Principles of hydrodynamic instability: application in coating systems

Part 2. Examples of flow instability

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ABSTRACT *With new high-speed paper machines, coating rheology and flow instability are critical issues. The ideal flow in a coating device is steady-state and two-dimensional. In other words, it would have no velocity components or momentum transfers in the cross-machine direction. In coating, complex fluids are applied to paper at high speeds under extreme flow conditions, so the problems of fluid mechanics and instability are often unique, requiring fundamental, in-depth analysis.*

Flow-through coating devices as nonlinear dynamic systems can only behave in a specific and consistent manner. By knowing the fundamental physical and dynamical principles (such as general behavior and transitions between steady and time-periodic states), we are better equipped to deduce and interpret information from experiments or computational analysis of instability problems. Mathematical examples are included to provide simple but practical working tools.

KEYWORDS
Blade coaters
Coating
Fluid dynamics
Fluid mechanics
Hydrodynamic stability
Rheology
Viscous flow

In this three-part study, we are examining the principles of hydrodynamic instability in coating systems. The purpose is to outline some techniques that are available for analyzing and predicting flow instability. In Part 1, the general principles of flow instability were outlined.

Here, in Part 2, we will look at examples of flow instability in coating systems, along with some relevant classical problems. By comparing practical instability problems in coating systems with relevant prototype problems, we explain the origin and mechanisms of the instabilities.

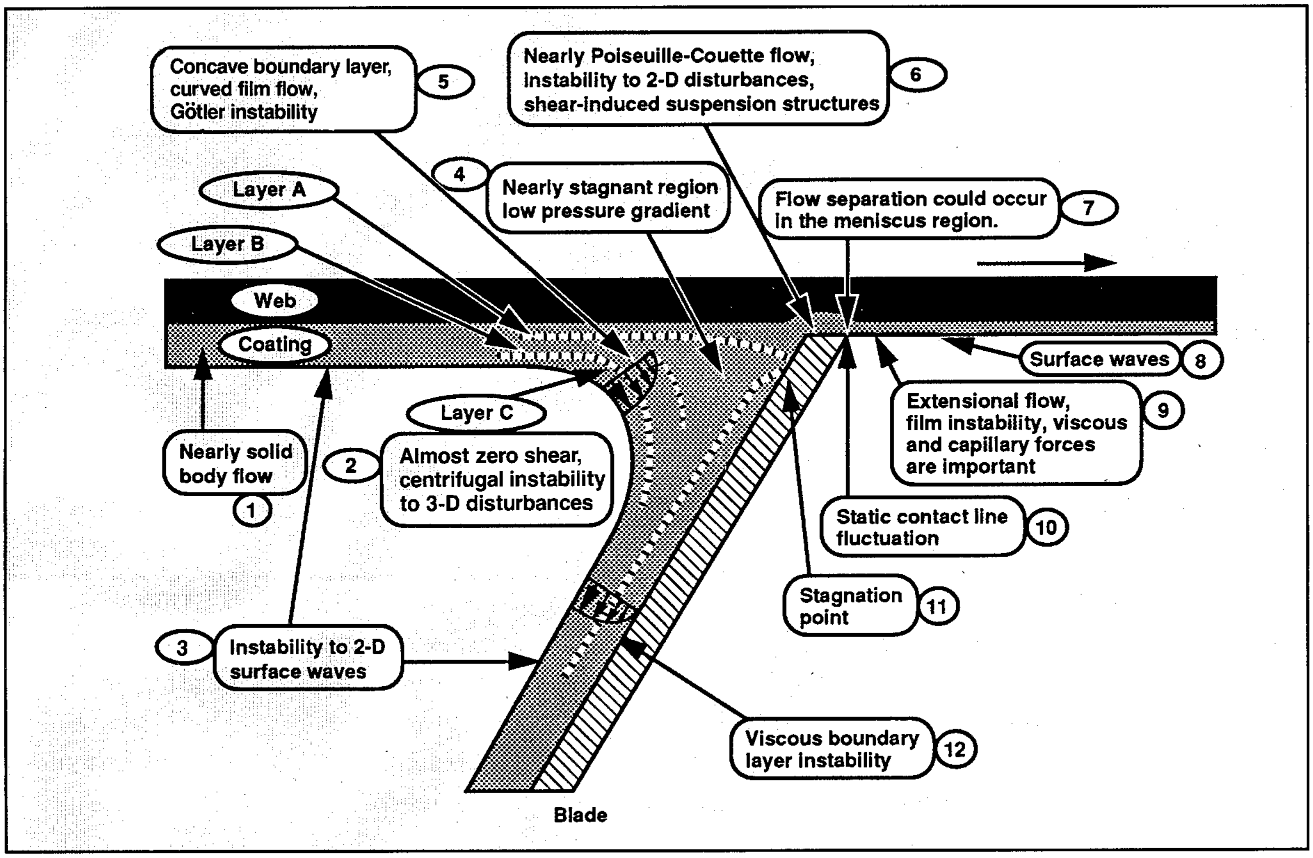
Flow instability in blade coating

Blade coating is the most popular mode for applying a coating on the surface of paper or linerboard, and the blade coater is a complex and interesting hydrodynamic device. The fluid, which is coating color or size, is forced from one extreme to the other, and the fluid particles experience changes in the shear rate from almost 0 s^{-1} upstream of the blade to nearly 10^6 s^{-1} under the blade in less than 10^{-2} s. Regions with enormous pressure gradients are adjacent to areas of nearly constant pressure, and differences in the velocity gradients are considerable.

