On the theory of thermocapillary, or Marangoni, convection in the capillary water on paper fibers

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ABSTRACT *Marangoni convection as an augmentor of intraweb heat transfer in situations involving a high temperature gradient is usually of fairly marginal importance. Marangoni convection is caused by surface tension (temperature dependent) gradients. Such convection, or similar convection caused by superheated capillary water explosions, may play a more prominent role in certain new drying processes characterized by high moisture contents and very high temperature gradients inside the web.*

KEYWORDS
Capillarity
Drying
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Marangoni
Paper webs
Press drying

The drying process inside the web, even at lower temperatures, is a complex process involving many mutually dependent factors or subprocesses (*1-11*). The authors of pertinent analytical studies (*5, 6, 8-11*) have developed mathematical models for the overall drying process based on differential formulas that represent certain of these subprocesses inside the web, taking into account various conservation principles.

An essential feature in all analyses of drying the paper web via hot surface techniques is the transfer of heat from the hot surface into the web interior. In addition to the vaporization-condensation mode, various researchers have considered thermal conduction through the fiber material, through the vapor, and through the intrafiber or interfiber water, or capillary water, wetting the outside of the fibers. Han (*5*) also considers that convective effects in the vapor ("microconvection") may often be important. The relative magnitude of thermal radiation usually is quite negligible (*5, 12*).

The models depicting the overall thermal conductivity of a wet fiber mat are usually built up on parallel or series arrangements of the three conducting phases: fiber, capillary water, and air-vapor gas. They also may be based on combinations of the parallel and series arrangements, as was the model evolved by Krischer (*12*). Han (*5*) discusses the suitability of several thermal conductivity models to most paper webs.

The possibility that Marangoni convection may augment to a considerable degree the conductive heat transfer through the capillary water in the drying of porous media is briefly brought up by Chen and Whitaker (*13*), but the authors then neglect Marangoni convection in their analysis of a drying situation with small temperature gradients. I observed the presence of Marangoni convection in the capillary water of paper webs subjected to strong temperature gradients (*14*). Quite intense circulation was seen under a microscope in free capillary water on the outside of the fibers near the edge of a paperboard sample when a heated soldering iron was brought into the vicinity. The convection was made visible by having insoluble powder particles in the water. It appeared probable that the water motion was caused only by the Marangoni effect.

In considering heat transfer in a wet paper web under conditions with a high temperature gradient (relative to that prevailing in conventional hot-surface drying), the question that arises is whether the heat flux due to Marangoni convection is proportional to the temperature gradient in the capillary water by a power higher than 1.0. If the power were greater than 1.0, the relative importance of Marangoni convection would grow with increasing temperature gradients, because "conventional" thermal conduction along that water is roughly proportional to the first power of the temperature gradient.

I resorted to an answer based solely on the results of a theoretical solution of the equations depicting Marangoni convection. A detailed solution of the pertinent equations (*15*), though worked out for a geometry and a situation that would have represented the ones I was considering, was too involved and difficult to apply readily. Therefore, I worked out an approximate solution (*16*). However, there are reasons to believe that the Marangoni effect will not be the only one responsible for capillary water convection in a hot web undergoing drying at a high moisture content and a high temperature gradient. This possibility was considered also (*16*).

Marangoni convection in capillary water is caused by a temperature gradient acting in a direction along the surface. Since the surface tension of water depends on its temperature (**Fig. 1**), a capillary surface at a lower temperature will pull on a neighboring surface at a higher temperature and cause convective motion (**Fig. 2**). The intensity of the resulting convection will depend not only on the temperature gradient but also on the viscosity of the water and the geometry of the convective system. The augmentary effect of Marangoni convection on heat transfer in capillary systems can be quite considerable as the convection carries cooler liquid into contact with the hotter surfaces and vice versa.

There are many analyses in the pertinent literature concerning Marangoni convection in specific situations (*15*). Before working out the approximate solution, however, I found no analyses on the extent of Marangoni convection or its effects as it occurs inside a drying paper web. The specific situation in which I thought that Marangoni convection could play an appreciable role was the hot-surface drying of paper coming from the press section at a dry content of about 40%.

