Predicting shear wrinkles in web spans

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ABSTRACT *Web quality and runnability are related to web wrinkling. A model that predicts the occurrence of shear wrinkles in web spans is presented. The model is verified by comparing predicted occurrences with experimental data.*

KEYWORDS
Analysis
Mathematical models
Webs
Wrinkling

Definition of the problem

Web troughs are defined as out-of-plane deformations of the web that occur between rollers in web-handling systems. Web wrinkles are defined as out-of-plane deformations of the web that are transferred over rollers from one web span to the next. Web quality is highly dependent on web wrinkling. Preventing wrinkles is important because they may cause permanent defects or tearing in the web. Wrinkles that are not responsible for a complete separation of the web can still lead to quality degradation in the winder as a result of the caliper variations caused by folded webs.

Wrinkles caused by lateral deformation of the web will form in directions not parallel to the machine direction (**Fig. 1**). Lateral deformations induce lateral shearing forces in the web, and the wrinkles that result are called "shear wrinkles." Web lateral deformations can be caused by:

1. Misaligned rolls
2. Controlled guide rolls
3. Shifting unwind and rewind stands
4. Interactions between the web and special rollers (such as curved axis rollers and concave rollers).

We have generated a model for both film and paper webs that predicts shear wrinkles as a function of lateral displacement, tension, caliper, span length, web width, and web material properties. The model has been veri-

Nomenclature

a = web span length

a_{mn} = amplitude of wrinkles

b = web span width

D = web bending stiffness = $Eh^3/[12(1-\nu^2)]$

E = Young's modulus

F = lateral force input to the web due to roll misalignment

h = web caliper or thickness

I = web area moment of inertia = $hb^3/12$

i = number of half waves, n, for a specific span that is wrinkled

L = web span length with no moment transfer

L_r = length of misaligned roll

m = number of half waves in the x direction

n = number of half waves in the y direction

N = load per unit length of web

w = deformation of the web normal to the web plane

σ = stress

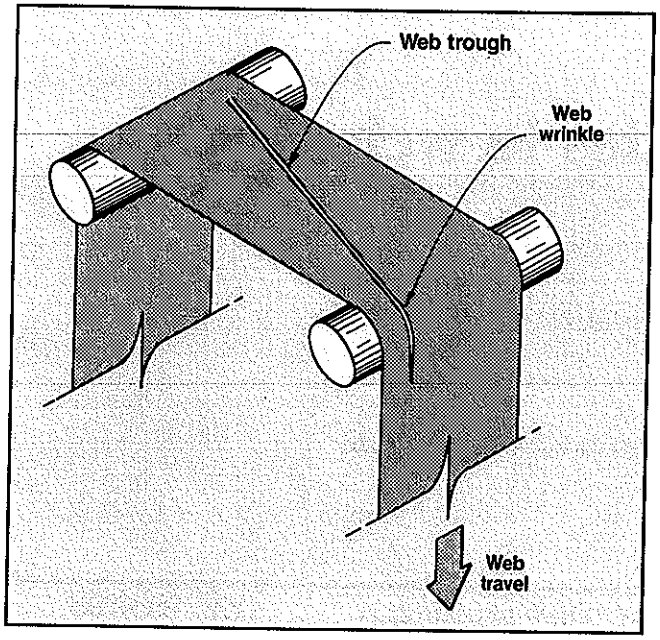
σ_{cr} = critical stress necessary to wrinkle the web