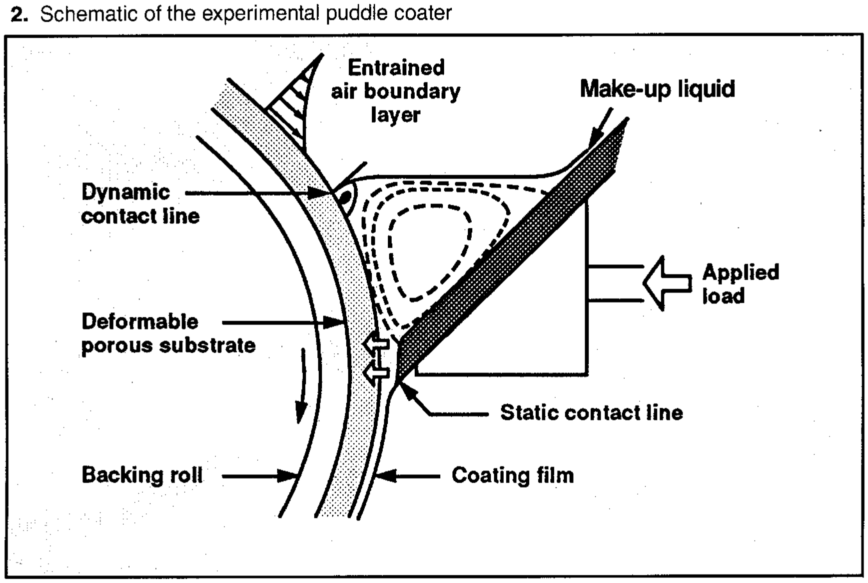
In addition to these complexities, one could hardly design a problem with more complex boundaries. The blade is flexible, the web is flexible and permeable, and there are free surfaces with static and dynamic contact lines. The expectations for an ideal coating device are also extreme. At high machine speeds, the flow needs to remain steady and two-dimensional for operating conditions to be ideal. In the two-dimensional ideal flow, there is no velocity component or momentum transfer in the cross-machine direction.

This "ideal" flow is disrupted by many types of instability mechanisms. The result is coating states that are unstable and three-dimensional, with consequences that are often intolerable. The extreme flow conditions,

1. Some stability characteristics of flow in blade coating with a roll applicator (See Pranckh and Scriven (2) for quantitative information.)



2. Schematic of the experimental puddle coater

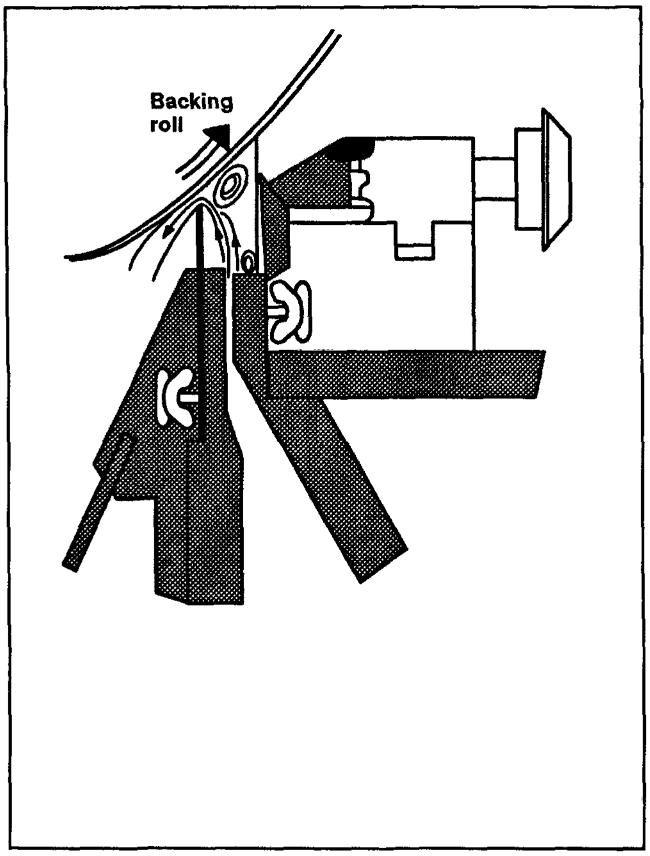


complex fluid properties, irregular flow boundaries, and increasing expectations create new challenges that demand advanced technological and scientific approaches. By combining results from carefully designed laboratory experiments with the well-known principles of dynamic systems, we obtain useful information for the design of improved systems. (These dynamic systems will be presented in Part 3.)