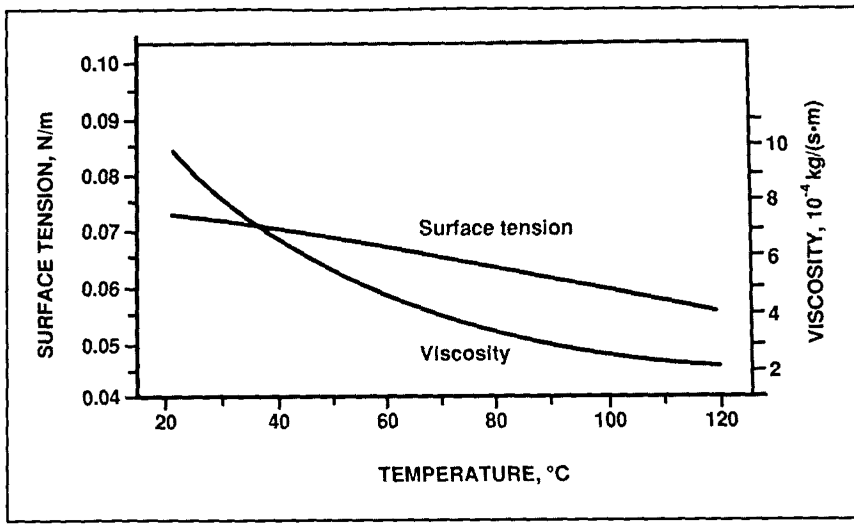
For many stocks, the critical dry content, or the dry content that marks the disappearance of "free water" from the fiber surfaces, is above 50% (*17*), although for certain other stocks the critical dry content could be as low as 32%. It seems, then, that there can be enough free water on the fibers, well into the drying section, at least on some paper machines, to warrant a study of the magnitude of the related Marangoni convection.

The approximate solution

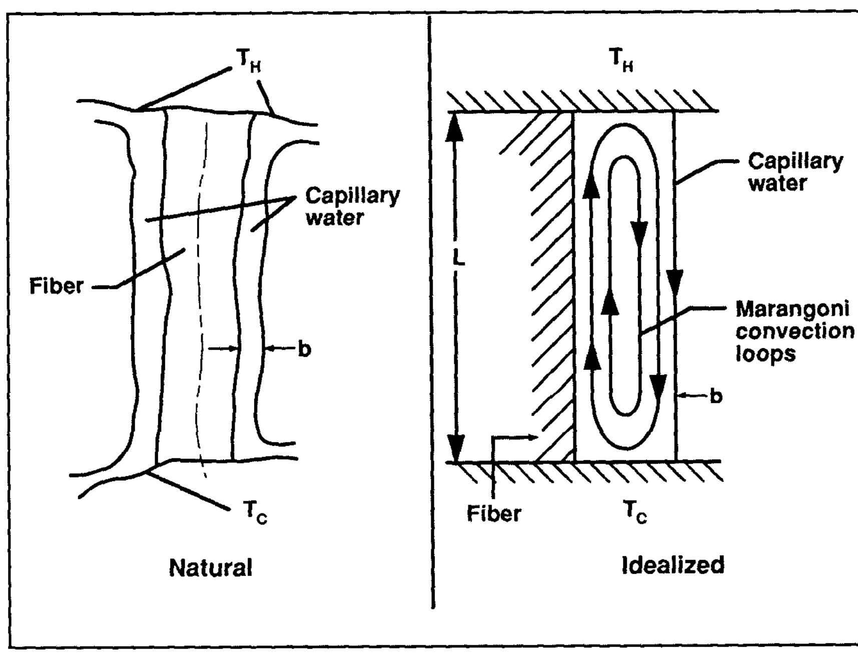
The equations were solved with numerical values depicting Marangoni convection in the axial direction in capillary water on the outside of a fiber, with a temperature difference acting from one end of the fiber to the other (*16*). The situation was idealized in rectangular, two-dimensional coordinates, as shown in **Fig. 3**.

