two of three consecutive points outside the upper and lower warning limits.

Alternatively, a simple scoring method can be used to better detect problems or changes. This method keeps a running point total, and action is taken whenever a point total of 8 is reached. This approach was presented last year by Jaehn (*4*).

These standard methods work well for many mill purposes but fail where the process is frequently altered under planned changes. These changes often fall into a category we may term "product changes." Product changes include grade changes, clothing replacements, or switches in critical raw materials, such as clay, sizing, or titanium dioxide.

Shewhart control charts do not work well when product changes affect the average running level and variability of the process. The usual solution to this problem is to keep separate control charts for each product and add process readings to the appropriate chart. This solution leads to a host of control charts, with too much time spent to maintain and review them. Consequently, opportunities to identify and solve process-related problems are often missed.

Here, we will look at three alternatives. When properly used, these techniques can reduce the number of charts and the associated costs. More importantly, these techniques improve the chances for identifying and solving problems early.

Techniques for expanding the use of SPC

The three techniques for expanding the application of SPC are:

1. Actual minus target
2. Percent of target
3. Z-scores.

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I. Calculation of Z-scores

Grade	Day	Shift	First-pass retention[a], %	Grade average	Grade std. dev.	Z-scores	$\bar{X}$	Range
~35 lb/3300 ft²	2	1	67.0	69.54	4.34	-0.585	-1.084	1.498
		2	67.0			-0.585		
		3	60.5			-2.083		
~35 lb/3300 ft²	3	1	70.5	69.54	4.34	0.221	-0.317	1.152
		2	68.5			-0.240		
		3	65.5			-0.931		
		Grade change on the paper machine						
~45 lb/3300 ft²	4	1	76.0	75.53	2.94	0.160	0.897	1.871
		2	77.0			0.500		
		3	81.5			2.031		
~45 lb/3300 ft²	5	1	77.0	75.53	2.94	0.500	-0.520	2.891
		2	76.5			0.330		
		3	68.5			-2.391		

[a]Readings average of duplicate measurements.

Each approach transforms the data to remove the effect of known process changes. Consequently, the underlying assignable (or special) causes of process variability can better be detected and removed.

In the first two approaches, each reading is assumed to have a target value which represents the ideal value for that reading. This target value changes simultaneously with the product. Examples of readings for product changes that have known target values are tests by paper grade, machine parameters for various grades, and furnish makeup.

The first technique charts the calculated actual values minus the target values obtained by:

$$\text{difference} = X_i - T_i \tag{1}$$

where

X_i = the ith process reading

T_i = its target.

In this case, positive or negative values come from process readings which are higher or lower than the target.

The second technique uses the percent of the target value, calculated by:

$$\text{percent} = 100\,(X_i/T_i) \tag{2}$$

where

X_i = the ith process reading

T_i = its target.

A percent of target value greater than or less than 100% indicates that the corresponding process reading was above or below the target value.

Control chart calculations are unchanged from this point on. Rather than using process readings, these calculated and adjusted values are used. Also, chart interpretation follows the usual techniques for Shewhart charts.

Assuming that a target value exists for a process response, which of these two approaches should be used? The aim of these calculations is to transform the readings so that they better fulfill the statistical assumptions inherent in the charts. When the process is under statistical control, with all assignable causes of variability having been eliminated, Shewhart charts expect the process to yield data with a constant mean and standard deviation over time. The appropriateness of either technique depends on how well the transformed data meet these assumptions when the process is in control.

One important question is, "Does the variability of the process readings change across the range of target values?" Often, process readings are more variable at higher target values. To answer this question, standard deviations of process data can be calculated for each product. If variability does increase with the target value, the use of "actual minus target" values will not remedy this problem. However, the "percent of target" approach may well do so. To confirm that this approach is appropriate, repeat the variability analysis on the "percent of target" values.