τ = web shear stress

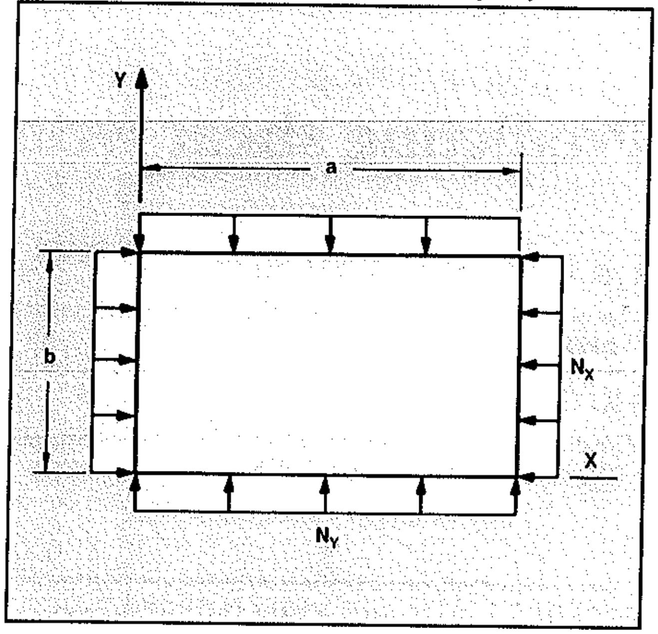
θ = angular misalignment of a roll

ν = Poisson's ratio.

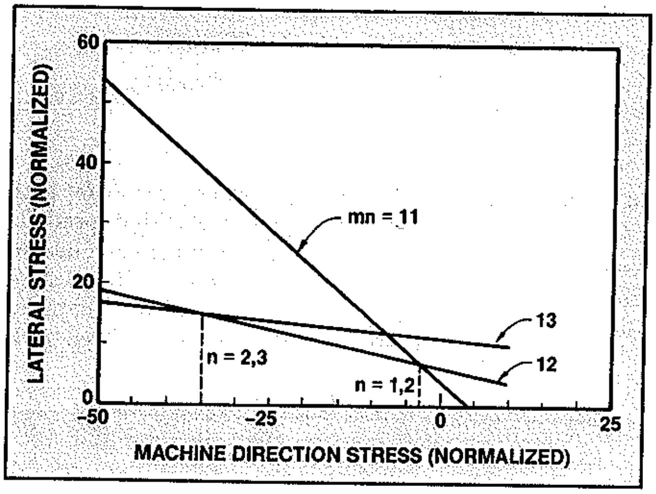
1. A shear wrinkle in a web line



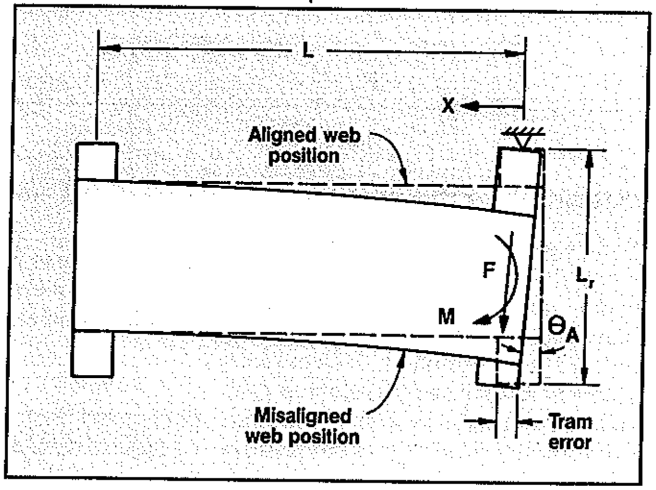
2. A web coupon with applied loads used in buckling analysis



3. Failure criteria



4. Lateral deformation of a web span



fied experimentally on both film and paper webs.

The basic value of this research is that it will allow web handling engineers to prevent shear wrinkling by:

1. Properly designing or modifying web lines
2. Setting realistic alignment tolerances of rollers in preventative maintenance programs
3. Troubleshooting via the model rather than by instituting immediate hardware changes, which are expensive and may not solve the problem anyway.

Deriving the wrinkling criteria equation

Webs are usually idealized as membranes with the capacity to resist tensile stresses but not compressive stresses. Real webs are composite membrane and plate structures that can react to compressive stresses until the buckling stress, though small, is surpassed. Web wrinkling is the result of compressive stresses in the web exceeding the buckling stress of the web.

Timoshenko (*1*) presents a solution for the buckling stress for a rectangular coupon. This solution allows a condition of tension on two opposing sides and compression on the other two opposing sides (**Fig. 2**). In keeping with Timoshenko's sign convention, positive stresses would induce compression into the rectangular coupon. The solution results from implementing the energy method, which states that when the work done by forces acting in the middle plane of the plate becomes larger than the strain energy of bending for any shape of out-of-plane deflection, the plate is unstable and buckling occurs.

