Predicting and verifying the best quality control under particular conditions

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ABSTRACT *By analyzing the measurements from a traversing sensor, the quality of the cross-direction profile control can be predicted and verified. The measured variables are resolved into their frequency component parts, and the signal is brought into correlation with the number of actuators available. A practical application of this method is presented without recourse to the "mathematical rigor."*

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Actuators
Basis weight
Control
Slice

Special analysis establishes the relationships between the behavior of a signal and its representation as the sum of a series of waveforms, each of a different frequency. These relationships can be important to the cross-direction (CD) profile control of web-formed products. Based on these relationships, signals of widely different forms and sources can be analyzed and interpreted with a generalized uniform method. The operator obtains an instrument that is easy to use, allowing him to identify the signal components and take the actions necessary to meet control requirements.

CD profile measurement and spectral analysis

As a parameter is measured, the sensor signals are recorded as sample data in a series. For example, the basis weight CD profile of a paper sheet is represented by dividing the web width into a series of equidistant elements. Each element is assigned a measured value. Depending on the resolution, the profile may consist of 128, 256, or 512 element values.

Such a profile is depicted in **Fig. 1.** At the top is shown a segment of the paper web, and the CD profile is shown below. The variables denote:

$2N$ = the number of elements

$x(\nu)$ = the profile values for each element

$\bar{x}(\nu)$ = the profile mean value

$x_a(\nu)$ = the profile deviations from the mean value.

Let us consider the deviations from the mean profile value. These deviations values in the chosen interval $(1 \ldots 2N)$ can be approximated through a set of trigonometric functions[1]:

(See equation 1)

where

j = imaginary unit

with the circular frequency ω as

$$\omega_0 = 2\pi/(2N\Delta l)$$
$$= \pi/(N\Delta l)$$

where

2π = number of radians in one cycle

Δl = the element width.

The harmonic coefficients in Eq. 1 define a discrete frequency spectrum for the sampled signal $x_a(\nu)$. Therefore, a CD profile measurement can be represented as the superposition of sine and cosine terms (Fourier analysis).

Power spectrum of a CD profile

In physics, the expression $x_a^2(\nu)$ often stands for a generalized form of power (energy). This is true when x_a represents the electrical current through a resistor. Its instantaneous power is then proportional to x_a^2, and the total power consumption is proportional to

$$\sum x_a^2$$

This energy or power consideration could be quantified as the mean square of a sampled signal (or the profile).

According with Eq. 1 follows the equation:

(See equation 2)

Due to the orthogonality property of the trigonometric functions, all sums vanish except those with terms a_n^2 and b_n^2 (i.e., $m = n$).

From Eq. 2, it follows that

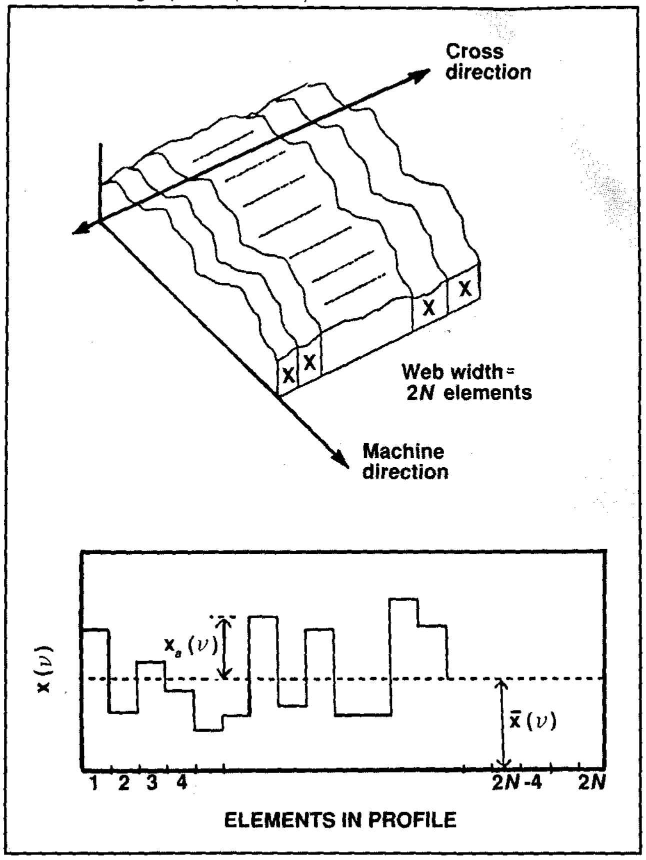
$$\bar{x}^2(\nu) = \sum_{\nu=0}^{2N-1}{}' \frac{1}{2}\left(a^2_\nu + b^2_\nu\right)$$
$$= 2\sum_{\nu=0}^{2N-1}{}' |c(\nu)|^2 \qquad (3)$$

