Computerized order combining for the three-knife corrugator

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Combining the benefits of sequential search and linear programming helps producers of corrugated boxes achieve their operating objectives.

One question frequently asked by producers of corrugated boxes has to do with the value of the third cutoff knife. It is clear that the addition of a third knife involves a significant capital cost and makes combining orders for the corrugator much more complicated. The major advantages of the third knife include reduced inventory and material-handling costs as well as reduced side trim. The producer must evaluate these advantages quantitatively to make a realistic return-on-investment calculation. The only practical way to do this is to use an effective computerized procedure that is capable of generating high-quality solutions to three-knife corrugator trim problems.

This paper describes a linear programming-based (LP) procedure for solving three-knife corrugator combining problems. This program is currently in use at a number of box plants around the world. These are plants that routinely run only one stock size on three-knife corrugators which are capable of processing rolls from a maximum of 2400-2600 mm (98-106 in.) in width.

Three-knife combining problem

Consider the problem of combining a set of customer orders with nominal requirements of R_i lineal meters of a blank of width W_i millimeters from a single stock size with a maximum usable width S millimeters. Let RL_i and RU_i be the lower and upper bounds on the number of meters required to satisfy order i. Commonly, RL_i could be equal to R_i and RU_i could be 110% of R_i.

If there is only one stock size, the important corrugator combining objective of minimizing the number of stock size changes vanishes. The other common corrugator combining objectives described below remain intact.

Minimize slitter changes

Each slitter change represents a possible disruption in the production process and the source of a possible error in the blank size or the location of the scores. Using a three-knife corrugator, it is theoretically possible to complete three orders in a single setup. This means that a pool of 24 orders could be produced in as few as eight slitter setups. An LP solution which guarantees minimum trim loss will generate one pattern per order size. The maximum number of slitter changes required for a pool of 24 orders should not exceed 24. Because each order is satisfied if the quantity scheduled is between RL_i and RU_i, small usage combinations from an LP solution can usually be discarded and the lost production picked up by increasing the use of other combinations containing the affected sizes. Therefore, an LP solution will generally involve less than one setup per size.

Minimum run between slitter changes

Minimum run between changes is especially important if the slitting and scoring heads are set manually. If slitter changes are made in rapid succession, the setup person will not have enough time to complete the required setups, and the corrugator will have to be slowed down or stopped to make time for the setup. This loss of production can be avoided by specifying a minimum run for each slitter setup. From an LP standpoint, a minimum run can be a major stumbling block and may result in underrunning one or more orders. For example, suppose the minimum run between slitter changes is 500 meters (1700 ft). Order A, which can be satisfied by producing between 1600 and 1760 lineal meters, appears twice in Setup 1, using 600 m, and twice in Setup 2, using 250 m. The total production of 1700 meters satisfies the requirement for Order A, but Setup 2 cannot be used and must be reduced to 0. If the use of Setup 1 cannot be increased by at least 200 m, then either a new setup must be generated to complete Order A or only a portion of Order A will be produced.

In an LP model, it is not possible to force a variable to be either 0 or greater than 500. The issue of a minimum run between slitter changes can only be dealt with indirectly by limiting the number-out of an order in a setup and restricting the orders that can be combined. The importance of controlling the number-out is easy to establish. Suppose the minimum run is 500 meters and Order B requires between 600 and 660 lineal meters of production. Any setup containing Order B at two or more out is guaranteed to be below the minimum run between slitter changes. The importance of restricting certain orders from being combined in the same setup is easy to establish also. Suppose Order C requires between 600 and

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660 lineal meters and Order D requires between 700 and 770 lineal meters. Clearly, any setup containing both Orders C and D either will be below the minimum run between slitter changes of 500 meters or result in Order D being produced partially.