To find the critical buckling load, the work done by the forces T is set equal to the strain energy in bending, V:

$$\Delta T = \Delta V \qquad (1)$$

In terms of forces N and out-of-plane displacements w, Eq. 1 becomes:

$$-(1/2)\iint[N_x(\delta w/\delta x)^2+N_y(\delta w/\delta y)^2]dx\,dy = (D/2)\iint\{[(\delta^2w/\delta x^2)+(\delta^2w/\delta y^2)]^2 - 2(1-\nu)[(\delta^2w/\delta x^2)(\delta^2w/\delta y^2) - (\delta^2w/\delta x\delta y)^2]\}dx\,dy \quad (2)$$

where

x = machine-direction coordinate

y = cross-machine coordinate.

The buckled out-of-plane shapes of the rectangular coupon are assumed to be represented by the equation:

$$w_{mn} = a_{mn}\sin(m\pi x/a)\sin(m\pi y/b) \quad (3)$$

In Eq. 3, we assume that the edges are simply supported. In this expression, m is the number of half waves in the x direction, and n is the number of half waves in the y direction. Substituting Eq. 3 into Eq. 2 yields Eq. 4:

$$N_x(m^2\pi^2/a^2)+N_y(n^2\pi^2/b^2) = D[(m^2\pi^2/a^2)+(n^2\pi^2/b^2)]^2 \quad (4)$$

A normalized stress σ_e is introduced as:

$$\sigma_e = \pi^2D/(a^2h) \quad (5)$$

Then dividing Eq. 4 by the caliper h and substituting the normalized stress yields:

$$\sigma_x m^2+\sigma_y n^2(a^2/b^2) = \sigma_e[m^2+n^2(a^2/b^2)]^2 \quad (6)$$

In applying Eq. 6 to spans in web lines, we will assume that the x direction will be the machine direction. The stress in the machine direction will be the stress associated with the web line tension. Since there is tension in the machine direction, no wrinkles will form perpendicular to the x axis. Thus, Eq. 6 can be reduced as follows by setting m, the number of wrinkle half waves in the x direction, equal to one:

$$\sigma_x+\sigma_y n^2(a^2/b^2) = \sigma_e[1+n^2(a^2/b^2)]^2 \quad (7)$$

The wrinkling criteria equation

Equation 7 is the basic equation for failure criteria used to indicate if wrinkling has occurred in the web. This expression shows the relationship between the machine-direction tensile stress σ_x, the lateral stress σ_y necessary to wrinkle the web, and the number of half waves n along the y axis that will be associated with this wrinkled condition. Further insight may be gained by substituting fixed values of n into Eq. 7 as follows.

For $n = 1$

$$\sigma_x+\sigma_y(a^2/b^2) = \sigma_e[1+(a^2/b^2)]^2 \quad (8)$$

For $n = 2$

$$\sigma_x+4\sigma_y(a^2/b^2) = \sigma_e[1+(4a^2/b^2)]^2 \quad (9)$$

For $n = 3$

$$\sigma_x+9\sigma_y(a^2/b^2) = \sigma_e[1+(9a^2/b^2)]^2 \quad (10)$$

Equations 8-10 now only contain variables σ_x and σ_y. All three expressions are plotted in **Fig. 3** for the case where $a = b$. Note the locations at which the lines representing $n = 1$ and $n = 2$ intersect. If σ_x is increased to a higher tension, it requires less σ_y stress to wrinkle the web if n becomes two. Since a decrease in σ_y corresponds to a lower energy state, the web would change its wrinkled shape from one half wave to two half waves at this intersection. By properly combining Eqs. 8 and 9 and then Eqs. 9 and 10, σ_y can be removed, and the value of σ_x at the intersection can be determined as follows.