The physical meaning of $c(\nu)$ is obvious. It is the contribution to the mean square of x_a, coming from the component with the frequency $\nu\omega_o$. For each frequency component $\nu\omega_o$ within the measured profile, an equal amount of actuator energy is needed to eliminate exactly that particular wave in the profile. This part expressed by $|c(\nu)|^2$ is measurable and is therefore strictly determinable.

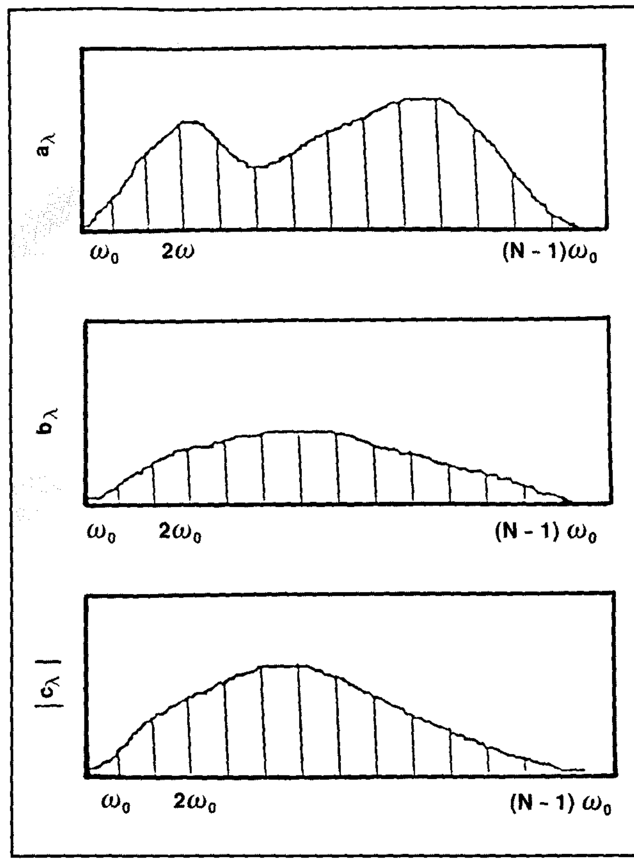
The coefficients a_λ and b_λ in the Eqs. 1-3 describe the discrete line spectrum of each harmonic, and their sum total yields the discrete power spectrum:

[1]Leonard, W.: Statistische Analyse linearer Regelsysteme, Teubner Studienbucher, Teubner-Verlag, Stuttgart, 1973.

1. The web width divided into 2N equal elements x (top) and a sample CD basis weight profile (bottom)



2. Discrete line spectrum of sampled signal



$$a_\lambda = \frac{1}{2N} \sum_{\nu=0}^{2N-1} x_a(\nu) \cos\left(\lambda \frac{\nu\pi}{N}\right) \quad (4)$$

$$b_\lambda = \frac{1}{2N} \sum_{\nu=0}^{2N-1} x_a(\nu) \sin\left(\lambda \frac{\nu\pi}{N}\right) \quad (5)$$

where

$\lambda = 0, \pm1, \pm2, ..., \pm N$

$c_\lambda = a_\lambda - jb_\lambda$

These properties are illustrated in **Fig. 2.**

The computed power spectrum is equal to the standard deviation, assuming that the profile mean value is zero (Parseval's theorem):

$$\bar{x}_a^2 = |c_\lambda|^2 = \sigma^2 \quad (6)$$

with

$$\sigma = \sum_{\nu=1}^{2N} \sqrt{\frac{1}{2N} (x_a(\nu) - \bar{x}_a)^2} \quad (7)$$

where $\bar{x}_a$ is the profile mean value.

Analyzing CD profile signals with Fourier transformation

The method of spectral analysis can be employed efficiently for CD profile control. In all cases, the variable to be controlled is measured and recorded as a sequence of sampled values. This recorded signal can be passed through a frequency analyzer to be broken down into its frequency components.

