The modeling of rotary lime mud filters

Kent R. Davey, George Vachtsevanos, and Jim C. Cheng
President, CEO, and research engineer, respectively, Intelligent Dynamic Systems, Inc., 2445 Old Covington Highway, Conyers, Ga. 30207

Jeffery S. Hsieh
Associate professor, School of Chemical Engineering, Georgia Institute of Technology, Atlanta, Ga. 30332

ABSTRACT *With the intent of more precisely controlling the lime mud filter, the authors propose and analyze a theoretical model that predicts the performance of the lime mud filter under various scenarios. The model solves the second-order partial-differential diffusion equation for water density with time and two first-order partial-differential mass exchange equations for alkali content in the mud with time. Variations in output, moisture, and alkali content of the mud are examined under a number of different conditions, such as changes in rotation speed, pressure, wash flow rate, volume of production, and blade position. The model is extremely valuable from a pedagogical perspective in dictating optimum control schemes.*

KEYWORDS
Analysis
Lime
Kilns
Mathematical models
Process control

Among the more troublesome problems associated with lime recovery is the temperature swings that occur in the kiln as a result of moisture variations in the feed mud. Because radiation is proportional to temperature raised to the fourth power, kiln temperature swings result in high energy losses. In addition, they also strain the kiln, reducing its lifespan. Density variations in the alkali content of the mud, on the other hand, cause operational problems. Too low an alkali content results in high dusting losses within the kiln because the calcium carbonate (the "dust") does not stick to itself effectively. Too high an alkali content yields too much sticking and often results in large balling or conglomeration effects down the kiln. An appropriate control goal is to maintain both the moisture and the alkali content at uniform levels, with the moisture at about 70% and the alkali at about 1.5%.

The control of lime recovery has been focused principally on the causticization and lime kiln areas (*1–4*). More recently, Uronen and Haataja (*5, 6*) have attempted a control based on a first-order pole representation of the filter as examined from a system's point of view. In their scheme, the water and alkali cycles are each represented as single pole plants. Sloan *et al.* (*7*) attempted to increase the efficiency of the filter through the use of dewatering additives. They focused on the long-term behavior of the system between scraping and blowing intervals and developed a total control strategy that is empirically based.

The two targets of keeping the solids content of the mud high and the sodium (that is, the alkali concentration) low are to some extent contradictory. As we shall see in a moment, aside from the wash flow, both the alkali content and solids content behave similarly under various operating conditions. One of the goals in developing this model is to obtain more insight into the parameters that can be changed to control either moisture content or alkali content individually or to change both. The final goal is to use the model information to design an adaptive multivariable control strategy for the mud filter and to provide a general control scheme based on that design.

Drum filtration process

Figure 1 shows the scheme of the lime cake filter. Lime mud slurry, consisting of calcium carbonate and water, is fed into the drum vat at a set rate. The incoming slurry is normally about 65% water, the high moisture content being necessary for pumping purposes. The outer radius of the drum is constructed of a fine wire mesh. During operation, a strong negative pressure is pulled by the vacuum pump on the interior of the cylinder drum. After a few cycles, following startup, a fine residual cake is established on the mesh.

Nomenclature

f = half rotational drum frequency

k = resistance factor relating pressure across the cake to fluid velocity through the cake (empirical quantity related to cake porosity)

K_D = diffusivity characterizing the diffusion of water into the porous cake medium (m²/s)

$k^{channel}$ = mass exchange coefficient from Kuo (*9*), governing the rate of change of alkali in the channel with time

k^{cake} = mass exchange coefficient from Kuo (*9*), governing the rate of change of alkali in the cake with time

k_s = chemical kinetic solubility coefficient relating $[Na]^{channel}$ to $[Na]^{cake}$ at $X = 0$

$[Na]^{channel}$ = concentration of sodium (mass/volume) in the channels

$[Na]^{cake}$ = concentration of sodium (mass/volume) in the cake

p = pressure

r_0 = characteristic radius of the duct for which the Hagan-Poiselle flow is applied

t = time measured from the instant that the cake leaves the vat

$\underline{t}$ = normalized time $\equiv f\,t$, so defined to be equal to unity when the cake reaches the blade

Th = total thickness of the cake

Th_0 = inner cake thickness – the distance between the blade and the filter mesh

Th_1 = outer cake thickness – the distance from the blade to the outer portion of the cake; this is the portion of the cake that drops into the kiln and will vary as the production level changes

v = convective fluid velocity dictated by the pressure differential across the cake (Eq. 3)