For intersection $n = 1,2$

$$\sigma_x = \sigma_e(1-4a^4/b^4) \quad (11)$$

For intersection $n = 2,3$

$$\sigma_x = \sigma_e(1-36a^4/b^4) \quad (12)$$

Thus, for a given value of σ_x, the half wave number n can be determined by searching for the bounding σ_x values that correspond to half wave number $n = i$. This determination is not complex since Eqs. 11 and 12, which are the bounds for σ_x when $n = 2$, can be expressed in a generalized form for $n = i$ as follows:

$$\sigma_e[1-i^2(i-1)^2(a^4/b^4)] > \sigma_x > \sigma_e[1-i^2(i+1)^2(a^4/b^4)] \quad (13)$$

With known web line stress, the half wave number Eq. 7 is employed to calculate $\sigma_{y,\,cr}$, the compressive lateral stress necessary to wrinkle the web. Solving Eq. 7 in terms of $\sigma_{y,\,cr}$ yields:

$$\sigma_{y,\,cr} = (b^2/i^2a^2)\{\sigma_e[1+(i^2a^2/b^2)]^2-\sigma_x\} \quad (14)$$

Calculating the compressive stress in the web

Now that a failure criteria equation has been established, expressions that relate web and web line parameters to this equation must be developed. A web span is shown in **Fig. 4**, which has undergone an end rotation and translation that could have been caused by a misaligned roll or an end-pivoted guide roll. If the web is treated as a multispan beam in bending, then shear must be constant in each span and moment will vary at most linearly from one roll to the next.

One of the fundamental laws of web handling is that for each span, the bending moment at the downstream roller is zero, provided there is no slippage across the downstream roll (*2*). This implies that immediately upstream of the misaligned roller, the condition of stress is such that there is a constant tensile stress σ_x (i.e., no bending moment) and a constant shear stress on each stress block that might be taken across the web width.

The second principal stress, which will always be compressive for the case described, can be calculated by the following equation (*3*):

$$\sigma_2 = (\sigma_x+\sigma_y)/2 - \{[(\sigma_x-\sigma_y)/2]^2+\tau^2\}^{1/2} \quad (15)$$

where $\sigma_y = 0$.

The second principal stress is the maximum compressive stress that the web must withstand. To determine the maximum shear stress which can be applied to the web prior to wrinkling, the critical stress $\sigma_{y,\,cr}$ necessary to initiate wrinkling can be computed from Eq. 14 substituted into Eq. 15 for σ_2 and rearranged to yield:

$$\tau_{cr} = (\sigma_{y,\,cr}-\sigma_x\sigma_{y,\,cr})^{1/2} \quad (16)$$

To relate this critical shear stress to the web and the degree of misalignment in the web line, a deflection analysis of the beam in Fig. 4 must be performed. The moment in the beam is:

$$M_{AB} = -F\,x - M \quad (17)$$

Invoking Castigliano's second theorem to determine the end rotation of the cantilever beam due to the lateral force F yields (*3*):

$$\theta_A = \delta U/\delta M = \int_L(M_{AB}/EI)(\delta M_{AB}/\delta M)\,dx = FL^2/2EI = (6\tau_{cr}L^2)/Eb^2 \quad (18)$$

which for small angles can be approximated as

$$\text{tram error}/L_r = (6\tau_{cr}L^2)/Eb^2 \quad (19)$$

where tram error represents the total

misalignment of the roll of length L_r from mount to mount in units of displacement.

In some cases, there is not adequate traction at the upstream roll to enforce the zero slope boundary condition and a condition in which moment and edge deflections are transmitted upstream into the next span. This condition is known as "moment transfer" between spans (*4, 5*). In moment transfer, the roll misalignment can be much greater than predicted by Eq. 19 prior to a wrinkle failure. As an example, **Fig. 5** is a beam model of a type of moment transfer.