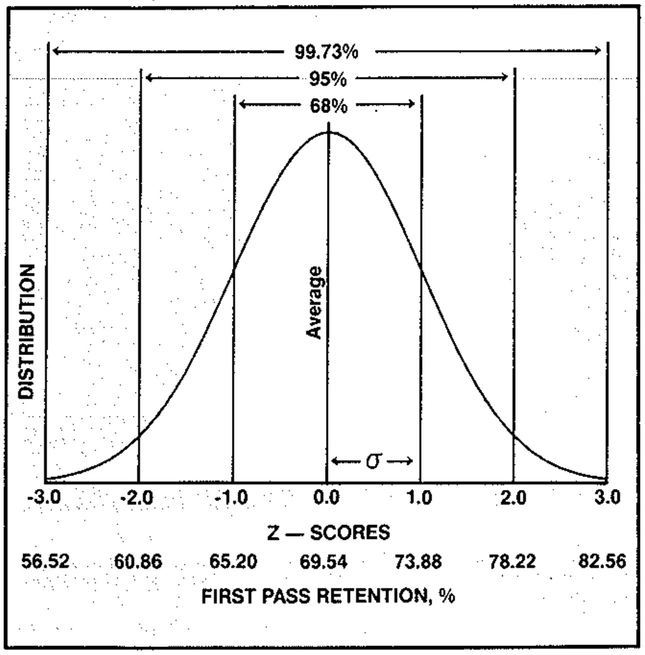
These approaches work well when a target value is available and when process variability is constant or closely related to the target value. Target values are often not available, however, and process variability may not behave in this manner. Therefore, a third approach is sometimes needed to properly monitor and control the process.

Z-scores

The technique

While the first two techniques use transformed variables in Shewhart control charts, the third technique uses transformed variables in a modified control chart. The transformed data are called "Z-scores," referring to readings from the normal or bell-shaped distribution,

1. A normal curve showing the relationship between first-pass-retention readings and Z-scores



II. Constants for Z-score control charts

Sample size n	$\bar{X}$ or X: Center line	$\bar{X}$ or X: UCL, LCL[a]	R or moving range: Center line	R or moving range: UCL	d_2
1[b]	0	±3.000	1.128	3.686	1.128
2	0	±2.121	1.128	3.686	1.128
3	0	±1.732	1.693	4.358	1.693
4	0	±1.500	2.059	4.698	2.059
5	0	±1.342	2.326	4.918	2.326
Other n	0	$\pm 3/n^{1/2}$	d_2	D_4d_2	d_2

[a]For the upper warning limit: UWL = 0.6667 UCL. For the lower warning limit: LWL = 0.6667 LCL.
[b]For individuals charts with moving ranges, n = 1.

which has a mean of zero and a standard deviation of one. Z-scores are calculated by:

$$\text{Z-score} = (X_i - \bar{\bar{X}}_p)/\sigma_p \tag{3}$$

where

X_i = the ith process reading

$\bar{\bar{X}}_p$ = its product average

σ_p = its product standard deviation, or "sigma."

More simply stated, a Z-score is the distance between a given process reading and its product average, measured in units of its product sigma. The product average and standard deviation values are statistics compiled from data collected for each product.

Like others, this transformation seeks to remove known and expected shifts in the data so that underlying patterns and trends can be revealed. Subtracting the average standardizes the level of the readings. Dividing by the standard deviation normalizes their variability, which is necessary because different products often experience different variabilities. These differences may result from the effect of the average level on the variability or from the ease or difficulty of maintaining process conditions.

An example

Let's say that a paper mill produces several grades of paper in the course of a month. One of its paper machines makes primarily two grades of fine paper: one at a basis weight of around 35 lb/3300 ft^2 (about 52 g/m^2) and the other at around 45 lb/3300 ft^2 (about 67 g/m^2).

Once each shift, operators measure first-pass retention by taking duplicate measurements and recording the average of the two. Analysis of first-pass-retention data on this paper machine showed that the lighter paper had an average value of 69.54 with a standard deviation 4.34, while the heavier paper had an average value of 75.53 with a standard deviation of 2.94.