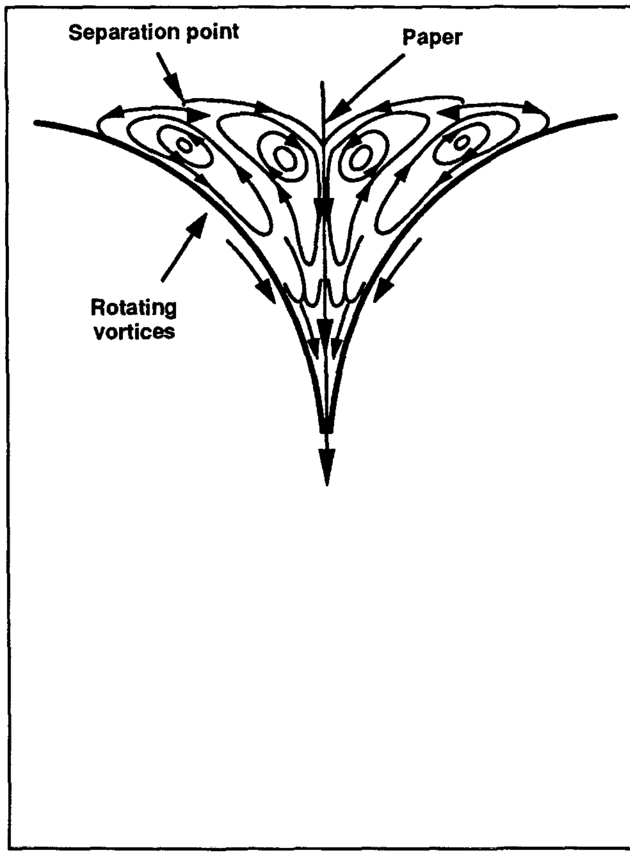
In computer-aided experiments, the governing fluid flow equations are solved accurately using robust computational techniques. The governing fluid flow equations are well established, and their solutions can accurately simulate the physical situation. This approach has proven useful in the analysis of the stability of slide coating flows, for example (*1*).

The computational analysis of blade coating is a difficult and time-consuming task, but the benefits are enormous. In contrast to experiments in which a limited amount of information can be extracted, an accurate or "exact" numerical solution of the governing equations of a fluid flow process provides complete information for every point in the domain. An example is the finite-element simulation of blade coating by Pranckh and Scriven (*2*), which is the most detailed analysis of blade coating currently available. Although only two-dimensional flow equations are solved in this simulation, the quantitative results also shed light on the relative magnitude of forces and on the hydrodynamic instability properties of the two-dimensional flow with respect to disturbances that induce three-dimensional flows.

3. Schematic of the short-dwell coater



4. Schematic of the size press



Blade coating with a roll applicator system

Consider the flow in a blade coating system with a roll applicator, as illustrated in **Fig. 1**. Various sections of the flow field exhibit unique characteristics. Here, the web is moving from left to right, carrying a thick layer of fluid with it. The fluid in this layer (Point 1) moves like a solid body and experiences almost zero shear. As this layer approaches the blade, it bifurcates into roughly three streams.

Layer A, adjacent to the web, continues into the nip of the blade (Point 6) and experiences an enormous increase in shear rate. Layer B, the middle stream, falls just below the separation line (Point 11), turns downward, and forms two boundary layers, one on the flat surface of the blade (Point 12) and a second next to the concave region of Layer C. The free-surface stream (Layer C) reflects down the blade under negligible viscous forces (irrotational flow).

In an incompressible fluid, a disturbance at one section propagates instantaneously throughout the media. Therefore, if three-dimensional disturbances grow upstream of the blade, their effects will propagate through the blade tip and may result in coating weight nonuniformities such as streaks. Let us look at the stability properties of these three layers.

Layer A enters a high-shear region under the blade and follows a nearly Poiseuille-Couette velocity distribution (*2*). Following Squire's theorem, the first instability will be due to a two-dimensional disturbance mode in the plane of the base fluid (Part 1). Further downstream in the meniscus region, however, other mechanisms such as visco-capillary instabilities become important.

The irrotational stream (Layer C) is under a centrifugal force and could become unstable to three-dimensional disturbances. According to Rayleigh's criterion (Part 1), if the radial gradient of the magnitude of the angular momentum becomes negative, then the flow could become unstable.

Let us assume that the curved section of this layer is circular, where $r = R_1$ and $r = R_2$ define its inner and outer boundaries, respectively. Also, let u_θ, the azimuthal component of velocity, be a function of r alone, and let other components of the velocity vector be negligible. The Rayleigh's criterion for stability reduces to

$$(r/u_\theta)\,(du_\theta/dr) > -1 \qquad (1)$$

This equation presents an approximation for the stability criterion of Layer C. A more accurate stability analysis can be obtained by actually calculating Rayleigh's discriminant along the streamlines given by Pranckh and Scriven (*2*).