For the computations, the two idealized fibers were designated by the subscripts A and B, respectively. Each wet fiber was assumed to have a dry content of 40% and a critical dry content of 50%. Each fiber was assumed to be made up of an inner

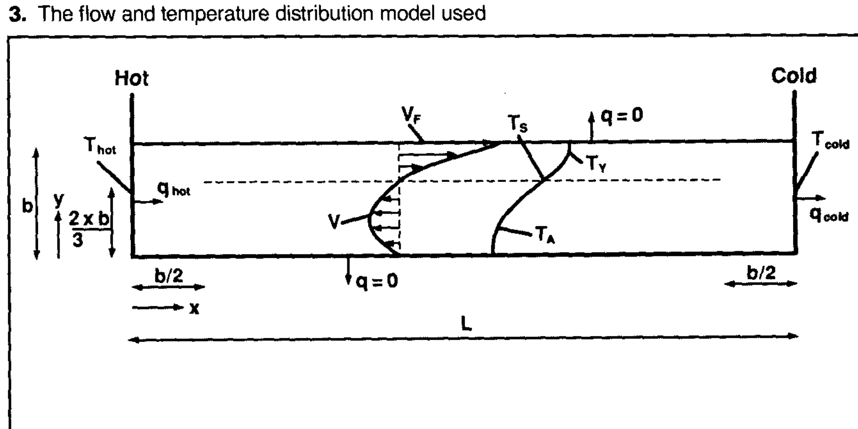
1. The surface tension and the viscosity of water



2. Fiber with free water on its outside and a temperature difference acting from end to end



3. The flow and temperature distribution model used



I. Dimensions of idealized wet fibers for the computations, μm

	Fiber	
	A	B
D_1	15	30
D_2	21.2	40.0
D_3	28.5	55.0
L	10–100	10–100
b	3.66	3.50

cylinder of diameter D_1 composed only of the dry material of the fiber, of "bound water" forming a cylindrical shell of inner diameter D_1 and outer diameter D_2, and finally of an outermost shell of "free water" of inner diameter D_2 and outer diameter D_3. The length of the fiber was L, and the thickness of the free water layer was designated as b. The temperatures of the ends were designated as T_H and T_C, or T_{hot} and T_{cold}. Naturally, in a real fiber, much of the bound water is inside the fiber, and the sharp division to a dry fiber cylinder and a bound water shell was only used in the model to arrive at a reasonable value for D_2. The dimensions chosen for Fibers A and B are given in **Table I**.

Because the mathematical model (*16*) is for two-dimensional, rectangular coordinates, b and L from Table I were treated directly as b and L of Fig. 3 when the mathematical model was applied. Since the b values are almost the same for Fibers A and B, there was no point in doing the calculations for both. The dimensions for Fiber A were used.

Results and indications from the mathematical model

In **Figs. 4 and 5** are shown some of the important results from the model. The R_Q values are rather low over the whole b/L range. (The R_Q value is the effective heat transfer by the Marangoni model divided by the heat transfer by conventional thermal conduction.) This may be explained as follows.

In the capillary water pool undergoing Marangoni convection, there is water motion in one direction (from the hot toward the cold end) at and near the free surface. There is an equal, but opposite, mass flow rate in the inner parts of the water pool, next to the solid surface. In the model of Fig. 3, the dashed line represents a surface of zero velocity. In relation to the flows, this surface could just as well be a real membrane. Essentially, then, we have a counterflow recuperative heat exchanger in the main part of the water pool (except for the end regions). For such a heat exchanger, the effectiveness, ϵ_S, is defined as the real heat transferred from the hot to the cold flow divided by the heat that would be transferred if there were no thermal resistance from the hot to the cold flow.

When ϵ_S is near 1.0 (as is the case here, as shown in Fig. 4), the average temperature of the fluid on the hot side will be only slightly above the average temperature on the cold side. The temperature distribution on each side will be linear from one end to the other. Thus, the average temperature of the water of the colder side approaching the hot end will be only slightly lower in temperature than T_{hot}, and there will be little heat transferred convectively at the hot end into the water. The situation at the cold end is similar.