Using this information, Z-scores were calculated for each reading, as listed in **Table I. Figure 1** shows the relationship between the readings for first pass retention and the Z-scores for the paper with a basis weight of about 35 lb/3300 ft^2. This diagram shows that a Z-score is just the number of sigma units by which a process reading falls above or below the average.

The Z-scores were then used to calculate the daily average values and the range values.

Control charts for Z-scores

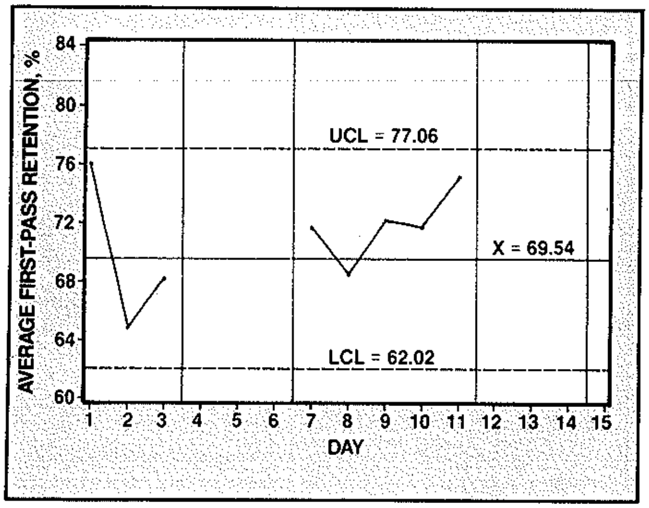
Maintaining charts

With the need to maintain product averages and sigmas and calculate the Z-scores, one begins to wonder if this approach is practical. Its advantage is that once the Z-scores are calculated, few calculations remain to be done because the control limits are always the same for any particular subgroup size.

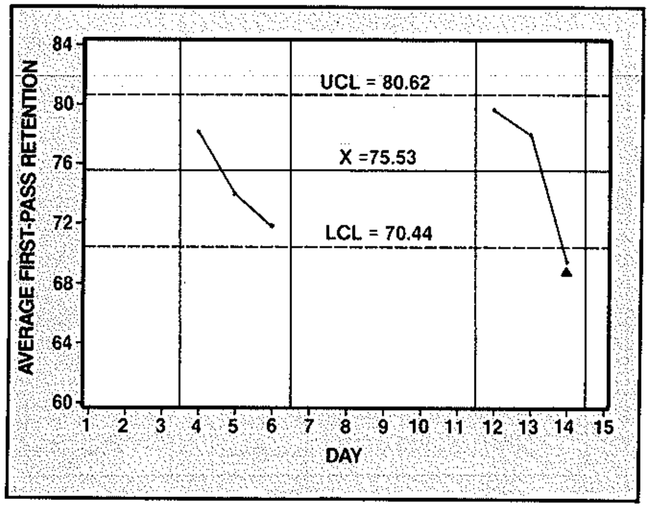
In **Table II**, taking n = 3 for a sample size n, the $\bar{X}$ chart's average line falls at zero and its upper control limit (UCL) and lower control limit (LCL) are at +1.732 and -1.732. The R chart's average and UCL lines fall at 1.693 and 4.358, respectively. Thus, for the price of more initial work, control chart calculations are eliminated. Only the Z-scores' $\bar{X}$ and R values remain to be calculated.

Another benefit from Z-scores is the ease of handling varying subgroup sizes. Suppose four readings were taken each shift, but occasionally one of the readings was unavailable or missed. Conventional methods make this situation somewhat awkward (*5*). In using Z-scores, however, a different subgroup size requires only the use of the tabled values for that sample size n.