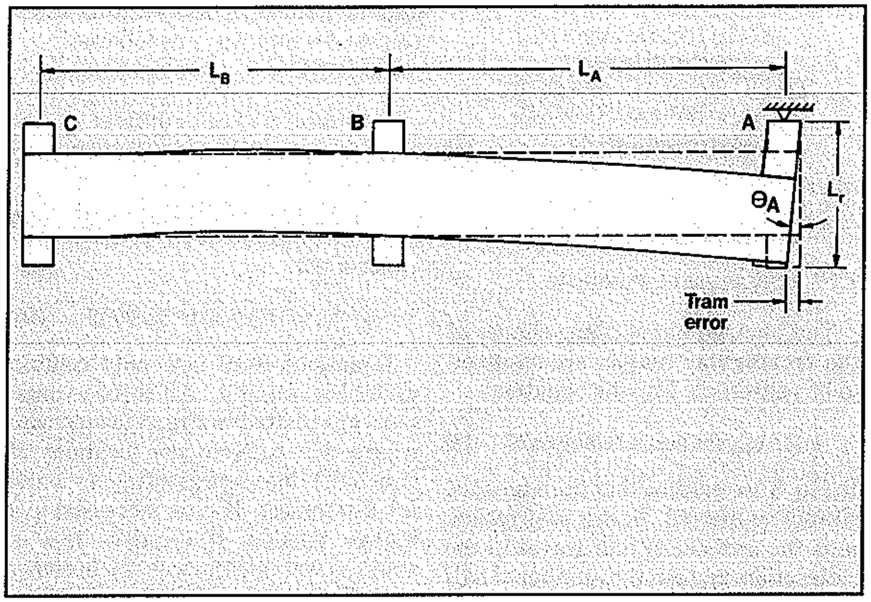
In this type of moment transfer, we assume that the web–roll traction is inadequate to restrain web rotation but is adequate to prevent lateral displacement of the web. Equation 20 was derived using the same technique we used to develop Eq. 19:

$$\text{tram error}/L_r = (6\tau/Eb^2)\,[L_A{}^2 + (L_A L_B)/2] \qquad (20)$$

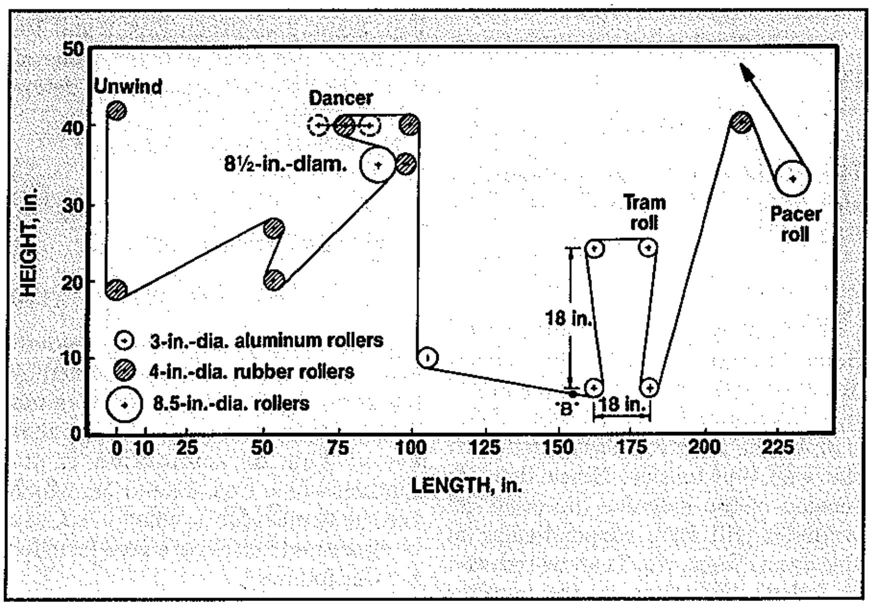
where L_A and L_B represent the lengths of the first and second spans upstream from the misaligned roll, respectively.

There are several other types of moment transfer that can occur between web spans. Which type exists is entirely dependent on the state of traction between roller and web. Equation 20 represents but one type of moment transfer and was presented here to help explain experimental data which will be discussed later.

5. Lateral deformation of a web span with moment transfer at Roller B



6. Tension-controlled web line



Experimental procedure

The experimental apparatus consisted of a test section which was installed in the tension-controlled web line shown in **Fig. 6** (*6*). The rollers in the test section were 3 in. in diameter with a root-mean-square surface finish of 32μ in. These rollers were arranged in a test section as shown in **Fig. 7**. The length of the end-pivoted roll L_r used to calculate the pivot angle was 20 in. The rolls in the test section were either 18 in. or 21 in. apart, and various web widths and calipers of polyester, polypropylene, and paper were tested.

We began each test by setting the line tension and velocity. A micrometer table was used to displace the end of the end-pivoted roll until a wrinkle formed in the test span. We then recorded the micrometer setting (equivalent to tram error) at which the wrinkle was formed. The line tension or the velocity was varied, and the test would be repeated.