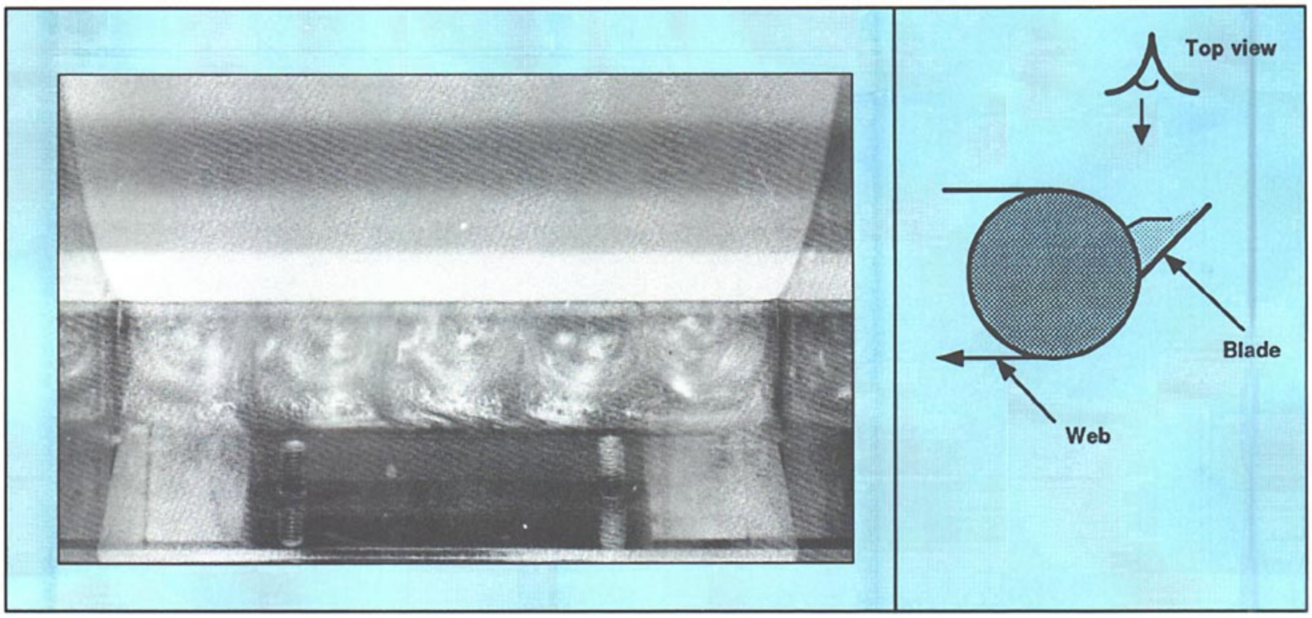
In the curved section of Layer B (Region 5), the velocity rapidly decreases to zero, forming a concave boundary layer susceptible to Görtler-Taylor type instabilities. This boundary layer, in conjunction with Layer C, roughly forms a free-surface curved film. This flow is similar to a free-surface film flow down a concave wall, which could become unstable to Görtler-type vortices in the concave region (*3*).

The critical Reynolds number Re is

$$Re = (U_0\delta/\nu)\,(\delta/R)^{1/2} \qquad (2)$$

where

5. Top view of the three-dimensional patterns in the pond of a puddle coater. The web moves down from the top of the picture at 500 ft/min (6).



R = radius of curvature

δ = film thickness

U_0 = average film velocity

ν = kinematic viscosity.

For the onset of Görtler-type vortices for a film falling under gravity alone, the critical Reynolds number is reported to be about 16.1 (*3*). This value is close to the value of 15.8 for the onset of Görtler instability in a boundary layer over a concave wall.

Compared to such a system, a blade coating system with puddle or short-dwell applicators has somewhat different stability properties. In addition to the local instability, the apparent two-dimensional flow in the pond of these systems can be globally unstable. In fact, the flow of Newtonian fluids in a cavity simulating the pond of a short-dwell coater has multiple stable steady states (*4*).

Centrifugal instability in the pond of puddle coaters, short-dwell coaters, and size presses

Puddle and short-dwell coaters, along with size presses, are used extensively in the industry for applying surface coatings. A description of these systems can be found in the review article by Eklund (*5*).

All of these systems share a common hydrodynamic feature—apparent two-dimensional rotating vortices (**Figs. 2-4**). These vortices are prone to instability and transition to three-dimensional patterns. As stated, flows with curved streamlines can destabilize to three-dimensional disturbances. It is this instability that replaces the ideal two-dimensional flow with the troublesome, three-dimensional flow patterns in many coating systems. A three-dimensional flow pattern in the pond contains CD velocity components that often lead to coating weight nonuniformities across the sheet.

The fact that three-dimensional disturbances in the pond upstream of the blade can lead to "streaking" defects in the coating film was reported first by Higgins (*6*) for a puddle coater. Higgins showed that a steady three-dimensional pattern (**Fig. 5**) can develop in the pond upstream of the blade as a result of the instability of the base state. Even more complex three-dimensional patterns can develop in the pond of short-dwell coaters.

The lower photograph in **Fig. 6** is the picture of one of the multiple, steady, three-dimensional patterns that can replace the apparent two-dimensional primary and secondary rolls (top) in a driven cavity simulating the pond of a short-dwell coater (*4*).