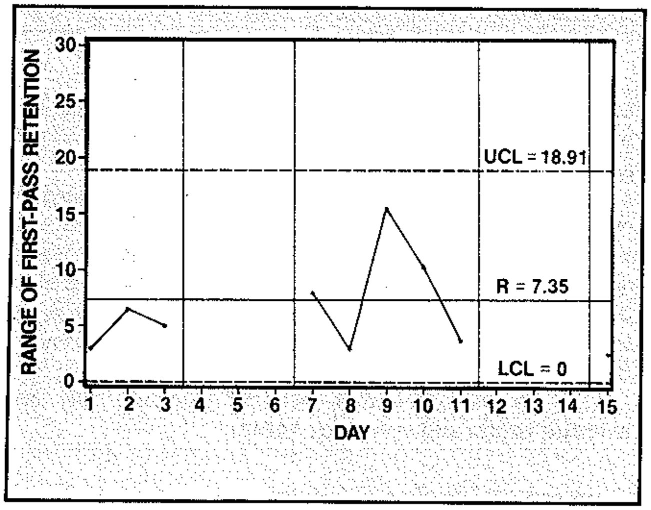
2. A typical X-bar chart (3σ limits for n = 3; basis weight of about 35 lb/3300 ft^2)



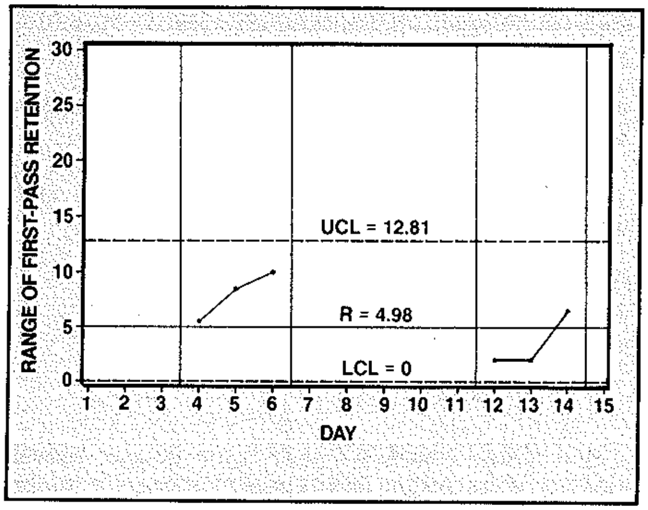
4. A typical X-bar chart (3σ limits for n = 3; basis weight of about 45 lb/3300 ft^2)



3. A typical range chart (3σ limits for n = 3; basis weight of about 35 lb/3300 ft^2)



5. A typical range chart (3σ limits for n = 3; basis weight of about 45 lb/3300 ft^2)



An example

Taking our same mill as an example, the plant personnel decided that they would make two sets of control charts with their first-pass-retention Z-scores. First, they would use X and moving range charts (n = 1) to spot big process problems in a timely fashion, during each shift. Then they would also produce $\bar{X}$ and R charts, grouping by the day (n = 3). According to Table II, the $\bar{X}$ chart's control and warning limits are ±1.732 and ±1.155, respectively. This chart was used to spot the more subtle problems or trends often missed by the individuals charts.

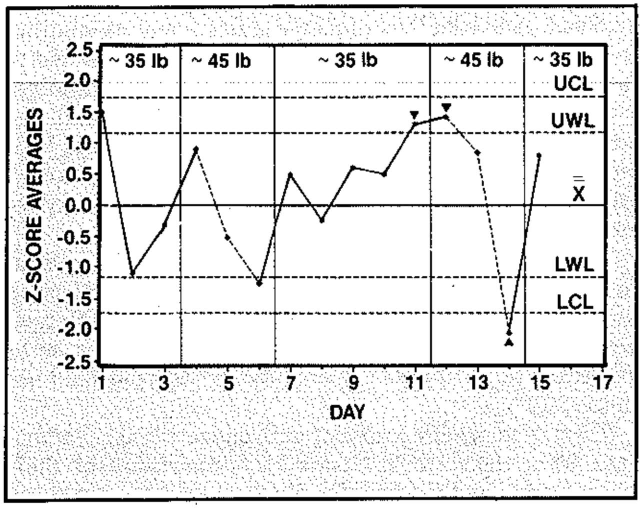
To compare Z-score $\bar{X}$ and R charts with the Shewhart charts, Shewhart charts were generated for the first 15 days of Z-score use. Control limits were calculated using historical data from the previous month. Separate charts were produced for the different grades of paper, as shown in **Figs. 2–5**. Figure 4 shows that on Day 14, the $\bar{X}$ fell below the lower control limit, indicating that first-pass retention was lower than usual. No other problem is evident in these charts.