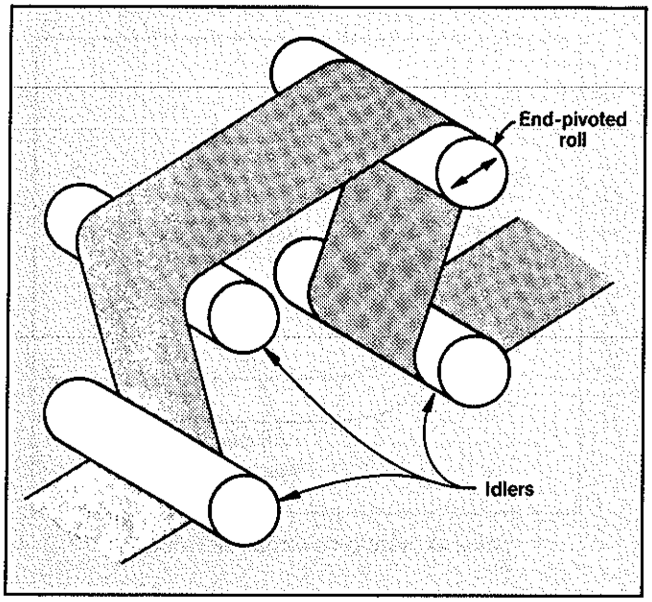
Experimental results and correlation to failure criteria

Many interesting phenomena became evident during these tests. The wrinkling effects of the length-to-width ratios (a/b) of the test span, web caliper, line velocity, average tensile stress (proportional to line tension), and tram error can be studied.

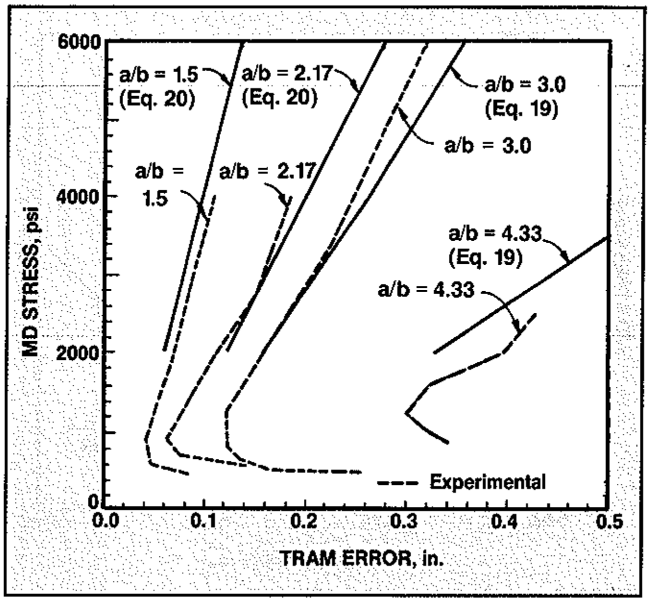
Figures 8–11 are plots of data taken on polyester, polypropylene, and paper. The general shape of the failure curve does not vary, but the location of the failure curve on plots of average tensile stress vs. tram error can vary, depending on the other parameters. Trends are shown on a generic failure curve in **Fig. 12** and are stated as follows:

1. As the length-to-width ratio increases, the curve moves horizontally to the right.

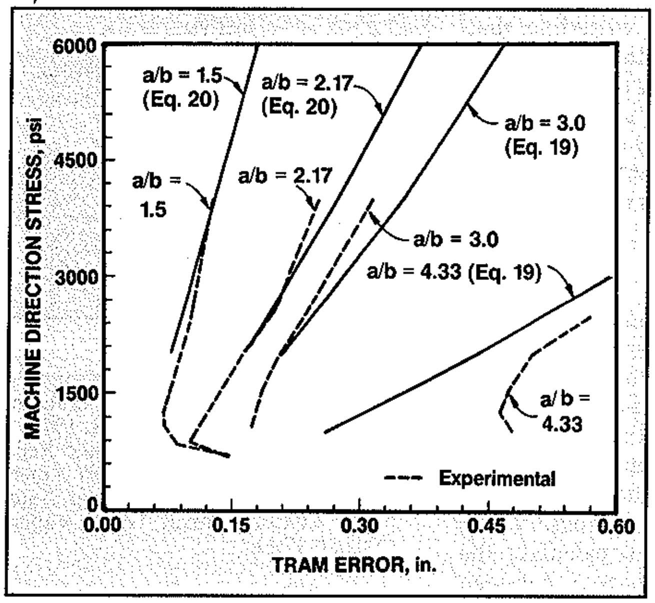
7. Test section



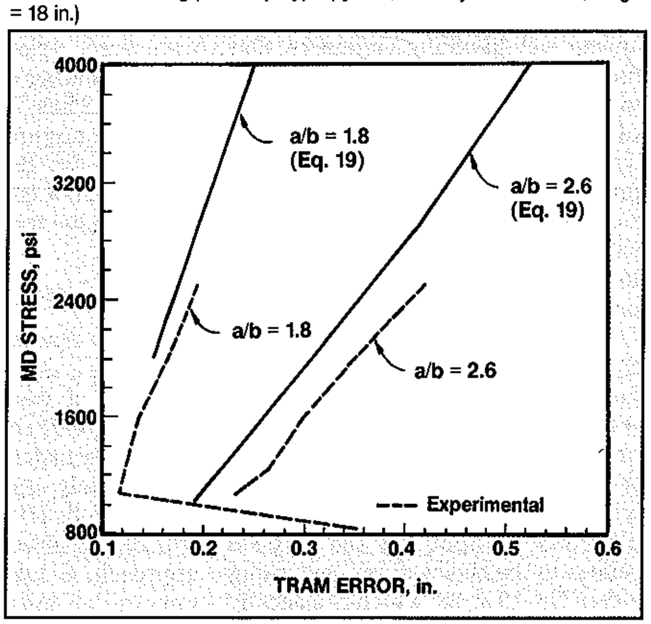
8. Shear wrinkling (79-gauge polyester, velocity = 200 ft/min, length = 18 in.)



9. Shear wrinkling (1.4-mil polyester, velocity = 200 ft/min, length = 18 in.)



10. Shear wrinkling (1.6-mil polypropylene, velocity = 200 ft/min, length = 18 in.)

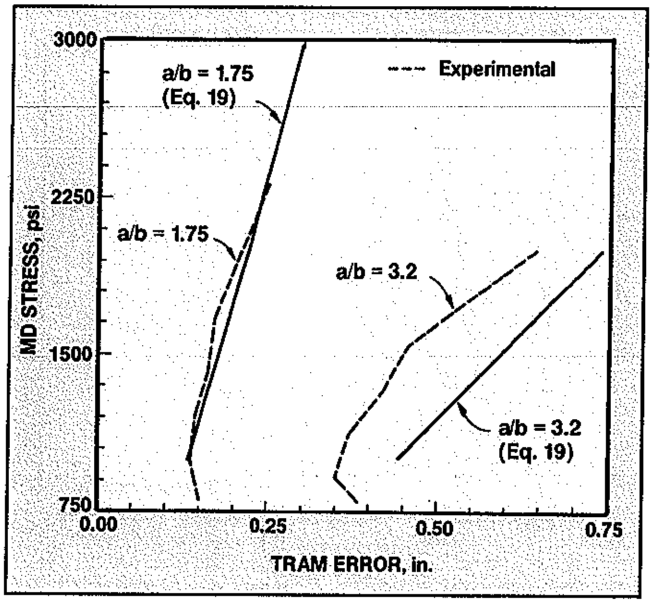


2. As web caliper (thickness) increases, the curve moves to the right again, but it has less effect than the length-to-width ratio.
3. As the web line velocity increases, the bottom of the L-shaped failure curve moves upward.