$\underline{v}$ = convective velocity normalized to the rotational time and cake thickness $\equiv v/(th\,f)$

x = variable designating a position in the cake as measured from the outside cake wall where $x = 0$

$\underline{x}$ = normalized position in the cake $\equiv x/th$, so chosen to be equal to unity at the inner filter screen

∇ = gradient operator $= \partial/\partial x$

$\underline{\nabla}$ = normalized gradient operator $= \nabla * \mathrm{Th}$

η = viscosity of the fluid flowing through the pores of the cake

ρ = composite density of the water in the cake, i.e., the mass of water per unit volume of the cake (the amount that could be evaporated by heating).

While the drum is in the slurry vat, calcium carbonate is pulled from the slurry onto the surface of the drum by virtue of the pressure gradient through the cake. The water is pulled through the porous medium into the inner region of the cylinder, and then it is pumped out along the axis where it is used in other segments of the recovery cycle. The drying of the cake starts when a segment of the filter cake exits the slurry vat. Water is pulled against the forces of diffusion towards the inner portion of the cylinder. Approximately half a revolution after exiting the slurry, the cake encounters a blade that removes the greater portion of the calcium carbonate cake, leaving only the residual cake on the filter.

Now containing only about 30% moisture, the lime mud falls into either a belt or screw feeder to be delivered into the kiln. The residual portion of the thickness of the cake designated Th_0 depends on the blade position. The additional thickness, designated Th_1, is that part delivered to the kiln at the production rate. Within a reasonable working range, Th_1 is set on the basis of the production rate. For instance, if the input mud slurry flow increases by 30%, so does the thickness Th_1, all other variables such as drum speed being held constant.

Just after exiting the slurry bath, the mud cake encounters a wash shower. This wash shower is used to keep down the alkali content of the mud on the cake. Too high an alkali content within the mud results in coagulation, or balling, of the mud as it flows down the lime kiln. Another consequence of the wash shower is, of course, that it increases the moisture content of the cake.

In operating the kiln, it is desirable to maintain the moisture content of the mud falling into the kiln at as low a percentage as feasible, while keeping the alkali content in the ranges of 0.5–1.0%. Fluctations in the moisture content of feed mud result in cyclic fluctuations of temperature down the kiln. These fluctations lead to several problems:

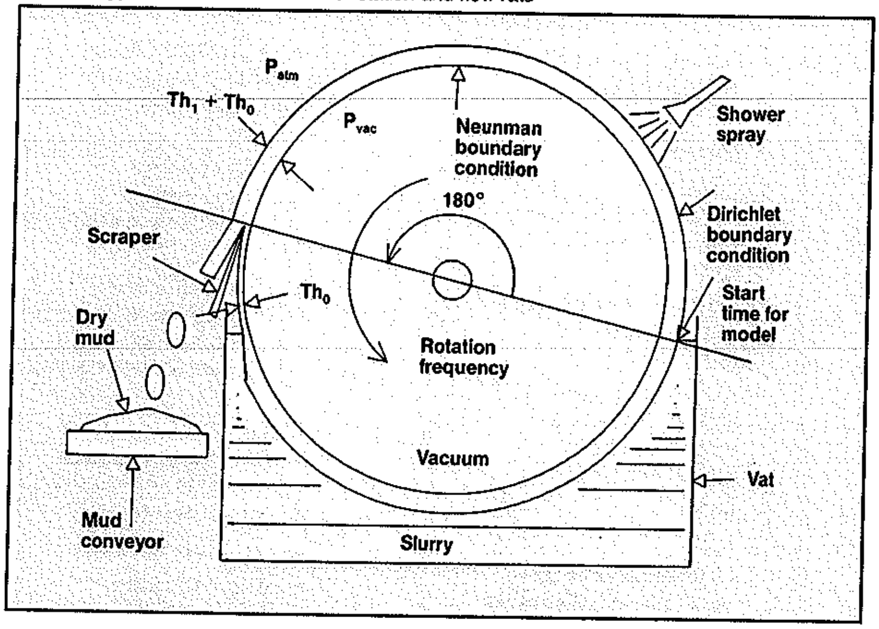
1. The operator has to alter the amount of fuel input into the kiln to match the moisture fluctuations.
2. The localized hot spots in the kiln cause long-range deterioration of the bricks more quickly.
3. A great deal of heat is lost with the fluctuations, because the heat loss through radiation increases exponentially as the temperature increases (*8*).