The frequency analyzer, realized as a program algorithm, computes the power spectrum $|c(\nu)|$ as well as the Fourier forms existing in the sampled signal.

According to Shanon's theorem, the highest harmonic is determined by the web-width subdivision in the $2N$ equal elements:

$$\omega_S = 2\pi/L \quad (8)$$

Then, since $L = 2N\Delta l$,

$$\omega_S = 2\pi/(2N\Delta l) \quad (9)$$

where

L = the web width

$2N$ = the number of the elements in cross direction

Δl = the width of each element.

The information obtained through this analytical method can be used for predicting or confirming the achievable control quality, **Fig. 3.**

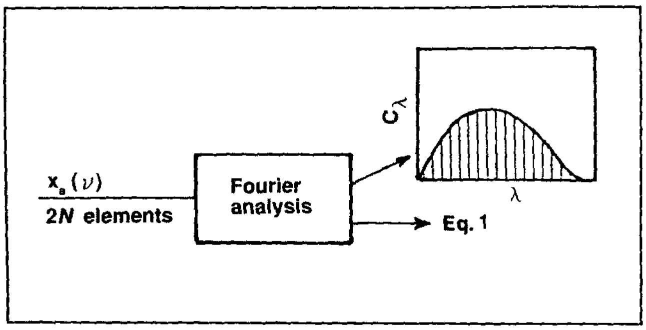
A criterion for quality control

After preprocessing by the frequency analyzer, the measured signal consists of a number of various sinusoidal frequencies. Each of these contributes with its energy to the total power spectrum or to the profile variances from a certain reference value (such as a mean value).

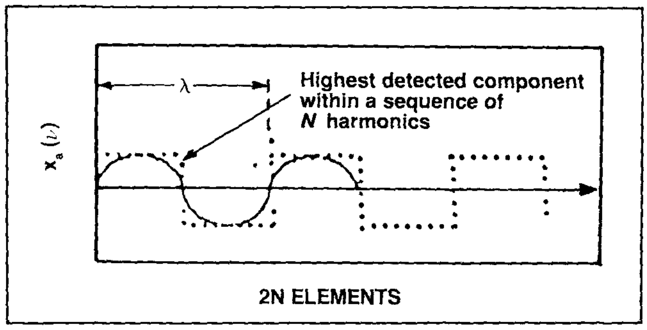
From a CD profile consisting of $2N$ elements, a series of N harmonics is obtained. To influence these harmonics in some way, we have to use a certain minimum number of actuators.

This means that for $2N$ elements, $2N$ actuators must be available. The reverse is also valid: if N actuator units are available, then up to $N/2$ harmonics can be controlled.

3. Fourier analysis realization of a sampled signal



4. Situation with 2*N* element resolution



Equations:

$$x_a(\nu) = \sum_{\lambda=-N}^{N} a_\lambda \cos\left(\lambda \frac{\nu\pi}{N\Delta l}\right) + \sum_{\lambda=-N}^{N} b_\lambda \sin\left(\lambda \frac{\nu\pi}{N\Delta l}\right) = \sum_{\lambda=-N}^{N} c_\lambda \cdot e^{j\lambda(\nu\pi/N)} \qquad (1)$$

$$\bar{x}_a^2(\nu) = \frac{1}{2N} \sum_{\nu=0}^{2N-1} \left\{ \left[\sum_{n=-N}^{N} a_n \cos\left(n \frac{\nu\pi}{N}\right) \cdot b_n \sin\left(n \frac{\nu\pi}{N}\right) \right] \times \left[\sum_{m=-N}^{N} a_m \cos\left(m \frac{\nu\pi}{N}\right) \cdot b_m \sin\left(m \frac{\nu\pi}{N}\right) \right] \right\} \qquad (2)$$

$$x_a(\nu) = \sum_{\lambda=-32}^{32} \left[a_\lambda \cos\left(\lambda \frac{\nu\pi}{N}\right) + b_\lambda \sin\left(\lambda \frac{\nu\pi}{N}\right) \right] \qquad (10)$$

The statement can be proven by an example. Assume that a basis weight CD profile is measured with a resolution of 256 elements. Its frequency analysis will yield 128 harmonics.

(See equation 10)

for $\nu = 1, 2, ..., 256$, and $\omega = 32\ (\nu\pi/N)$. When the power spectrum is depressed nearly to zero for all harmonics up to the 32nd, further efforts to improve control quality will be unsuccessful and will only be a waste of time and energy.