The Z-score $\bar{X}$ chart for this same period (**Fig. 6**) showed another problem not detected by the Shewhart charts. On Days 11 and 12, the $\bar{X}$s were both above the upper warning limit, meaning that first-pass retention was higher than in the past. This problem was missed by the Shewhart charts because of the grade change between Days 11 and 12. The Z-chart thus led production personnel to check the feed system for the retention aid. They found that the feed pump was out of calibration, feeding too much chemical. By recalibrating the pump early on Day 13, they were able to remedy the problem.

The Z-score $\bar{X}$ chart also detected the problem on Day 14. Examination of production records revealed that the shift from Day 13 to Day 14 was caused by a change in the forming section's foil configuration early on Day 14. On identifying the cause, the change was reversed during the grade change. Consequently, Day 15 showed no signs of problems.

The R chart (**Fig. 7**), however, indicated no variability changes throughout this period.

6. A Z-score X-bar chart showing signs of process problems.

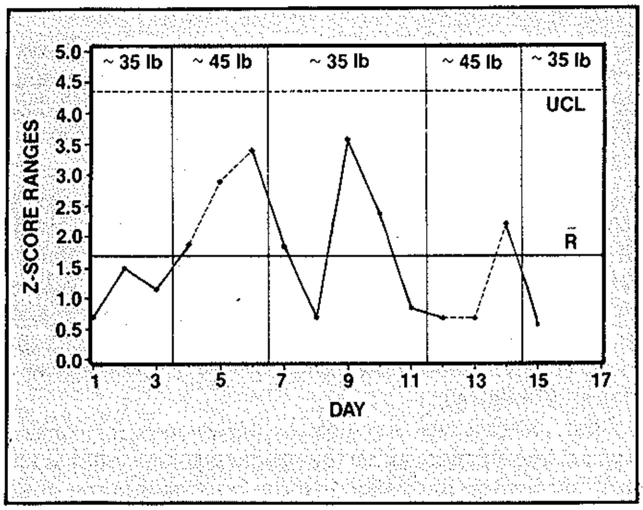


III. Calculation of revised averages and sigmas

Grade	N_x	$\bar{\bar{X}}$	N_r	$\bar{R}$[a]	σ
~35 lb/3300 ft²					
Current period	46	69.78	40	4.80	4.25
Historical data	50	69.54	45	4.90	4.34
Revised values	96	69.66	85	4.85	4.30
~45 lb/3300 ft²					
Current period	52	76.21	45	3.10	2.75
Historical data	53	75.53	46	3.32	2.94
Revised values	105	75.87	91	3.21	2.85

[a]Calculated using moving ranges, so d_2 = 1.128.

7. A Z-score range chart showing no evidence of variability changes.



Establishing product averages and sigmas

The most demanding part of using Z-score charts is the establishment and maintenance of the product averages and sigmas. These values are established through the use of recent data. The initial average value of $\bar{X}$, or $\bar{\bar{X}}$, is calculated by averaging available readings for the product. To obtain the product's σ, these steps are followed.

First, calculate the subgroup or moving range values. Moving ranges should not be calculated across "breaks" in the process data, which occur during product changes and downtimes.

Second, calculate $\bar{R}$, the average of the calculated subgroup moving ranges.

Third, calculate σ, according to Eq. 4:

$$\sigma = \bar{R}/d_2 \tag{4}$$

where the d_2 values (Table II) are indexed by n.