The underlying question in our research is how to best maintain the moisture content of the lime mud and its alkali content close to the target concentrations. The questions are, then, which variables do we have to alter, and what effect do they have on moisture and alkali content?

The alterable parameters are:

1. Drum speed
2. Wash flow rate
3. Pressure differential

1. Drum-type mud filter with variable rotation and flow rate



4. Volume rate of production
5. Blade position.

Our objective was to determine the effects of changes in these parameters on the moisture and alkali content of the lime mud as it drops into the kiln.

Mud cake filter model

Moisture content equations

Within the cake, moist calcium carbonate on the drum filter acts like a porous medium. One can view the cake as a resistance that impedes the flow of water by a pressure differential. Because the lime mud is packed with a greater packing density near the inner radius of the cake, the resistance is nonisotropic, increasing somewhat linerarly towards the inner radius.

Jacobi (*8*) shows the pressure to vary nearly quadratically through the cake. The resistance is caused by the flow of a viscous fluid through the porous channels. The conventional approach to this problem attempts to relate this resistance to void fraction and cake porosity. Here, we seek a more fundamental and less empirical approach.

The equation governing the laminar flow of fluid through a porous medium is the Navier-Stokes momentum equation:

$$\rho[(\delta v/\delta t) + (\vec{v} \cdot \nabla \vec{v})] = -\nabla p + \eta \nabla^2 \vec{v} \qquad (1)$$

The first two terms on the left-hand side represent inertial forces. These forces are quite small compared to the pressure and viscous forces on the right-hand side. The problem degenerates to a balancing of the pressure and viscous forces. The approach taken in most problems of this nature is to view the cake as composed of ducts. A familiar parabolic Hagan-Poiselle flow characterizes the parabolic velocity profile in each duct. The average velocity down any one duct is balanced by the pressure gradient down the duct, with the ratio of the velocity to pressure being the equivalent resistance of the medium (*9*).

The result of this exercise (*8*) is that the gradient of the pressure along the radial dimension is given by:

$$dP/dX = (\eta/r_0^2)v = (\text{resistance})v \qquad (2)$$

where X is the radial dimension through the filter cake. According to Jacobi, the resistance term varies nearly linearly, increasing towards the inner radius. The pressure varies quadratically, increasing towards the inner radius, and the velocity is, for the most part, uniform through the cake. This velocity varies approximately inversely as the square of the thickness of the cake:

$$v \simeq [k/(Th)^2]\,(P_{out} - P_{in}) \qquad (3)$$

In reality, the effective resistance, which we have lumped into the factor k, is quite a complicated parameter. Its value depends on the average size of the pores in the cake, the molecular composition of the medium, the average density of calcium carbonate in the cake, and the ambient temperature. A more detailed discussion of this parameter is found elsewhere (*8*).

After several hours of filter operation, the inner mesh begins to become clogged. There is, thus, a long-term time constant associated with the parameter k. After a period of 1-4 h, the filter is scraped by lowering the blade on to the mesh to clean off some of the residual cake clogging the mesh. The scraping lowers the resistance of the cake considerably. Eventually, however, more clogging builds up, and it cannot be removed by scraping. Every 2-5 days, either the mesh of the filter screen is blown out by high pressure air jets, or the sheet is dropped by releasing the vacuum pressure. In addition to this cleaning, another cleaning occurs every two weeks, in which the screen is sprayed with acid. For the purpose of modeling, these second-order clogging effects occur on a long enough time scale to be ignored.

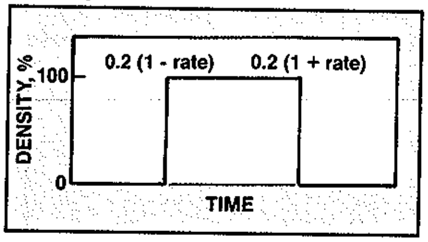
When the filter is operating properly with no buildup of material in the vat, mass conservation dictates that the amount of calcium carbonate entering the vat is in the form of a slurry and that it leaves the filter cake in the form of dried mud. A less efficient means of control is to monitor and adjust the level of the vat. The filter is not operating properly when the vat level rises. When the filter is clean, a higher vat level (commensurate with a higher production rate) will yield a higher moisture mud cake.