Minimize side trim

Minimizing side trim is easily accomplished using LP. The primary benefit of using LP is that the LP minimum trim loss solution establishes whether or not the orders will combine in an acceptable manner. Subject to minor adjustments in trim loss made possible by producing order i in the interval from RL_i and RU_i, the LP solution indicates the best trim loss that can be attained. If the LP solution results in an unacceptable amount of trim loss, then there is no way to combine the order pool in question. This is very important because it eliminates the possibility of wasting time searching for a solution that does not exist.

The LP solution also provides information through the shadow prices or dual variables regarding which orders are difficult to combine and which are easy. The shadow prices can be used in a sequential procedure to help avoid making the mistake of combining orders one way when they must be combined in another to attain a reasonable level of side trim.

Order contiguity

Keeping all the blanks for each order localized in the run is important in virtually all corrugated plants and especially so in plants which make extensive use of conveyors. Completing each order as quickly as possible once it has been started can be a difficult task because the timing of order completion cannot be controlled directly in an LP solution procedure. Minimizing the number of slitter changes makes it easier to achieve this objective because it limits the average number of setups in which each order can occur. If 24 orders are satisfied using eight setups for a three-knife corrugator, each order must be completed in one setup and order contiguity is no problem. At the other extreme, if 24 orders are satisfied using 24 pattern setups with each using all three knives, on average each order will be in three setups. In this case, order contiguity may be impossible to attain because there may be no way to sort the setups so that there are no large gaps between the setups which contain the first and last occurrence of each order.

I. Pool of orders to be combined

Order number	Order quantity	Width, mm	Length, mm
1	13,333	1608	916
2	4,050	736	1,166
3	1,000	686	1,226
4	3,500	602	772
5	500	576	1,784
6	5,000	550	890
7	2,000	546	660
8	3,000	533	1,182
9	2,000	530	1,816
10	1,000	526	1,664
11	700	499	1,214
12	1,000	495	1,240
13	500	489	1,330
14	14,000	486	1,182
15	4,500	483	1,052
16	5,000	478	1,192
17	2,000	474	1,258
18	8,000	469	1,182
19	5,000	467	1,182
20	11,000	456	1,346
21	1,000	452	1,276
22	4,000	450	1,326
23	30,000	446	1,126
24	2,000	445	660
25	11,700	426	1,344
26	1,000	411	878
27	5,000	393	1,068
28	6,000	392	1,142
29	8,000	390	1,180
30	2,000	367	1,047
31	5,000	340	1,276
32	10,000	308	842

Solution procedure

The following procedure for solving the three-knife corrugator combining problem is similar to the new approach for solving paper machine roll trim problems (*1*).

Step 1

Solve the following LP problem using a Gilmore-Gomory-type (*2*, *3*) LP algorithm. Minimize $\Sigma_j X_j$ subject to $\Sigma_j A_{ij} X_j = RU_i$ for all i ($X_j \geq 0$), where $\Sigma_i A_{ij} W_i \leq S$ for all j [$\Sigma_i \delta(A_{ij}) \leq 3$] and $\delta(a) = 0$ (if $a = 0$) and $\delta(a) = 1$ (if $a > 0$).

The LP solution establishes whether or not the order pool has an acceptable solution. If not, the nature of the problem can be easily identified and steps taken to adjust the order pool before proceeding. If the LP indicates an acceptable solution may be possible, the shadow price, U_i, for each order is saved. The shadow prices are valuable because they identify which sizes are easy to combine and which are difficult. Orders for which the ratio U_i/W_i is larger are difficult to combine, and those for which this ratio is smaller are easier.

Step 2

Starting with the orders which require the fewest lineal meters, search for setups which complete at least two orders. Note that the LP solution will have one setup per order. Because, at most, three orders can be combined, any setup which completes at least two orders can partially produce, at most, one order. Order contiguity also is satisfied because any setup that has only one partially produced order can be sorted so that it is adjacent to setups which produce the balance of that order.

The risk in generating setups in this sequential manner is that one of the first setups may contain all easy-to-combine sizes which are needed to trim the difficult-to-combine sizes. The constraints of the dual to the LP problem given above can be used to help eliminate this risk.