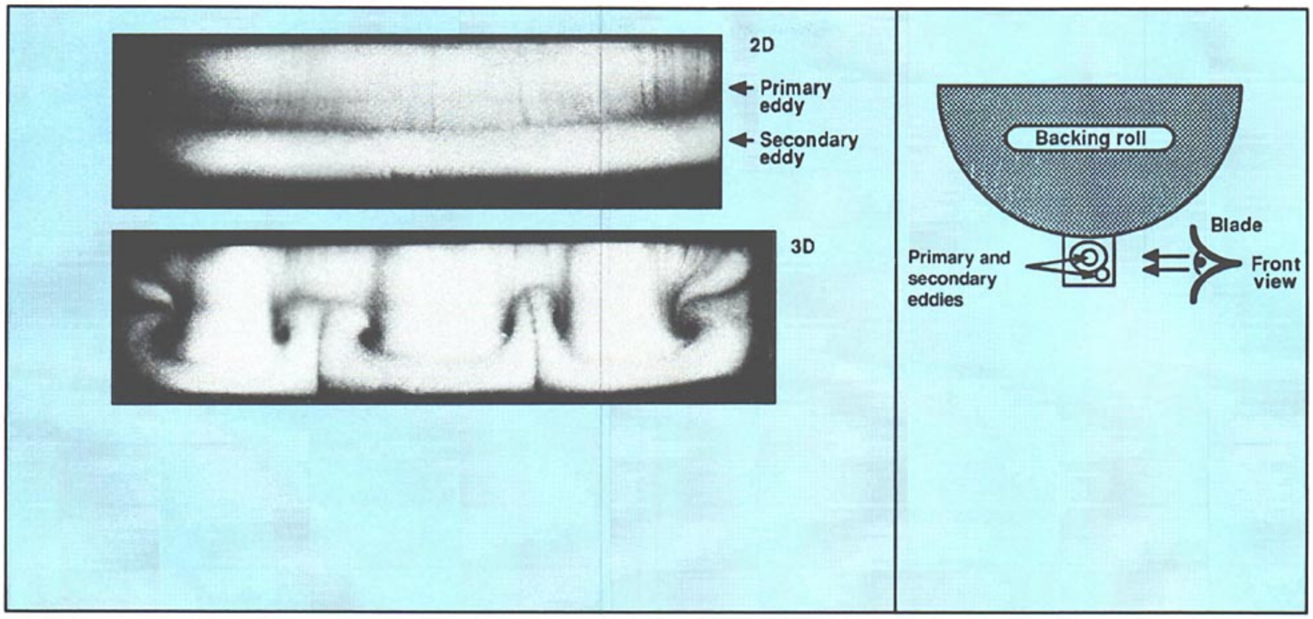
It would be interesting to examine these multiple states and obtain the bifurcation diagram for the flow structures. If the three-dimensional states represent isolated solutions of the governing equations, then standard branch tracing techniques may not be sufficient to completely map the solution structure. (We will consider this further in Part 3.)

For linear instability, the disturbance structure could be steady or time-periodic, which results in either a steady or time-periodic three-dimensional flow. By simply rescaling and adding the destabilizing disturbance to the two-dimensional base flow, we can closely approximate the three-dimensional flow pattern near the point of onset. In the case of transition from steady to time-periodic flow (a Hopf bifurcation) in a different problem, this technique has proven quite effective and useful in understanding and explaining the instability mechanism (*7*).

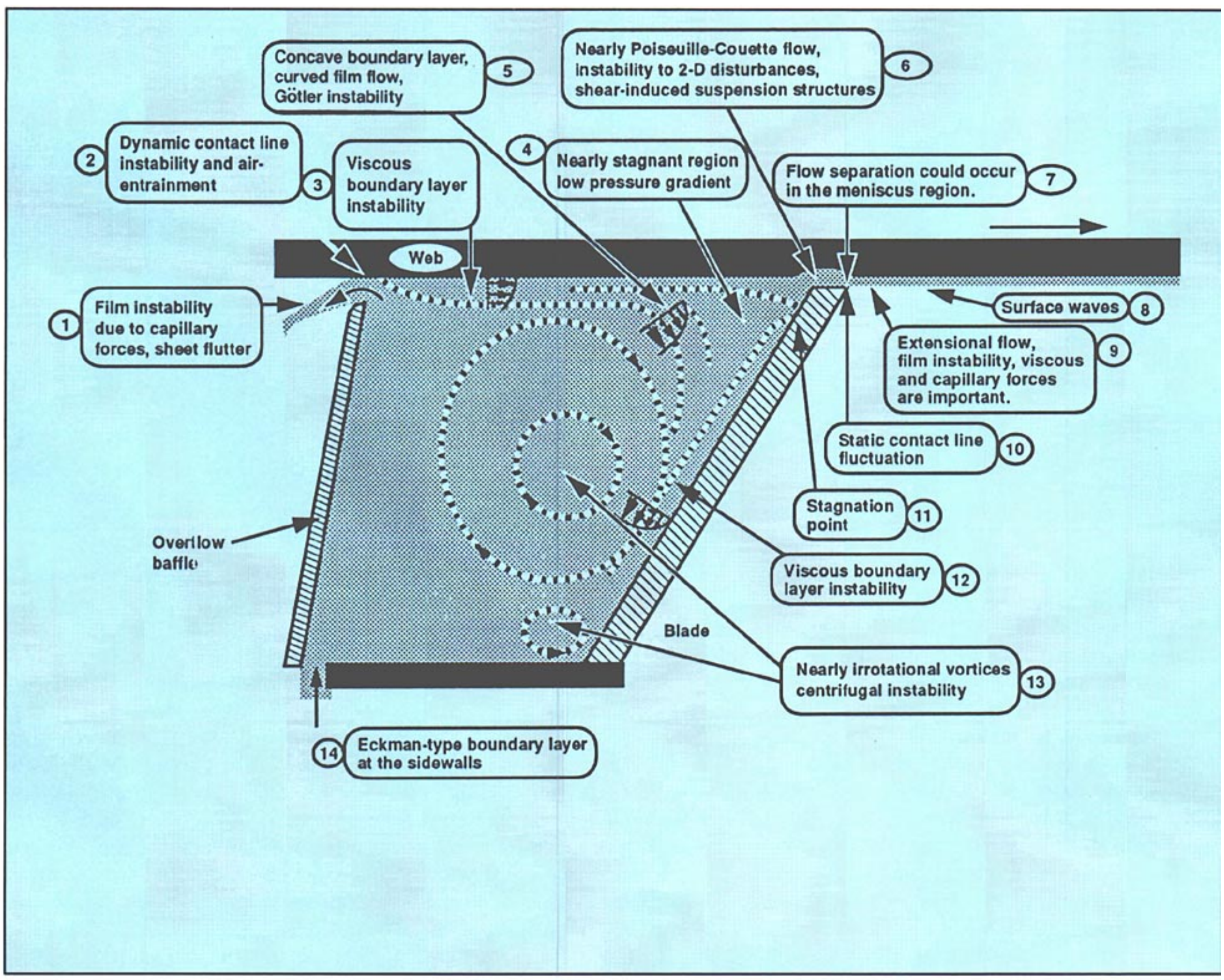
Little is known about the flow instability of non-Newtonian fluids and suspensions in these surface application systems. How can more advanced systems be developed in a rational and cost-effective manner without a fundamental understanding of these issues?