This situation is represented in **Fig. 4**. In this case, an actuator unit must operate on each partial half-wave, to compensate the deviation from the reference value. The effectiveness of control depends, however, on the equidistant sampling of the measured signal. This means that an increase in the number of elements for profile resolution results in a greater number of identifiable harmonics; an increase in the calculated variance should also be expected.

To demonstrate, let us consider an example from actual practice. The results in the example were obtained from the Lippke CD profile control installed in the Haindl No. 6 paper machine at Schongau. The CD control for basis weight has been designed to control a paper web 6 m wide that has been divided into 128 data boxes. For this control, 63 slice actuators were available. The design goal was a 2σ minimizing and a halving of the peak-to-peak value, both in the cross direction.

The demand for minimizing is in fact a good criterion for property variations in the cross direction, but it is by no means an adequate one unless the CD profile resolution has been taken into consideration in light of the number of actuators available. The spectral analysis method offers, then, the final control criterion needed to make a decision when the control algorithms have reached their peak performance as limited by the number of actuators installed.

With a resolution of 128 CD elements, the Fourier transformation yields a power spectrum of 64 components. Accordingly, the existing actuators can in the best case depress the first 32 harmonics. Only the 2σ part contributing to these harmonics can be compensated, because the number of the 63 actuator units will hardly be sufficient for more harmonics control.

The situation is depicted in **Figs. 5-7**. The first analysis (Fig. 5) in the phase with CD control off yields a total 2σ of 1.07. After the control was switched on (Fig. 6), the CD variance decreased to 0.98. The study of the obtained spectrum reveals an interesting situation.

While the harmonics up to the 32nd order and their contribution to the 2σ-value come under lively control changing shape and magnitude, the part beyond remains almost stable. A stepwise control optimization followed by spectral analysis led after 60 min to the situation shown in Fig. 7. The 2σ has been reduced to 0.72. The associated power spectrum has been depressed in the 32 first harmonics to very low values. However, the part beyond still remains high, and the high magnitude of the 37th harmonic especially remains unchangeable. This harmonic is responsible for the significant increase in the 2σ value.

After this phase, further control optimization had brought no further improvement. The final results show a reduction of 50% in 2σ. That was the point the control algorithms had reached with the 63 available units. Their maximum performance had been reached, and thus their capabilities were exhausted.

The applied method is also used as a predictive tool to overcome problems mentioned by Slowik.[2] The method offers the information needed to attain realistic goals for a certain CD control. The usefulness in that case is twofold:

1. Saving time to learn the dynamical behavior of the plant to be controlled
2. Offering realistic specification requirements even in quantitative form concerning the reductions of CD variability under certain conditions (i.e., actuator number vs. measured elements).

For these purposes, the measured profile is analyzed and transformed in its harmonics. Based on the obtained 2σ and power spectrum (the situation as represented by Fig. 5), the quality of control can be predicted. Once the purchaser has defined his requirements for the number of actuators, the distance can be calculated. To do so, a straightforward reversing of the calculation presented here is necessary.

For the No. 6 paper machine, 74 actuator units should have been provided to eliminate the most disturbing 37th harmonic and thus decrease the 2σ to 0.4.

Application on a paper machine

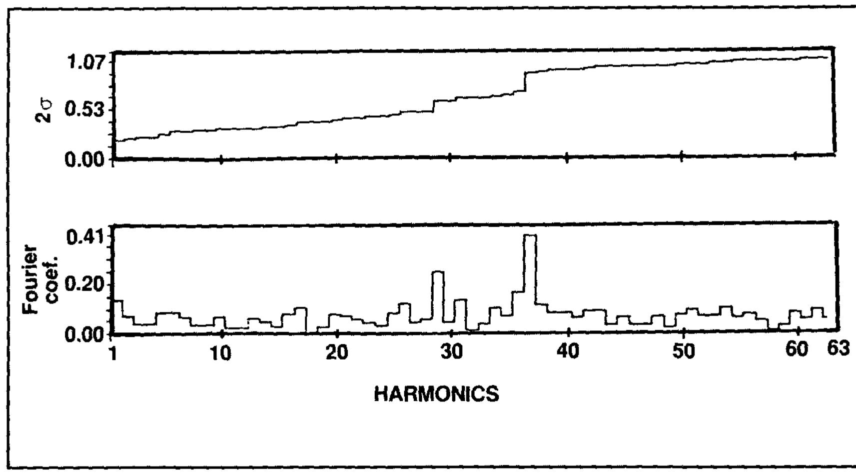
Spectral analysis can be applied efficiently to CD profile control to make predictions in advance or to confirm the accuracy of the control results.