The dual constraint is $\Sigma_i A_{ij} U_i \leq 1$.

For all the setups in the optimal LP solution, this relationship is satisfied as an equality. To eliminate accepting setups which contain only easily combined orders (those which have small values of U_i/W_i), reject any setup

II. Setups to be used

Setup	Usage	Trim loss	Order number	Size		Number out	Cuts	Percent of order
1	916	31	9	526	1664	2	550	110.0
			16	478	1192	2	768	30.7
			26	411	878	1	1043	104.3
2	1452	25	7	546	660	1	2200	110.0
			16	478	1192	3	1218	73.1
			24	445	660	1	2200	110.0
3	682	23	12	495	1240	2	550	110.0
			13	489	1330	1	512	102.6
			17	474	1258	2	542	54.2
4	1152	38	4	602	772	2	1492	90.4
			17	474	1258	1	915	45.8
			30	367	1047	2	1149	110.0
5	625	48	4	602	772	1	809	24.5
			11	450	1326	4	471	47.1
6	935	25	4	576	1784	1	524	104.8
			11	499	1214	1	770	110.0
			22	450	1326	3	705	52.9
7	3901	19	8	533	1182	1	3300	110.0
			9	530	1816	1	2148	107.4
			19	456	1346	3	2898	79.0
8	3467	37	6	550	890	1	3895	77.9
			18	469	1182	3	2933	110.0
			20	456	1346	1	2575	23.4
9	1335	20	6	550	890	1	1500	30.0
			27	393	1068	4	1250	100.0
			32	308	842	1	1585	15.9
10	1649	29	19	467	1182	1	1395	27.9
			23	446	1126	3	1464	14.6
			32	308	842	2	1958	39.2
11	4555	39	14	486	1182	4	3950	110.0
			19	467	1182	1	3850	77.0
12	1042	35	15	483	1052	5	990	110.00
13	2805	20	23	446	1126	3	2491	24.9
			28	392	1142	2	2456	81.9
			32	308	842	1	3331	33.3
14	1928	24	1	1608	916	1	2104	15.8
			25	426	1344	1	1434	12.3
			28	392	1142	1	1688	28.1
15	10384	26	1	1608	916	1	11336	85.0
			25	426	1344	1	7726	66.0
			29	390	1180	1	8800	110.0
16	1349	34	3	686	1226	1	1100	110.0
			21	452	1276	1	1057	105.7
			25	426	1344	3	1003	25.7
17	5194	36	2	736	1166	1	4454	110.0
			23	446	1126	3	4612	46.1
			31	340	1276	1	4070	81.4
18	1824	18	23	446	1126	4	1619	21.6
			31	340	1276	1	1429	28.6
			32	308	842	1	2166	21.7

which completes two or more orders but does not satisfy the following lower bound, $\Sigma_i A_{ij} U_i \geq 0.95$.

Step 3

After all the setups which complete two or more orders have been found, combine the remaining requirements using the same LP solution procedure described in Step 1. If no satisfactory solution is found, drop the last setup found in Step 2, add these requirements to the remaining set of requirements, and resolve the LP. Repeat this process until a satisfactory LP solution is found for the remaining requirements. A satisfactory LP solution must be found when all the setups are dropped because it will be the same as the solution found in Step 1.

Step 4

Add all setups which meet the minimum run length requirement found in Step 3 to those remaining from Step 2 (those that were not dropped in Step 3). Use the difference between RU_i and L_i for all orders to increase the use of the active setups to drive those orders whose production is below RL_i as close to RL_i as possible. If all orders are produced between RL_i and RU_i, go to Step 6; otherwise go to Step 5.

Step 5

Attempt to generate new setups for those orders whose procedure is below RL_i. Combine those orders with other orders whose production is below RL_i and any orders whose production is above RL_i but where RU_i minus current production is greater than the minimum run between slitter changes.