The total thickness of the cake can be represented as

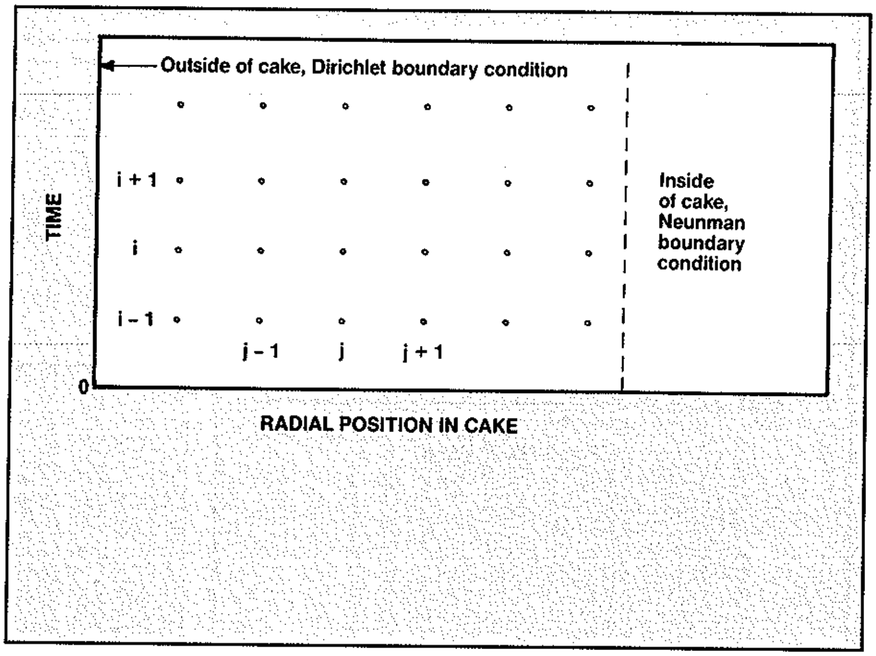
$$Th = (Th_0 + Th_1)\,(\text{production rate/rotation rate}) \qquad (4)$$

where the variable thickness on top of the residual cake (Th_1) is altered by the rotation rate and the production rate. A change in either one of these rates alters the thickness and, thus, changes the velocity of fluid flow through the cake. The equation of momentum has been taken care of in Eq. 1. The equation that must be used to determine the density of the water independent of the flow through the cake is that of mass conservation

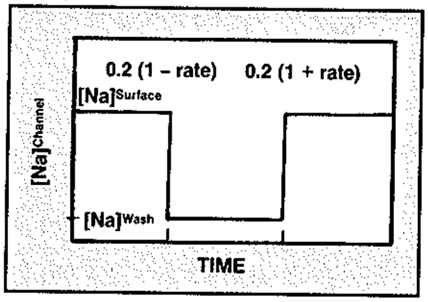
2. Boundary condition on water density at the outer edge of the filter cake vs. time



3. Finite difference grid used to solve for the density of water, where the solution marches up the time axis one row at a time



4. Boundary condition on the sodium channel density at $x = 0$ for all time



$$\partial\rho/\partial t + \vec{v} \cdot \nabla\rho = K_D\nabla^2\rho \tag{5}$$

where

ρ = density of water

K_D = diffusivity of water in the cake.

The characteristic diffusive velocity of the problem is given by the diffusivity D divided by the characteristic length, which in this case is the thickness. The characteristic diffusion velocity is typically a tenth of the effective velocity v

$$K_D/Th = v/10 \tag{6}$$

It is convenient for this problem to declare nondimensional variables for time, space, and velocity as follows:

$\underline{t} = ft$

$\underline{X} = X/Th$

$\underline{v} = v/(Thf)$

$\nabla = \underline{\nabla} \times Th$

$$f = \text{rotation frequency}/2 \tag{7}$$

Here, f is the rotation frequency, X is a dimension through the thickness of the cake, and ∇ is the gradient operator, which in this case is defined as

$$\underline{\nabla} = \partial/\partial\underline{X} \tag{8}$$

Using the nondimensional variables, we now write Eq. 5 as

$$K_D/[f(Th)^2]\underline{\nabla}^2\rho - \underline{\vec{v}} \cdot \underline{\nabla}\rho - (\partial\rho/\partial\underline{t}) = 0 \tag{9}$$

Note that f is the rotation frequency divided by 2. Therefore, the normalized time that it takes for the filter cake to move from the air slurry interface up to the blade where it is taken off is equal to 1.