The classical flow problems reviewed in Part 1 show that two-dimensional rotating flows can readily be replaced by three-dimensional

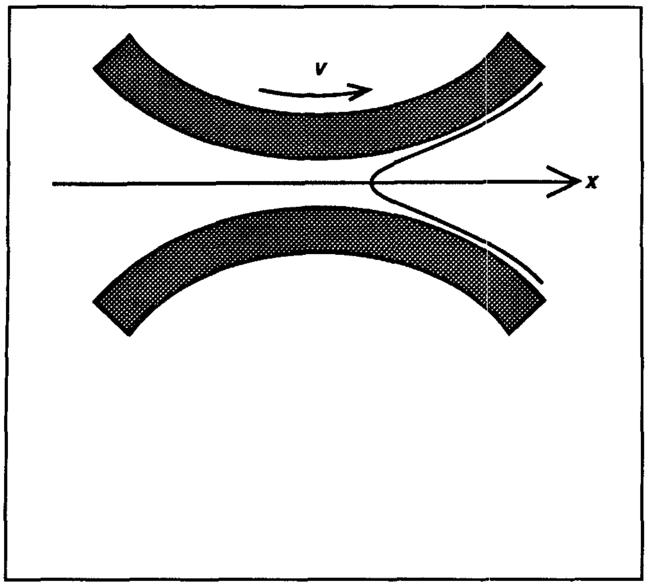
6. Illustration of multiple steady states in the pond of a short dwell coater. Top photo: a steady state that appears almost two dimensional. Bottom photo: a second steady state with complex three-dimensional patterns that can replace the two-dimensional state at identical operating conditions. The pictures are the front view of flow in a rectangular cavity simulating the pond of a short dwell coater (*4*).



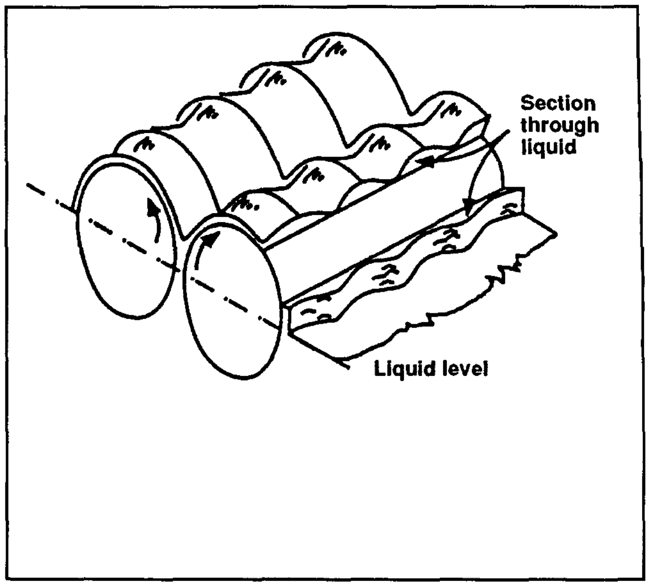
7. Some stability characteristics of flow in blade coating with a short dwell time applicator