Step 6

Sort the setups to keep each order in adjacent setups if possible. There are many different ways to do this. One effective way is to count the number of setups in which each order appears. Those that appear in only one setup are not a problem. Those orders that appear in two or more setups should control the way the setups are sorted. First bring together those setups which contain orders that occur in only two setups, and then focus on orders that appear in three setups and so forth. This will tend to minimize the number of orders which do not appear in adjacent setups.

Example

An actual order pool (with size in millimeters) is shown in **Table I**. Each order can be overrun up to a maximum of 10%. The orders are to be combined using a single roll-stock size of 2450 mm (100 in.). The minimum acceptable side trim is 18 mm (0.75 in.), and the maximum is set at 50 mm (2 in.). The objective at this plant is to keep side trim well below 2%.

A solution for this order pool using the procedure described above is shown in **Table II**. The setups have been sorted to achieve order contiguity. Setups 1-9 and 16 were generated in Step 2. Note that Setups 1, 2, 3, 6, 7, and 16 each contain two complete orders. Setups 4, 5, 8, and 9 do not contain two complete orders. Each of them, however, did complete two orders. This can be seen by observing Setups 3 and 4. Setup 3 contains two complete orders and a portion of the order requiring a blank which is 474 × 1258. The balance of this order is contained in Setup 4 along with another complete order. Setups 10-15, 17, and 18 were generated by solving an LP as described in Step 3. In this case, the setups with usage levels above the minimum run between slitter changes of 400 meters completed all of the remaining orders within the 10% overrun limit, and no additional setups had to be generated. There are no incomplete orders. The total run is for 45,191 meters, and the trim loss is 1.16%. The number of setups is 18, which is only 7 more than the theoretical minimum of 11.

Table III shows a summary by order which indicates the percentage of each order's nominal requirement that has been produced and the setups in which each order is produced.

Note that only two orders (446 × 1126 and 308 × 842) are not completed in adjacent setups. Of the 30 other orders, 19 are completed in one setup, 10 are completed in two adjacent setups, and one is completed in three adjacent setups.

III. Order summary

Order number	Order quantity	Width, mm	Length, mm	Percent made	Setups
1	13,333	1,608	916	100.8	14,15
2	4,050	736	1,166	110.0	17
3	1,000	686	1,226	110.0	16
4	3,500	602	772	108.3	4, 5
5	500	576	1,784	104.8	6
6	5,000	550	890	107.9	8. 9
7	2,000	546	660	110.0	2
8	3,000	533	1,182	110.0	7
9	2,000	530	1,816	107.4	7
10	1,000	526	1,664	110.0	1
11	700	499	1,214	110.0	6
12	1,000	495	1,240	110.0	3
13	500	489	1,330	102.6	3
14	14,000	486	1,182	110.0	11
15	4,500	483	1,052	110.0	12
16	5,000	478	1,192	103.8	1, 2
17	2,000	474	1,258	100.0	3, 4
18	8,000	469	1,182	110.0	8
19	5,000	467	1,182	104.9	10, 11
20	11,000	456	1,346	102.5	7, 8
21	1,000	452	1,276	105.7	16
22	4,000	450	1,326	100.0	5, 6
23	30,000	446	1,126	107.3	10, 13, 17, 18
24	2,000	445	660	110.0	2
25	11,700	426	1,344	104.0	14, 15, 16
26	1,000	411	878	104.3	1
27	5,000	393	1,068	100.0	9
28	6,000	392	1,142	110.0	13, 14
29	8,000	390	1,180	110.0	15
30	2,000	367	1,047	110.0	4
31	5,000	340	1,276	110.0	17, 18
32	10,000	308	842	110.0	9, 10, 13, 18

Conclusion

The three-knife corrugator can result in lower side trim, greater corrugator productivity, and greatly simplified inventory replenishment. The keys to achieving these benefits are an order pool which averages more than four cuts across the width of the corrugator and a capability to quickly generate high-quality solutions to the order pools to be scheduled. This paper has presented an effective approach to solving these problems. This approach combines the benefits of sequential search and linear programming to achieve the operating objectives of the modern corrugated producer.□

Literature cited

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