The method presented has been successfully applied to the CD profile control for basis weight on paper machine No. 6 at the Haindl paper mill at Schongau.[3] The method can be an answer to the problems reported

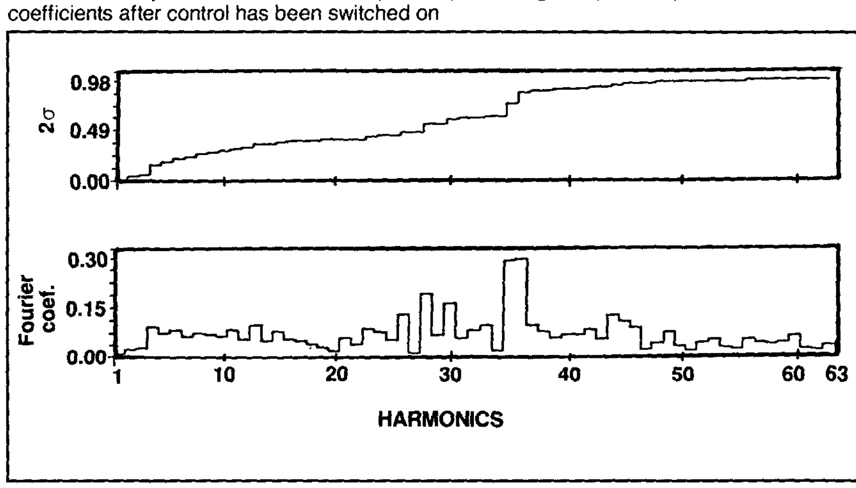
[2]Slowik, M. A., Tappi J. 72(6): 57(1989).

[3]Haindl Schongau PM6: trim = 6000 mm, speed = 1250 m/min, basis weight = 40-54 g/m², newsprint production = 160,000 tons/year.

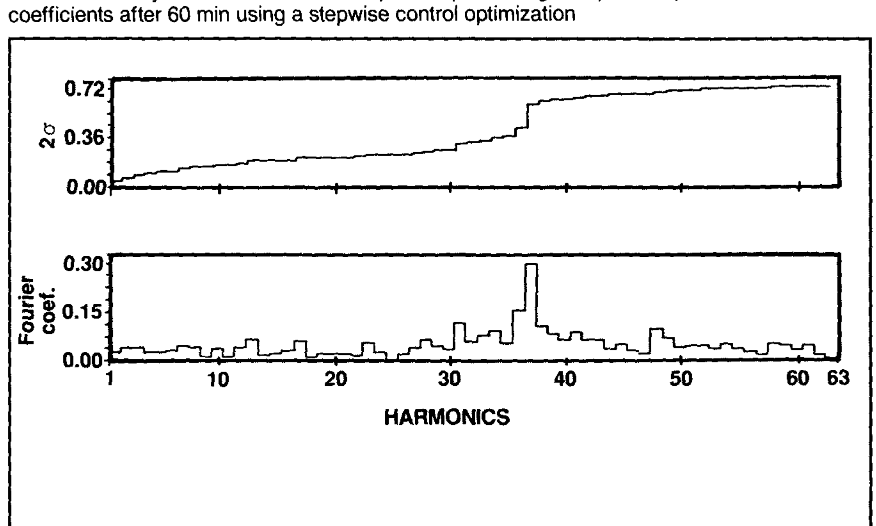
5. Fourier analysis results on the CD profile with 64 actuator units and 128 data elements. The upper illustration shows the power spectrum (2σ) computed with the control not active. The lower illustration shows the obtained Fourier coefficients.



