Exponentially weighted schemes for continuous processes control

M. Pepio and C. Polo

Laboratorio de Estadistica, E.T.S.E.I.T.- U.P.C., Terrassa, Spain

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The assumptions underlying most statistical methods for process control are that the process mean and variance are constant and that the observations are independent. In other words, the process is assumed to remain on target and in a state of statistical control unless a special event occurs.

Under these assumptions, the logical approach is to devise a test to detect special events as early as possible, look for assignable causes, and correct the process back to target by removing the causes. The control schemes of this kind have been successful in the parts industries, in which the assumptions are quite reasonable and the cost of adjusting the process is frequently substantial.

However, the continuous-process industries such as chemicals, fibers, pulp and paper, and mineral processing do not sustain these assumptions since many of the quality measurements exhibit drifts with time. A common source of disturbances that cause the process characteristics to wander is variation in raw material, such as wood variations in pulping and papermaking. Although it is desirable to eliminate these variations, it is not economically feasible to do so. Goods of high quality must be made even in the face of such disturbances.

Frequently in the process industries, the only noncapital cost involved is expense resulting from being off target. Therefore, a control system needs to be able to make adjustments to compensate for the drifting behavior. It should also alert operators when a special cause of disturbance makes an adjustment inappropriate and requires corrective action.

Method

The process quality variable or response, Y, depends on a controlling input, X, of the process. Through appropriate adjustments, the controlling input allows the system to compensate for disturbances and respond to setpoint changes.

This dependence and the drifting behavior of the process can be represented by the model:

$$Y = x'\boldsymbol{\beta} + \theta = \beta_0 + \beta_1 x + \ldots \beta_p x^p + \theta \quad (1)$$

where the regression $x'\beta$ must be estimated under controlled conditions and θ is a random variate dependent on raw material variations, measurement error, adjustments in previous process steps, and variability not explained by the regression function.

At the ith measurement,

$$Y_i = x'_i\boldsymbol{\beta} + \theta_i \quad (2)$$

giving

$$\theta_i = Y_i - x'\boldsymbol{\beta}$$

The forecast for the drift, $\tilde{\theta}_{i+1}$ will be

$$\tilde{\theta}_{i+1} = \lambda\theta_i + (1 - \lambda)\,\tilde{\theta}_i \quad (3)$$

for

$$0 < \lambda < 1$$

where the exponential weight λ smooths the influence of drifts on the forecast. The absolute error of the previous forecast is given by

$$\epsilon_i = |\theta_i - \tilde{\theta}_i| \quad (4)$$

If no adjustment is made ($x'_i = x'_{i+1}$) the response can be forecasted as

$$\tilde{Y}_{i+1} = x'\boldsymbol{\beta} + \tilde{\theta}_{i+1} \quad (5)$$

With T as the process target and D as an absolute deviation from which the process is adjusted, if $|\tilde{Y}_{i+1} = T| > D$, then the Eq. 6 determines the regressor level x'_{i+1} to compensate for the forecasted drift:

$$x'\boldsymbol{\beta} = T - \tilde{\theta}_{i+1} \quad (6)$$

Verification

The parameter λ is found in minimizing the mean squared error, MSE, or the mean absolute error, MAE, of the data collected over a period long enough to ascertain the drift range, during which time all those personnel involved should be in agreement that the process is in control.

$$MSE = \sum_{i=1}^{n} \epsilon_i^2/n \quad (7)$$

$$MAE = \sum_{i=1}^{n} \epsilon_i/n \quad (8)$$

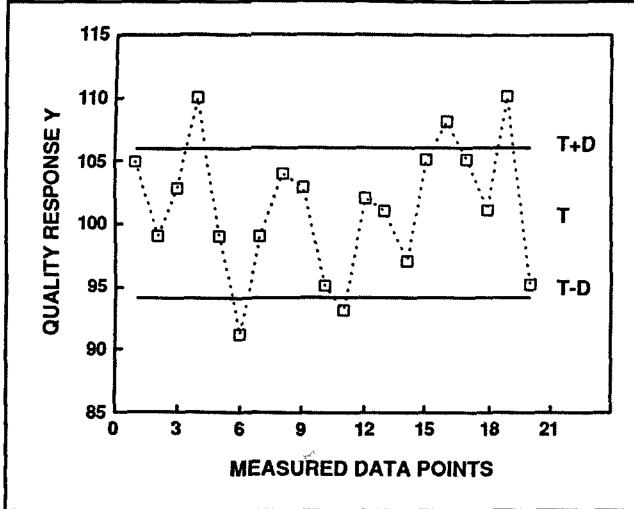
Thus, it is worthwhile to use the data to devise a control scheme that allows us to verify if the process really was working consistently during the collecting period. If it was working steadily, the data are used in managing the process. If the control scheme reveals that the process was irregular, then the data are thrown out. In either case, as soon as new data are collected, the parameter λ has to be evaluated anew.