Equation 9 is a second-order partial-differential equation. At time $t = 0$ when the segment of the filter leaves the slurry vat, the initial condition we use is that the cake moisture is uniform through the thickness of the cake and equal to the moisture content of the vat.

This assumption is reasonably good and was tested by varying the starting densities. We found numerically that 50% variations in starting densities resulted in only 8% changes in the final moisture content. It is still necessary to impose boundary conditions on the inner and outer layers of the cake with time. We shall define the outer portion of the cake as X = 0 and the inner portion as $\underline{X} = 1$. **Figure 2** shows the boundary condition used on the outer radius $\underline{X} = 0$.

The surface concentration of water changes from 0% to 100% when the wash flow is turned on. Increasing the wash flow rate is simulated by varying the time over which this boundary condition is turned on. As the flow rate is increased, the window, over which the 100% condition is turned on, increases proportionally. The wash rate parameter varies from zero to unity.

The problem is laid out as a finite difference grid and is solved implicitly (**Fig. 3**). That is, the system solution is solved step by step through an implicit solution.

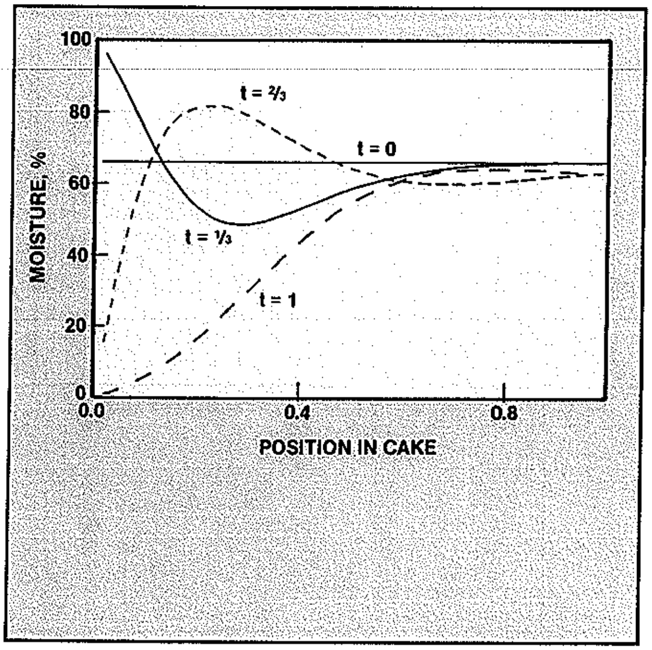
$$+ K_D/[f(Th)^2]\{[(\rho_{i+1,j+1} + \rho_{i+1,j-1}) - 2\rho_{i,j}]/\Delta\underline{X}^2\} - v[(\rho_{i+1,j+1} - \rho_{i+1,j-1})/2\Delta\underline{X}] - (\rho_{i+1,j} - \rho_{i,j})/\Delta\underline{t} = 0 \tag{10}$$

Equation 10 represents one such nodal equation that is written for every point across the thickness of the cake. At the inner radius, we assume a Neunman condition—that is, that the rate of change of slope in the density is zero.