The historical data should contain at least 20 readings per product. Out-of-control points may be eliminated from these calculations if the upset's cause has been identified and eliminated from the process (*5*).

Once product averages and sigmas are calculated, the use of the Z-score control charts can begin. Before initiation of these charts, however, plans for periodic review and updating of the product averages and sigmas should be made. For most operations, reviews should be performed on a monthly or quarterly basis.

Updating product averages and sigmas

Calculations

At each review, product $\bar{\bar{X}}$ and $\bar{R}$ values should be calculated as described above, using data from the period. Again, each product should ideally have at least 20 readings. New overall statistics should then be obtained using both the current and previous values. To update the overall product average and sigma values, these steps should be followed.

First, calculate $\bar{\bar{X}}$ and $\bar{R}$ for the current period. Second, determine N_x, the number of readings, and N_r, the number of ranges, for the current period.

Third, calculate the new $\bar{\bar{X}}$ and the new $\bar{R}$, which are the updated overall average value and average range value, respectively:

$$\bar{\bar{X}}_{new} = [(N_{xt}\bar{\bar{X}}_{old}) + (N_x\bar{\bar{X}})]/(N_{xt} + N_x) \tag{5}$$

$$\bar{R}_{new} = [(N_{rt}\,\bar{R}_{old}) + (N_r\,\bar{R})]/(N_{rt} + N_r) \tag{6}$$

where

N_{xt} = total number of readings used to calculate $\bar{\bar{X}}_{old}$

N_{rt} = total number of ranges used to calculate $\bar{R}_{old}$

Fourth, calculate the new σ:

$$\sigma_{new} = \bar{R}_{new}/d_2 \tag{7}$$

where the d_2 values are listed in Table II.

Fifth, update the values of $\bar{\bar{X}}_{old}$, $\bar{R}_{old}$, N_{xt}, and N_{rt} for use at the next review. The new product average is $\bar{\bar{X}}_{new}$, and the new sigma value is σ_{new}.

An example

At the same mill's first review period one month after starting the Z-score control charts, $\bar{\bar{X}}$ and $\bar{R}$ values for first pass retention were calculated for the current month for each grade, as shown in **Table III**.

Using these values and the historical values, revised values were calculated. For example, using the equations for $\bar{\bar{X}}_{new}$, $\bar{R}_{new}$, and σ_{new} for paper with a basis weight of about 35 lb/3300 ft^2:

$$\bar{\bar{X}}_{new} = [(50 \times 69.54) + (46 \times 69.78)]/ (50 + 46) = 69.66 \quad (8)$$

$$\bar{R}_{new} = [(45 \times 4.90) + (40 \times 4.80)]/ (45 + 40) = 4.85 \quad (9)$$

$$\sigma_{new} = \bar{R}_{new}/d_2 \times 4.85/1.1284 = 4.30 \quad (10)$$

These new $\bar{\bar{X}}$ and σ values were used to calculate the Z-scores during the second month.

Monitoring product averages and sigmas

As part of each review, the periodic $\bar{\bar{X}}$'s and $\bar{R}$'s can be examined to determine if long-term changes or trends are occurring in the process. If such shifts are detected, $\bar{\bar{X}}_{new}$, $\bar{R}_{new}$, N_{xt}, and N_{rt} should be revised to include only periods since the change. These shifts or changes are often caused by changes in furnish, stock, equipment, or procedures that have resulted in quality improvements.

Benefits and advantages

Although these approaches add some new complexity to an operation's SPC work, there are three major benefits that far outweigh the inconveniences.

First, these techniques allow SPC monitoring of many process variables not currently examined. Previously, the techniques were not useful for detecting process difficulties because the process shifts often hid them. Now these variables become useful because the effect of these product changes has been removed.

Second, these techniques improve the chances of identifying process problems despite product changes. Ordinary techniques generally require a separate control chart for each product. Trends or problems that occur around a product change are thus difficult to catch. Such trends or problems are more quickly and easily detected using the single chart.