Figures 4 and 5 show that there is a strong trend toward increasing R_Q as b/L increases. The figures also show that R_Q increases roughly with the square of the temperature difference. These are trends similar to those obtained for Marangoni convection with liquid drops that undergo condensation (*18*). The two curves in Fig. 5 differ only with respect to the temperature at which the process takes place. The higher viscosity at the lower temperature slows down the Marangoni convection and lowers R_Q.

So far, we have examined only the magnitude of the augmentary effect of Marangoni convection on heat transfer along a fiber in a web. For overall heat transfer in a web, the possible augmentary effect at fiber-to-fiber contact points also should be taken into account. For paper webs in the ordinary density range of 400 kg/m^3 to 900 kg/m^3 (*19*), the effective thermal conductivity of paper behaves mostly as if the paper were made up of parallel rods of the conducting phases in the direction of the flux (*5, 19*). In other words, the contact resistances between the fibers are relatively small compared with the resistances along those imaginary rods. Under these circumstances, it is easy to conclude that Marangoni convection in the vicinity of the contact points could contribute little to the reduction of the overall thermal resistance, no matter how intense and effective that convection might be locally.

The conclusion to be drawn is that the augmentary contribution of heat transfer by Marangoni convection is very slight—usually negligible—for paper webs of ordinary density at moisture levels that prevail at the entrance to the conventional drying section, with a temperature gradient in the z direction that does not exceed 2×10^5°C/m.

The possibility that convection caused by explosion may occur in web capillary water

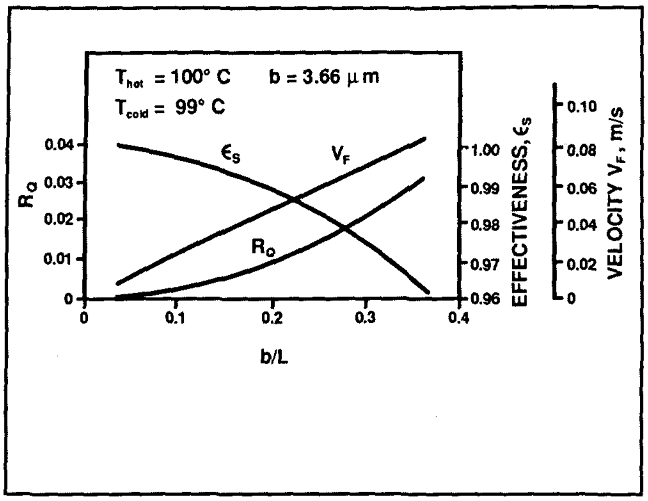
The convection currents observed in the free capillary water in a wet paperboard web subjected to a high temperature gradient were thought to be caused by the Marangoni phenomenon (*14*). There is, however, another possible mechanism for generating such convective capillary currents.

It is possible that water trapped in small intrafiber or interfiber apertures or cavities will be heated to a superheated temperature, with the immediate evaporation of this liquid taking place explosively. In fact, such minute explosions almost certainly do occur in the vicinity of the heated surface.

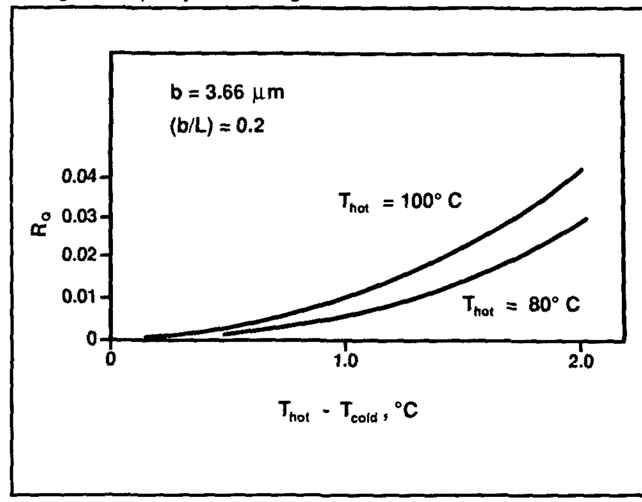
An explosion of this sort may splatter small droplets all around if it is strong enough, but it certainly will cause convective motion in the capillary water continuum where the explosion occurs. The sonic and phase boundary motion effects of bubble formation and growth associated with boiling have been studied extensively (*20*), and similar effects will prevail in the situation considered here.