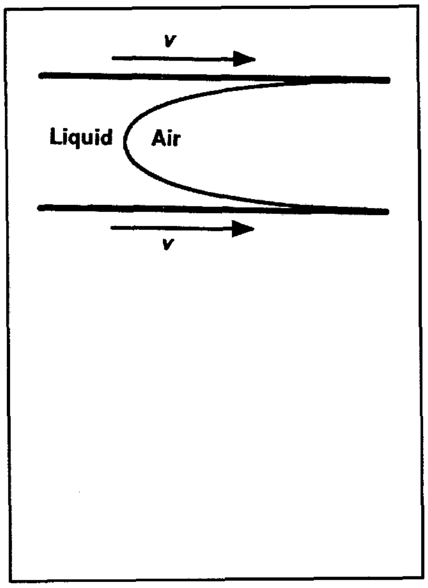
8. Forward roll coating



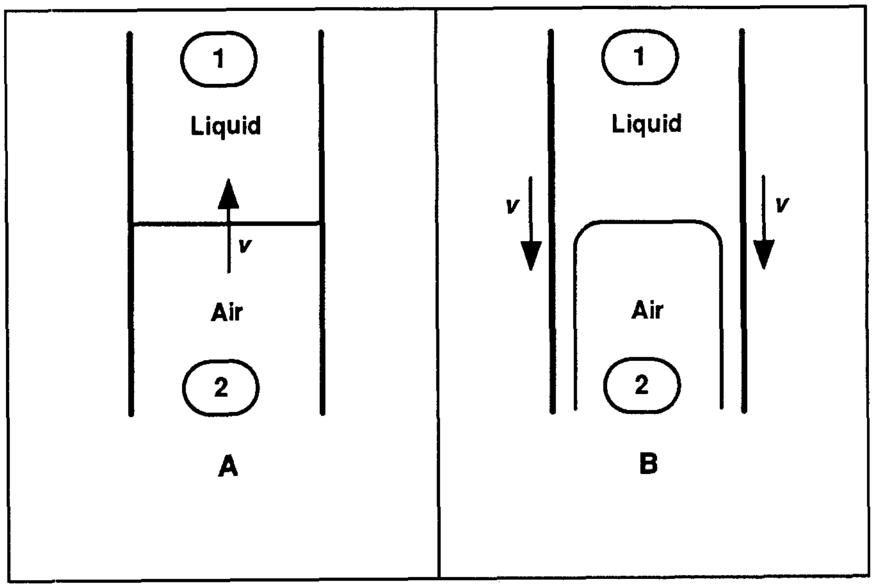
9. Schematic of ribbing lines in forward roll coating



10. A meniscus formed between two parallel solid surfaces



11. Liquid displacing air between closely spaced parallel solid surfaces



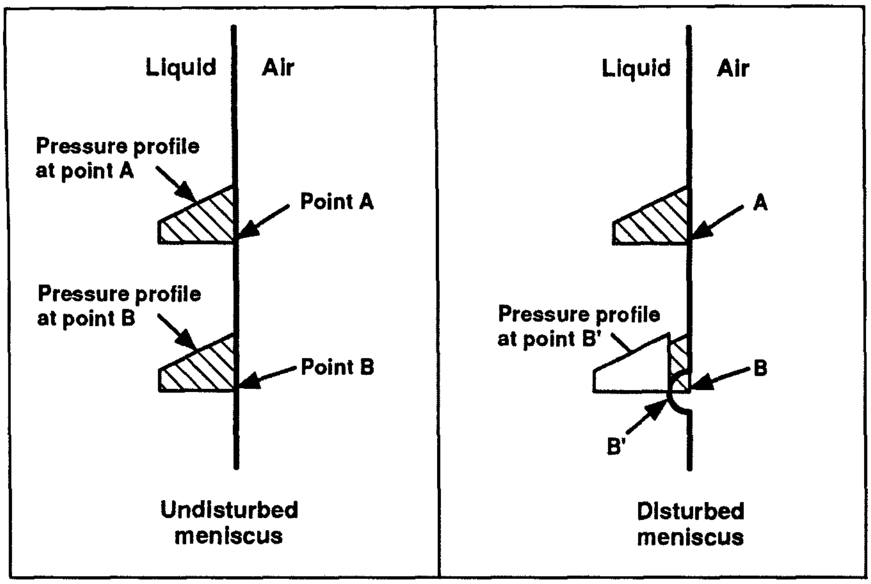
flow patterns due to centrifugal instability. Size presses, puddle coaters, and short-dwell coaters are the most vivid examples of processes that include apparent two-dimensional rotating fluid vortices that are vulnerable to centrifugal instabilities. Continuing work on short-dwell coaters has shown a variety of interesting and unexpected results in terms of local and global stability properties. Possible instability mechanisms that can result in intolerable operating conditions in this system are outlined in **Fig.** 7. The non-Newtonian effects have a dramatic influence on the stability properties of recirculating flows and should be considered in future studies.

Ribbing lines in forward roll coating

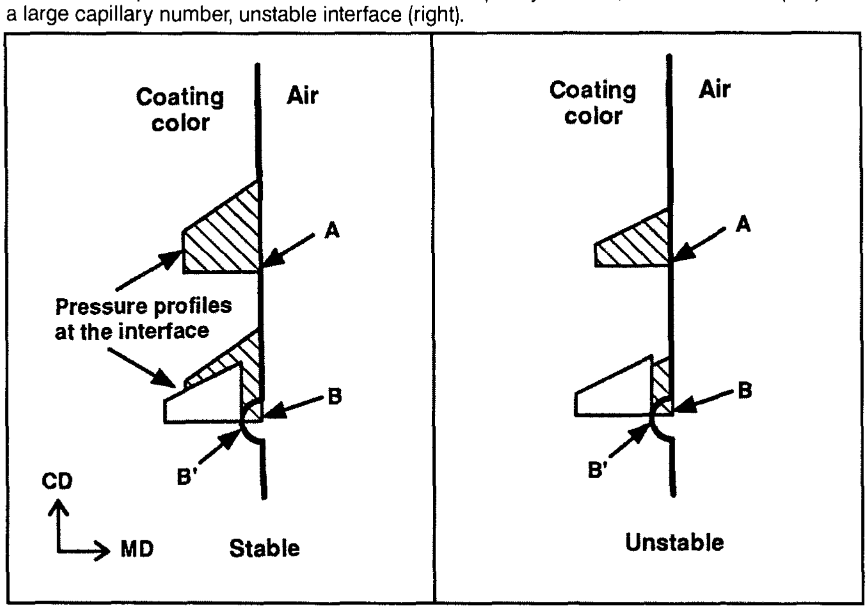
Ribbing instability in roll coating is a well-known phenomenon that has been extensively studied. [For a detailed discussion, see the review article by Ruschak (*8*).] A meniscus forms in the diverging section of the nip, as shown in **Fig.** 8. The ideal condition is achieved when a steady two-dimensional flow field is maintained—that is, when there is no flow in the cross-machine direction. However, this two-dimensional flow destabilizes and gives rise to a three-dimensional flow pattern. In this transition, it turns out that the flow becomes spatially periodic in the cross-machine direction, and therefore the free surface becomes ribbed and uneven (**Fig. 9**). This usually results in unacceptable operating conditions.