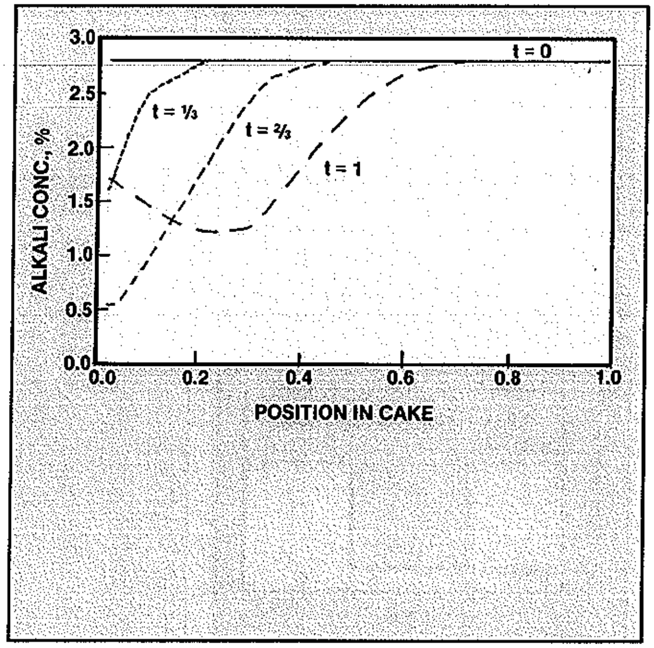
Prediction of alkali content

The prediction of the alkali content in the mud is described by a different set of equations, which is to be expected since sodium is removed from the cake by dissolving. Kuo (*10*) was among the first to describe the filter cake alkali process using the equations of mass exchange. The problem is examined from the perspective of having a fluid moving in the channel, with characteristic velocity v, past a fixed film cake. Ions are exchanged between the

5. Water concentration in the cake vs. the normalized position in the cake for various times



6. Alkali concentration in the cake vs. the normalized position in the cake for various times



fixed cake and the moving fluid. The mechanism of removing sodium is thus one of sodium interchange between a moving channel fluid and a fixed film making up, effectively, the walls of the channel itself. The two equations describing this process are:

$$\partial[\mathrm{Na}]^{cake}/\partial t = k^{cake}\,(\mathrm{Na}^{channel} - \mathrm{Na}^{cake}) \quad (11)$$

$$(\partial[\mathrm{Na}]^{cake}/\partial t) + \vec{v}\cdot\nabla[\mathrm{Na}]^{channel} = k^{channel}\,([\mathrm{Na}]^{cake} - [\mathrm{Na}]^{channel}) \quad (12)$$

We have written the equations for the sodium ion, but they are valid for any ion. In this case, the two variables referred to are the concentration of sodium per unit volume in the cake and the concentration of sodium per unit volume in the channel. Only the channel equation has the convective velocity term, since it is the only one that is moving. We are only interested in the concentration of the cake sodium.

What is the best way to solve this pair of equations? When the boundary conditions are uniform with time, the solution of Eqs. 11 and 12 is that of a pair of Bessel functions using normalized parameters (*10*). On the other hand, when the boundary conditions are varying with time, it is more appropriate to consider a numerical solution paralleling that for water. We choose to solve the pair of equations using the method of characteristics (*9*). This method is simply a technique of defining the rate of change of the variables of interest along certain lines which move with time. Equation 11 is uniform with time and can be described simply as

$$d\mathrm{Na}^{cake}/dt = k^{cake}\,(\mathrm{Na}^{channel} - \mathrm{Na}^{cake}) \quad (13)$$

However, Eq. 12 can be more conveniently described using two equations:

$$d\mathrm{Na}^{channel}/dt = k\,(\mathrm{Na}^{channel} - \mathrm{N_a}^{cake}) \quad (14A)$$

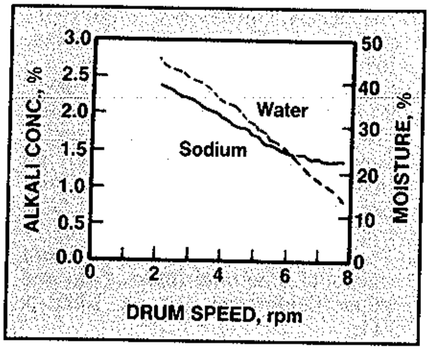
$$dX/dt = v \quad (14B)$$

This pair of equations simply states that the channel sodium changes with time according to Eq. 14A along the line Eq. 14B. One simply moves into a Lagrangian framework to describe the rate of change of the sodium in the channel with time. The problem degenerates to one of record keeping, simply knowing where you are along the path x, so as to appropriately time step Eqs. 13 and 14.

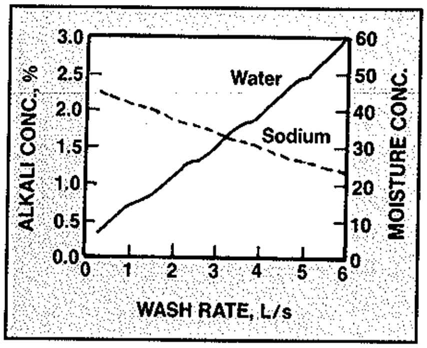
The outstanding question yet to be settled is what the appropriate upstream boundary condition is for both the sodium in the cake and the sodium in the channel (**Fig. 4**). Paralleling the argument above, we assume that the sodium in the channel and the stream are equal to that of the vat at time $t = 0$, the time at which the segment of the cake being analyzed leaves the vat. It is necessary to continually update the upstream boundary condition on the concentration of sodium in the moving channels. During normal conditions outside of the wash flow, the outer rim of the cake is drying with time. The sodium in the cake is diminishing because the concentration of the sodium being convected towards the inner chamber of the drum is less than that of the cake: its sodium content is always a fraction of that in the cake.