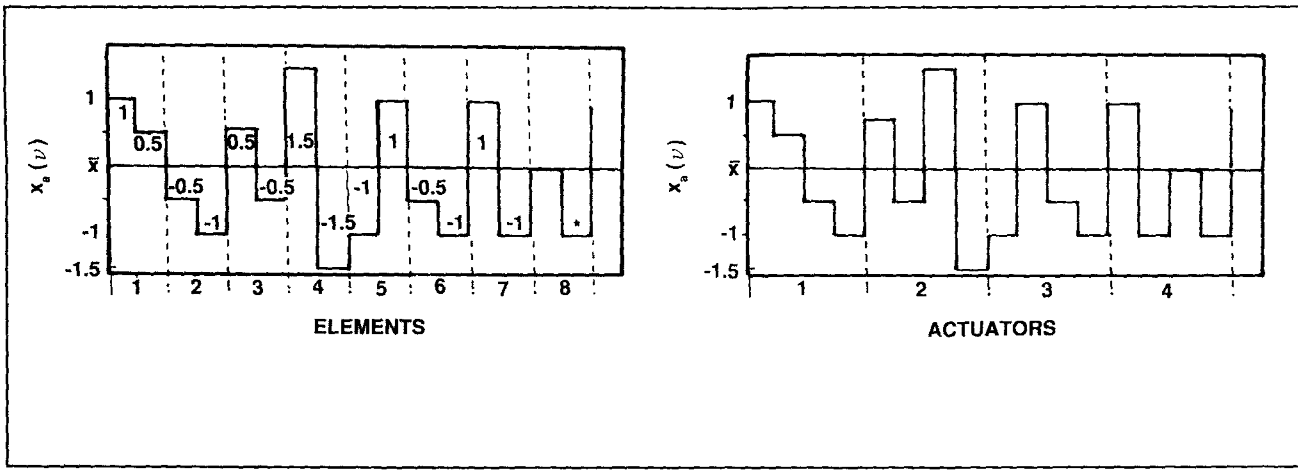
6. Fourier analysis results on the CD profile, presenting the power spectrum and Fourier coefficients after control has been switched on



7. Fourier analysis results on the CD profile, presenting the power spectrum and Fourier coefficients after 60 min using a stepwise control optimization



8. Computational differences in the CD variance between an element-mapped profile and an actuator-mapped profile



by Slowik,[2] who described the difficulties in specifying requirements for CD control and in quantifying the results.

By utilizing the signal analysis results, the optimum number of actuators required for a target control quality can be specified. Alternatively, with a set number of actuators, the best achievable control quality can be calculated in advance.

The method can also be used as an effective tool in regard to control guarantees given to the customer. For example, a reduction of the CD profile variance to much less than 50% in the region where the actuators can be effective still makes a reasonable safeguard for a guarantee.

Finally, there are differences between CD profile resolution with respect to the number of actuator units and the resolution in its CD elements. In any case, an "actuator-mapped" CD profile always shows much less CD variance than an "element-mapped" profile.

Generally the variance in the cross direction behaves proportionally to the profile resolution. A decrease of resolution leads to a decrease in the cross variance and vice versa. The reason for this lies in the average building and the resulting suppression of counteracted deviations with different direction. These deviations do still exist in the actuator mapped representation, but their appearance in the mathematical calculation is suppressed although the profile offers just the same quality as the "element mapped" CD profile.

To show this, the two kinds of profile representation are depicted in the **Fig.** 8. For the element-mapped profile, the variance is

$$\begin{aligned}\sigma_{el} &= [(1/2N)\Sigma\,(\overline{x} - \overline{x}^2)]^{1/2} \\ &= [(1/8)\cdot 1.93]^{1/2} \\ &= (0.24)^{1/2}\end{aligned}$$

For the actuator-mapped profile, the variance is

$$\begin{aligned}\sigma_{act} &= [(1/2N)\Sigma\,(x - \overline{x})^2]^{1/2} \\ &= [(1/4)\cdot 0.19]^{1/2} \\ &= (0.049)^{1/2}\end{aligned}$$

These simple arithmetic computations yield

$$\sigma_{el} > \sigma_{act}$$

because the differences with different signs counteract each other.

During the CD profile application on the PM 6, a reduction of the element-mapped 2σ to 30-40% was always achieved in a short period of time. An actuator-mapped variance reduction to less than 50-40% was achieved without any problems.

Summary

The spectral analysis offers a decision criterion for CD profile control. The importance of this criterion is twofold:

1. It can be used to predict the expected control results with the actuator system available and is helpful in preparing specifications.
2. It can be used as a criterion for confirming the quality of the control quality.

This analysis can be applied in reducing CD variance. As a result of its effectiveness, the method has become a fixed component of our CD profile control system.□

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