Some of the convective motions I saw in wet paperboard webs may have been caused by the superheated liquid explosions. A phenomenon that led to this supposition is discussed in the *1989 Engineering Conference Proceed-*

4. R_Q, ϵ_S, and V_F as functions of b/L. ϵ_S is the effectiveness of the heat exchanger on both sides of the dashed line of Fig. 3.



5. R_Q as a function of $T_H - T_C$. R_Q is the ratio of heat transferred by Marangoni convection to that transferred by conventional conduction through the capillary water in Fig. 3.



ings (*16*). The mathematical model analysis shows that with conventional paper or paperboard drying, rapid convection loops set up in the free capillary water on the outside of fibers, caused either by a thermocapillary force or by superheated water explosions, will have only a very slight effect on the overall heat transfer in the web.

A look at the convective current effects in press drying

In a typical press drying situation, such as in a heated platen press experiment (*21*), a Convac (Condebelt) experiment (*14*), or an impulse drying experiment (*22*), the conditions which led to the conclusion that capillary convection currents are of little effect are no longer valid. The density of the web can exceed 1000 kg/m^3, the initial moisture content can be as high as 70%, and the *z*-directional temperature gradient can exceed 2×10^{6}°C/m. Thus, the effective R_Q value, resulting from intense Marangoni convection, can be quite large, perhaps greater than 1. It is also almost certain that superheated water explosions will cause much convective motion then—possibly more than any other force.

Conclusions

Marangoni convection in the free water on the outside of a typical paper web fiber usually has a negligible augmentary effect on the overall heat transfer in the web under the conditions of conventional cylinder drying. At best, the effect is fairly marginal if the cylinder temperature is raised to unusually high levels (perhaps up to 180°C) with a high free moisture content (say, 35% dry content) in the web.

An interesting feature demonstrated by the mathematical model is that there can be rather intense Marangoni convection currents in the capillary water, but these currents do little good in transferring heat from the hotter regions of the web to the colder regions. Since there is motion in one direction in one part of the capillary water cross-section and the same mass rate of flow in the opposite direction in the rest of the cross-section, a counterflow heat exchanger exists. This heat exchanger is highly effective; consequently, the average temperature of the water on the hot side is only slightly above the average temperature on the cold side. The temperature of the cold-side flow approaching the hot end of the convection pool is thus only slightly below that of the hot end, and little heat will be picked up by the rapidly moving convective current at that end.

The relative importance of Marangoni convection under conventional conditions is at best marginal. However, the rate of Marangoni heat transfer from the hot end and to the cold end increases rapidly, with an exponent much higher than one of the ratio of pool depth to pool length, as that ratio increases, if the end-to-end temperature difference remains constant. Similarly, if the geometry remains constant, the net Marangoni heat transferred increases with roughly the second power of the temperature difference.

There is reason to believe that convection currents in the web-free capillary water can also be generated by superheated water explosions. These most likely occur in intrafiber or interfiber cavities near the heated surface. The augmentary effect of the explosion-generated convection currents on the overall heat transfer would suffer also from the effectiveness of the heat exchange between the heated, moving water and the cooler water coming to replace it.

Though there is no definite evidence, the results obtained and the information available suggest that convection currents caused either by Marangoni forces or superheated water explosions do play a role in the heat transfer within the web in various types of press drying or impulse drying. In these, the *z*-directional temperature gradient inside the web can be as high as 2×10^{6}°C/m (about 20 times as high as in intense conventional cylinder drying). In many of these new processes, the moisture content of the web can also be high—perhaps as high as 70%. Both of these conditions would strongly favor the effectiveness of the capillary convection process in transferring heat and also would tend to induce superheated capillary water explosions near the hot surface.□

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