The important parameter in this

12. Pressure profiles at an undisturbed and disturbed meniscus between two parallel moving solid surfaces



13. Pressure profile at the meniscus in a small capillary number, stable interface (left) and a large capillary number, unstable interface (right).



problem is the capillary number, Ca, which is defined as $\mu V/\sigma$, where μ and σ are the viscosity and surface tension of the fluid, respectively, and V is the roll speed.

For rolls with equal size and speed, the meniscus forms close to the center of the nip for a sufficiently large capillary number. This flow becomes unstable, and ribbing lines appear when the capillary number exceeds a critical value. In the limit, this could be approximated as a meniscus forming between two nearly parallel planes moving with equal speed, V, as shown in **Fig. 10**. The following question remains: What is the physical reason for the instability? In other words, why does the two-dimensional flow cease to exist beyond a critical capillary number?

The answer can be obtained from the classical instability analysis of Saffman and Taylor (*9*), who considered the stability of air-liquid interface between two closely spaced parallel solid surfaces. As **Fig. 11** illustrates, Fluid 2 is displacing Fluid 1 through a sharp interface. Initially, the interface is assumed to be a straight line (Fig. 11A) moving upward with velocity V.

Using standard linear stability analysis (reviewed in Part 1), Saffman and Taylor showed that the interface becomes unstable to infinitesimal disturbances if

$$[(\mu_2/k_2) - (\mu_1/k_1)]\, V + (\rho_2 - \rho_1) g < 0 \qquad (3)$$

where

μ = fluid viscosity

k = permeability of the medium to the fluid

ρ = fluid density

g = acceleration of gravity.

The subscripts 1 and 2 refer to the upper and lower fluids, respectively.

In many situations, such as with some coating flows, the acceleration due to gravity g may not be important. If we choose Fluid 1 to be liquid and Fluid 2 to be air and if we fix the frame of reference relative to the meniscus, then Fig. 11B represents an approximation of the forward roll coating problem, since for sufficiently large capillary numbers, the coating meniscus is near the nip, where the roller surfaces are nearly parallel.

The linear stability analysis of Saffman and Taylor shows that if the pressure gradient, dP/dX, in the liquid adjacent to the meniscus in Fig. 11 is positive, then the flow is unstable and ribbing lines appear. Using matched asymptotic expansion, Ruschak (*10*) showed that the pressure gradient in this situation is always positive, and the flow is therefore unstable.

Here, the instability mechanism is pressure driven (*11*) and can be explained in the following way. Consider two neighboring points, A and B, on an undisturbed meniscus, as illustrated in **Fig. 12** (left). There is a positive pressure gradient in the machine direction (i.e., $dP/dX > 0$). Now let a minor disturbance pull the surface by a small amount from Point B to Point B′. The pressure profile remains about the same but shifts upstream by a small length BB′. This disturbance creates a pressure gradient in the cross-machine direction. Since the pressure at the undisturbed neighboring points, such as Point A, is now lower than pressure at B′, this forces flow from disturbed points toward the undisturbed regions and reinforces the disturbance, causing a positive growth rate. The disturbance structure with the most destabilizing wavelength will grow first.

The fact that the meniscus in a forward roll coating is not always unstable (in contrast to Saffman and Taylor's model) indicates that the diverging surfaces of the rolls stabilize the flow at lower capillary numbers. The stabilizing mechanism is attributed to the change in the radius of curvature of the disturbed section of the meniscus. Since for diverging surfaces, the free surface at Point B′ in **Fig. 13** will have a smaller radius of curvature than Point A, this decrease in radius of curvature will reduce the pressure by an amount proportional to the surface tension. If the surface tension is large (small capillary number), then the decrease in pressure at the disturbed location due to a reduction in radius of curvature will be greater than the increase due to the shift in pressure profile; therefore, the flow will be stable. For large capillary numbers, the influence of the surface curvature on the pressure will be small, and the flow will become unstable.

Wetting line instability and air entrainment

In addition to flow instability and nonuniformities in coating weight distribution, most high-speed coating systems suffer from an additional constraint—air entrainment at the contact line between liquid and the substrate. Air entrainment results in severe limitations on flexibility and the speed of coating and may form a bottleneck for the on-machine coaters.

The physics of dynamic wetting is not completely understood. In general, the contact line is a straight line where air, liquid, and solid meet. As the speed of the substrate increases, the contact angle approaches 180°, at which point the contact line destabilizes and breaks into V-shaped (sawtooth) structures. These V-shaped structures grow in size with further increases in speed, and finally air bubbles penetrate the liquid from the downstream tip of these structures. Flow visualizations of this process (*12*) show that two or more of these V structures coalesce, form, and release a large bubble and then resume their original shape in a time-periodic manner.

Currently, there is no theory that can fully explain the physics of wetting. The assumption of no slip at a solid wall results in a dynamic incompatibility in the system; the equations become singular at the wetting line. Various models (e.g., Navier slip condition) can be used in numerical simulations to treat the dynamic contact line in an approximate but consistent manner. A general background on contact angles and wetting is given in the review article by Dussan (*13*). Recent progress in the understanding of dynamic wetting and air entrainment in thin film coating is presented by Scriven (*14*).□

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