The control scheme

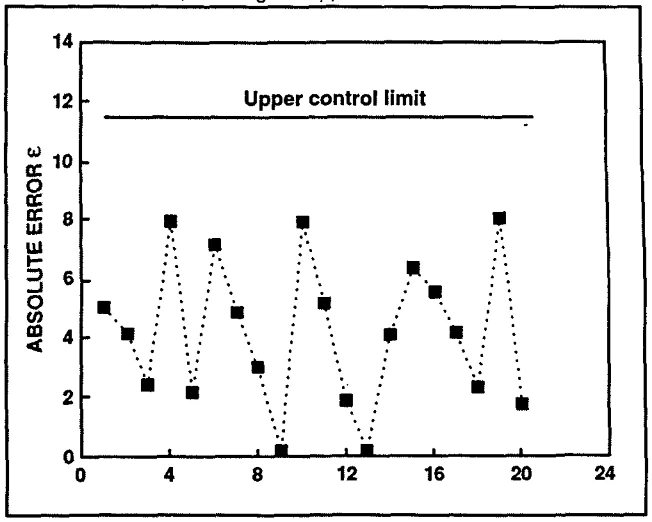
It is doubtful that θ and hence the forecasted error would be normally distributed, because the various causes of the observed variability do not all have a reduced (small) effect. Therefore, it is advisable to have recourse to a distribution-free method.

Let M be the median of the drift forecast absolute error. Let Q_1 and Q_3 be the first and third quartiles of

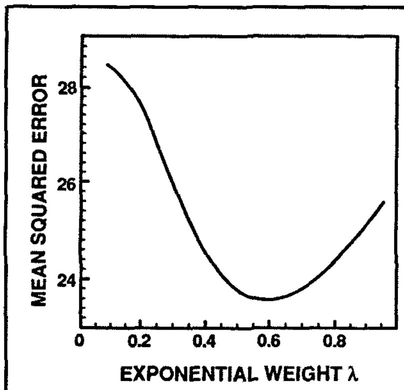
1. Data plot



3. Control scheme, showing the upper limit



2. Mean squared error as a function of λ



I. Data collected to set up the control scheme with forecasts, drifts, and absolute errors for $\tilde{\theta}_0 = 0$

Datum no.	*Input X*	*Response Y*	*Forecast* $\tilde{\theta}_i$	*Drift* θ_i	*Absolute error* ϵ
1	10	105	0	5	5.0
2	10	99	3	-1	4.0
3	10	103	0.6	3	2.4
4	10	110	2.04	10	7.96
5	8	99	6.82	9	2.184
6	8	91	8.13	1	7.126
7	10	99	3.85	-1	4.851
8	10	104	0.94	4	3.06
9	10	103	2.78	3	0.224
10	10	95	2.91	-5	7.91
11	10	93	-1.84	-7	5.164
12	11	102	-4.94	-3	1.934
13	11	101	-3.77	-4	0.226
14	11	97	-3.91	-8	4.091
15	11	105	-6.36	0	6.364
16	11	108	-2.55	3	5.546
17	10	105	0.78	5	4.218
18	10	101	3.31	1	2.313
19	10	110	1.92	10	8.075
20	8	95	6.77	5	1.77

errors, ϵ_i, associated with the collected data when the method is applied with the selected λ. Then, an acceptance interval $M \pm 2(Q_3 - Q_1)$ can be used for monitoring the process.

Evidence of variation outside the common-cause system is obtained when a plotted error value falls outside the control limits. This event indicates that the process should be investigated for special causes. If appropriate, action should be taken to bring the monitored process back to control. Taking into account that an absolute error cannot be negative, the lower limit of the interval can be fixed at zero.

This control scheme will be helpful for removing disturbances rather than compensating for them with feedback adjustments by means of the regulating input X. The scheme will not conceal information that might be used for quality improvements.

Application

A process quality measurement Y has a target $T = 100$ and an admissible deviation $D = 6$, giving specification limits of 100 ± 6. Outside these limits, the process is adjusted back to target by the regulating input X, according to the regression function

$$x'\beta = 50 + 5x \qquad (9)$$

Table I shows the input values, X_i, and the responses, Y_i, as collected to set up the control scheme. These data are plotted in **Fig. 1**.

The setting X was only modified when Y was falling outside the specification limits (Measurements 4, 6, 11, 16, and 19). **Figure 2** shows the development of the mean squared error as a function of λ. The minimal value of the mean squared error is obtained for $\lambda = 0.6$. For $\tilde{\theta}_0 = 0$ and this value of $\lambda = 0.6$, the drifts θ_i, the forecasts $\tilde{\theta}_i$, and the absolute errors ϵ_i have been calculated and are shown in Table I.

Then

$$M = (4.091 + 4.218)/2 = 4.155$$

$$Q_1 = (2.184 + 2.313)/2 = 2.248$$

$$Q_3 = (5.546 + 6.364)/2 = 5.955$$

and the acceptance interval $M \pm 2(Q_3 - Q_1)$ is [0; 11.569], where the lower limit is taken as zero since an absolute error cannot be negative.

This control scheme can be applied to verify that the process was working under stable conditions by plotting the drift forecast absolute errors, as shown in **Fig. 3**. Because the plotted points show a shape rather being homogeneous, this scheme could be applied to check the absolute error of forecast in this circumstance from this point on, while monitoring the process with a chart like that of Fig. 1.□

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