It is necessary to have an upstream boundary condition on sodium. This is a solubility condition that answers the question of how much cake sodium is dissolved in the water at the outer portion of the cake. This is an issue of chemical kinetics, and the ratio $[\mathrm{Na}]^{channel}/[\mathrm{Na}]^{cake}$ equals the kinetic reaction coefficient k_s (a function of temperature). We have carried out these coefficients with $k = 1/2$. When the wash flow is turned on, this boundary condition drops to a low value, equal to the sodium content in the wash water. This boundary condition remains on during the time that the outer portion of the cake is bathed by the wash, similar to the condition shown in Fig. 2. Again, we use a normalized wash rate parameter that varies from zero to unity to represent the change in the wash flow rate. The mass exchange coefficient k^{cake} and $k^{channel}$ are functions of the total wash flow. These coefficients must there-

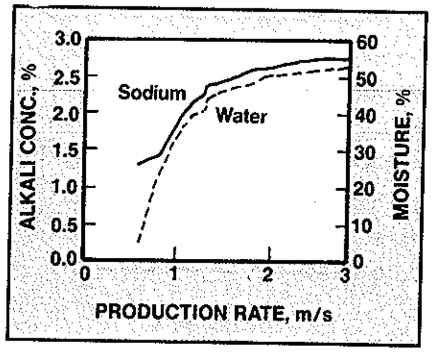
7. Alkali and moisture variations vs. drum speed



8. Alkali and moisture variations vs. wash rate



9. Alkali and moisture variations vs. production volume



fore be increased by a rate proportional to the increase in the volume flow rate through the pores.

Solution

The solution for both the moisture and alkali content of the mud on entering the kiln is simulated by Eqs. 10, 13, and 14. Time $t = 0$ represents the time the cake (fully formed in thickness) leaves the vat. The clock begins incrementally to increase towards time $t = 1$, which represents the point at which the blade removes the cake. At the point of removal, we calculate the average water and alkali content of the mud deposited into the kiln.

This calculation is done simply by averaging over all concentrations from $X = 0$ to the point $X = (Th - Th_0)$. When solving for alkali content, the concentration along the channel after every time step must be shifted inward to account for the convective displacement given by Eq. 14B. For a given number of spatial points N

$$\Delta \underline{X} = 1/N \tag{15}$$

The time step Δt is chosen to be equal to the spatial step size ΔX divided by the characteristic velocity

$$\Delta \underline{t} = \delta \underline{X}/\underline{v} \tag{16}$$

Thus, the total number of time steps is simply equal to 1 divided by Δt or

$$\text{time steps} = \underline{v}/\Delta \underline{X} \tag{17}$$

Results

Figures 5 and 6 show simulation results for water and sodium concentration at normalized times $\underline{t} = 0$, 1/3, 2/3, and 1. Note the predominant effect of the convective term in both figures. In computing average moisture and sodium concentration, we assume throughout that the blade is positioned at $\underline{X} = 1 - Th_0/Th$. We are now in position to compute the desired effects of changes in various variables.

Figure 7 shows the effect of rotation rate on moisture and alkalinity. Because of the quadratic dependence of the velocity v on thickness, increasing the rotation rate must yield a dryer cake with time. The same pressure gradient is removing water from a thinner cake much more effectively over a shorter period of time. This trend is only slightly mimicked in the case of sodium; the mass exchange between the fixed cake and the moving channel stream makes the effective velocity a second-order effect as opposed to first-order one.

The effects of wash rate variations on both moisture content and alkalinity are shown in **Fig. 8**. The reduction in alkalinity caused by increased wash flow is dramatic. As expected, however, this reduction is paid with the rather heavy price of increasing the moisture content of the cake. One control strategy is to increase the rotation rate at the same time one increases the wash flow rate, to offset the increase in moisture content. Uronen (*6*) records on a local basis an 8% increase in moisture for a 5% increase in wash flow rate. These data are a bit higher than our prediction, which on a more expanded scale is 5% for moisture and 5% for the flow rate.