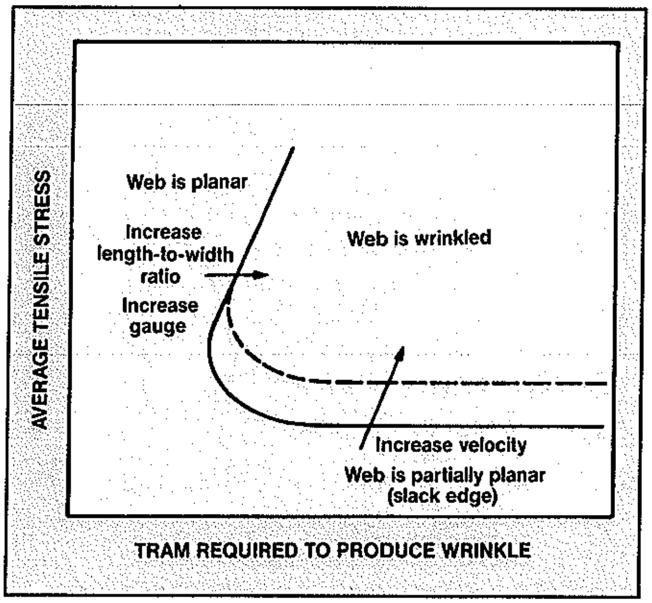
The area included within the L-shaped curves is a region in which wrinkling has been initiated. The area to the left of the curve is a region in which the web is in a planar state in which bending stresses within the web limit the lateral web displacement. The area below the curve represents a region in which part of the web is planar but one edge is slack. The state of the web in each of the three regions shown in Fig. 12 is depicted in **Fig. 13**.

From the aspect of retaining high web quality, it would be preferable to have every span in the web line operating in the region to the left of the L-shaped failure curves. This region of the parameter space represents a planar web with no slack edges which would be easier to process in coating, calendering, printing, laminating, and other operations that must be free of shear wrinkles.

Returning to Fig. 12, we see that the optimal condition would be to have high length-to-width ratios of the web spans and a thick web to make the region to the left of the L-shaped curve as large as possible. Although the first parameter can be controlled to some extent in web lines, the second parameter cannot be controlled in that the thickness depends on the product



11. Shear wrinkling (4-mil crepe paper, velocity = 100 ft/min, length = 21 in.)



12. The effects of length-to-width, gauge, and line velocity on the failure curve

specifications. This observation does, however, reinforce the adage passed among web-handling engineers, that thinner webs are much more difficult to handle than thicker webs of the same material. Thus, if a technique were developed that allowed us to predict the vertical portion of the L-shaped curves, most of the shear wrinkling problems would be solved. In fact, the failure criteria equation predicts just this.

Predictions of the failure criteria equation have been overlaid on the failure curves (Figs. 8–11) obtained in the laboratory. The failure criteria predictions correlate quite well with the experimental data for the three web materials studied. The only difficulty in applying the criteria relates to the choice of implementing either Eq. 19 or 20, which is basically a decision regarding whether moment transfer is occurring or not. This decision can be made only with knowledge of the available traction between the web and the upstream roll, which is a complex issue even with uniform web stresses across the roll. It becomes even more complex when trying to address the state of traction for a web with varying machine-direction stresses in the cross-direction caused by web bending.

This complexity will be a topic of future research. However, the failure criteria method is useful if Eq. 19 is employed exclusively, which is a conservative approach because Eq. 20 will always predict a higher allowable tram error prior to wrinkling than Eq. 19, holding all other parameters constant. Thus, if an engineer needs to set roll alignment tolerances for a web line, Eq. 19 will either yield an accurate or a conservative value for the allowable tram error.

Conclusions

A failure criteria equation has been established that accurately predicts the onset of shear wrinkling in web spans, provided that the state of moment transfer is known at the roll immediately upstream of the misaligned roll. If the state of moment transfer is unknown, a conservative approach is taken to calculate the amount of misalignment between rolls that can exist without wrinkling occurring.□

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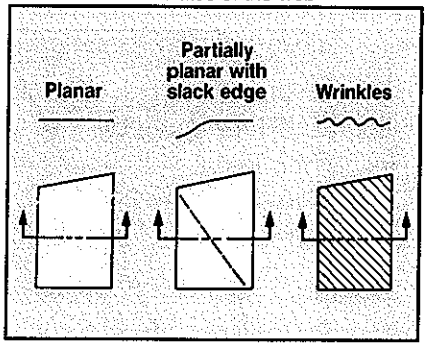
13. Mechanical states of the web



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