Third, these techniques often result in time savings as well. As mentioned, the usual techniques often lead to a multitude of control charts, which often becomes a nightmare for data management and control chart maintenance. These techniques replace many control charts with just one, saving time, improving information, and reducing confusion.

Common obstacles to overcome

As with any new technique, some obstacles must be overcome before successful implementation is possible. These obstacles fall naturally into two categories: those that relate to operators and hourly personnel, and those that relate to the professional staff, including engineers, supervisors, and managers.

Operators

For operators, the primary obstacles are the calculations involved. All three techniques require some calculation before the operator can calculate and chart the $\bar{X}$ and R values. The actual-minus-target values technique requires only one operation, a subtraction. On the other hand, the percent-of-target technique and the use of Z-scores both require two mathematical operations. Most importantly, the ability to properly handle positive and negative numbers in calculations of $\bar{X}$ and R is essential.

The best way to overcome these obstacles is to first reduce the needed calculations to a "cookbook" of clear step-by-step instructions including as little mathematical jargon as possible. Second, supply operators with inexpensive and simple calculators for their personal use on the job.

Third, hold practice sessions with operators using actual data from their work. Stress the importance of their participation and the benefits for them and the company. Also, offer ample help and encouragement. Be patient with those slow to catch on.

Professional staff

For the professional staff, obstacles center on additional demands on their time. For the actual-minus-target and percent-of-target techniques, this demand is minimal. The only time spent is to organize practice sessions and ensure that the proper target values are used.

Implementation of Z-scores, however, can demand a great deal of time, especially if Z-charts are not slowly phased in. Time is needed to obtain and use the historical data necessary for calculating average and sigma values for each product. Additionally, the periodic review and revision of these values requires time. To overcome these potentially large obstacles, the following recommendations should be followed.

First, select a few, key process parameters for consideration. Second, determine the products to be included for each parameter, and prioritize the implementation of Z-score charts for these parameters. Next, obtain the necessary historical data and perform the calculations for $\bar{\bar{X}}$ and σ one parameter at a time.

Study the observed average and sigma values to determine if the actual-minus-target or percent-of-target approaches will work just as well. Establish the frequency of the average and sigma value reviews. In most settings, monthly or quarterly reviews are appropriate. Improve or automate recordkeeping to simplify the periodic review, and systematize the review process until the responsibility can be passed on to a capable assistant.

Basically, the recommendation is to start small in key areas and gradually expand. The results of not doing so could be disappointing and embarrassing.

Computer-assisted SPC

Presently, many SPC programs that can be run on personal computers can aid the in implementation of ordinary SPC techniques (*6*). These programs will produce the necessary control charts, though with widely varying degrees of difficulty. Good SPC software should:

- Produce accurate Shewhart control charts, even if the subgroup size varies
- Require little training for effective use of the software
- Be quick and easy enough for operators to enter their own process readings
- Be simple enough for operators to produce their own control charts
- Provide data security against inappropriate copying or deletion.

However, to facilitate implementation of these techniques, the software should also allow the user to specify the $\bar{\bar{X}}$ and σ values for the control charts. (For the Z-score charts, set $\bar{\bar{X}}$ to zero and σ to one.) Finally, the software should perform the calculations needed to produce the transformed values, thus relieving operators of this task.

Summary

The three techniques of actual minus target values, percent of target value, and Z-scores can expand the use of SPC. The first two techniques are simple and easy to incorporate into an existing SPC program. The third requires more work and planning, but it can also reduce current SPC workloads and provide better capabilities for detecting problems.

These three techniques can help open the door for some previously locked-out breakthroughs in quality improvements. These breakthroughs were precluded because product changes often obstructed the effective detection of problems and their solution. With this obstacle removed, additional quality and productivity improvements become a possibility. The realization of these potentials, however, requires commitment, attention, training, and time.

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Received for review Dec. 8, 1987.

Accepted Feb. 26, 1988.