It is important to ask a more basic question of what happens to both moisture and alkalinity as the production rate increases. **Figure 9** shows this effect. Increasing the volume flow will increase the thickness Th_1 and will thus dramatically reduce the effectiveness of the filter in removing both sodium and moisture. At large production rates, the cake begins to saturate in moisture and sodium content.

The effect of increasing pressure is shown in **Fig. 10**. Because of the secondary effect of velocity in Eq. 12 on the sodium in the cake, the effect of pressure in reducing sodium is less than that of the more pronounced reduction seen in moisture content.

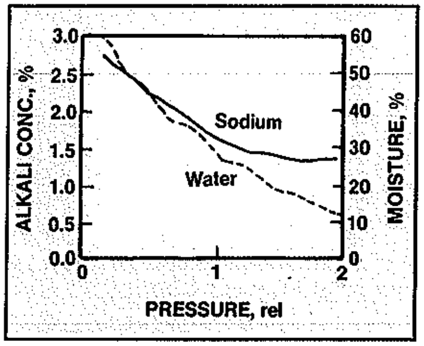
As is readily apparent from these figures, the data by Jacob and Uronen shows *no consistency*. In addition, their results are only semi-empirical and are given over limited ranges. It is our contention that these results are unreliable. It is unfortunate that additional model data are not available in the literature accessible in the field.

Another viable control strategy is to vary the position of the blade. Mass balance dictates that the thickness Th_1 will not vary as long as the rotation rate in the flux of mud into the slurry remains constant. As the blade is lowered towards the filter, the total thickness of the cake changes only because the thickness Th_0 is getting smaller. This reduction in total thickness has a dramatic influence on the velocity of the fluid through the cake. The influence of both as the blade thickness Th_0 is varied is shown in **Fig. 11**.

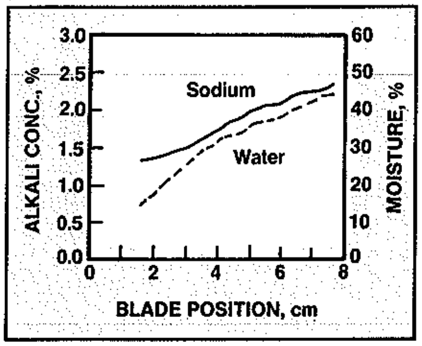
Conclusions

We have presented a model that is capable of predicting both the moisture and alkalinity of a mud cake filter under various operating conditions. The model serves pedagogically to indicate various control scenarios for maintaining the moisture and alkalin-

10. Alkali and moisture variations vs. pressure in the drum



11. Alkali and moisture variations vs. blade position



ity at desired levels. In particular, we see various mechanisms for decoupling the controls to act on both sodium and moisture independently. The fact that increasing wash flow increases the moisture while decreasing the sodium is a predominant focus for the decoupling mechanism.

The magnitude of various changes is also indicated by the models. Changes in pressure and blade height, which alter the velocity of fluid through the cake, have a more predominant influence on moisture than they do on alkalinity. These tools will be useful in developing control mechanisms for the mud cake filter.□

Literature cited

1. Bingham, L. and Angevine, P., Tappi 60(11): 111(1977).
2. Mehra, N., Tappi 62(9): 47(1979).
3. Ashworth, R., Tappi 61(5): 116(1978).
4. Leiviska, K., Modeling and control of a lime kiln, thesis for Licentiate of Technology, University of Oulu, Dept. of Process Engineering, Finland, 1976.
5. Haataja, R., Leiviska, K., and Uronen, P., Modeling and optimization of mud filter and lime kiln, The 5th International IFAC/IMEKO Conference on Instrumentation and Automation, Belgian Institute of Automatic Control, 1983, p. 115.
6. Uronen, P., Modeling and advanced control of lime mud filters, Pulp and Paper Conference, Canada, 1985, p. 611.
7. Sloan, J., Coursey, D., and Drengler, R., 1988 Annual Meeting Conference Proceedings, TAPPI PRESS, Atlanta, p. 67.
8. Jacobi, Kraft recovery system, Ph.D. thesis, Purdue University, Lafayette, Ind., 1973.
9. Melcher, J. R., Continuum Electrical Mechanics, MIT Press, Cambridge, Mass., Ch. 11, 1981.
10. Kuo, M. T., AICHE J. 6: 566(1960).

Received for review Apr. 3, 1988.
Accepted May 24, 1988.