

Roll cockle—following its evolution

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ABSTRACT *Roll cockle—the surface buckling that can form on the exterior of finished paper rolls prior to sheeting—is a significant source of economic loss within the paper industry. The roll surface buckles in response to constrained hygroexpansion in the cross-machine direction. Buckle development is a function of sheet thickness, elastic modulus, and roll radius. The greater the roll radius and the thinner the sheet, the more likely the roll is to form the characteristic pattern of alternating-diamond buckles on its surface.*

Image-correlation analysis, a noncontact strain-measurement method, was used to track the evolution of roll cockle in a roll of toilet tissue. Buckle formation was examined by developing a time sequence of strain profiles for the exterior of a roll exposed to an increasingly humid environment.

KEYWORDS

Cockle
Dimensional stability
Humidity
Image analysis
Lightweight papers
Optical instruments
Tissue

Mills that produce lightweight paper are familiar with the problem of roll cockle, a repetitive pattern of alternating diamonds on the surface of a finished roll. The diamond-shaped deformations on the roll surface usually are about an inch long, and in severe cases, the pattern can penetrate the roll to a depth of 4 in.

Roll cockle is the result of compression failure caused by constrained hygroexpansion of the paper cylinder. Hygroexpansion, the driving force, is limited to the exterior of the paper roll, which can expand up to ten times more in the cross-machine direction (CD) than in the machine direction (MD). The hygroexpansion is constrained because the interior layers of the paper roll are beyond the point of moisture penetration and thus do not expand.

The occurrence of roll cockle increases with increasing roll radius, suggesting that roll radius influences roll-cockle formation. Mills often remove the outer layers of cockled paper until the roll surface is smooth. However, a new pattern of alternating

diamonds is generated when the roll is returned to storage, suggesting the time-dependent nature of cockle formation.

Theoretical evaluation of cockle in a paper roll suggests two equations, one defining the period of the repeat pattern and another defining the minimum critical buckling stress needed to initiate compression failure (1). Minimum critical buckling stress is defined in Eq. 1.

$$\sigma_{cr}(\min) = Et/r[3(1-\nu^2)]^{1/2} \quad (1)$$

where

E = elastic modulus of sheet in the CD
 t = thickness of sheet
 ν = Poisson's ratio for sheet
 r = radius of paper roll

Thus, the minimum critical buckling stress is a function of the sheet's elastic properties as well as the roll structure or radius.

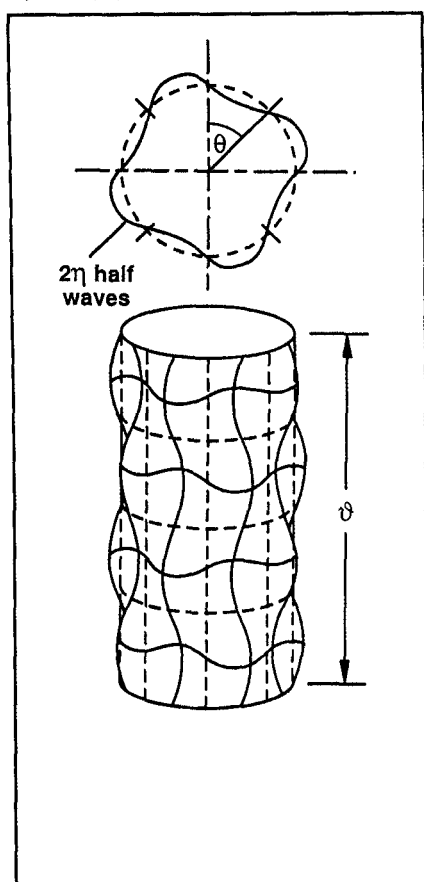
The formation of the buckled configuration of alternating diamonds on the surface of the paper roll is a complex humidity- and time-

dependent phenomenon. Buckle formation can be examined using an image-correlation analysis technique that was originally developed to measure strain fields (2-9). Examination of the dimensional changes on the exterior of a paper roll as it is transformed from a smooth cylinder to a cylinder with an alternating series of diamond depressions and ridges can yield some interesting insights into the material properties of paper and the structural properties of the roll.

Experimental

The change in the shape of a roll of toilet tissue was measured in an environment of increasing humidity. The experimental approach can be illustrated by placing a damp towel in contact with a roll of toilet tissue for 10 min. The roll of toilet tissue was chosen as a surrogate for a roll of lightweight paper because it is easier to use in the laboratory. Preliminary measurements taken on the tissue roll showed that expansion in the CD can exceed 6%, while expansion in the MD

1. Changes in cylinder circumference and length following hygroexpansion of surface layers of paper



2. Boundary of images presented in Figs. 3 and 4, with x representing the CD and y representing the MD

is 1.5%. However, a disadvantage of tissue material is that its material properties in compression are very difficult to define.

The formation of the buckling pattern of alternating diamonds on the exterior of a roll of paper is a complex phenomenon that requires significant bending and stretching of the paper roll surface. During hygroexpansion of the toilet tissue roll, strains may appear to be both positive and negative. This is due to the sinusoidal-like change in the roll radius that occurs upon hygroexpansion. **Figure 1** shows how hygroexpansion-induced changes in the length and circumference of the cylinder affect roll radius. The strains that transform the initially smooth cylinder into a cylinder with a pattern of alternating-diamond buckles were measured from the side of the cylinder, as seen in **Fig. 2**.

The image-correlation analysis system measures a three-dimensional change in shape by measuring only two dimensions. Therefore, the tran-

sition zones between the sinusoidal peaks and valleys appear to be shortened in the two-dimensional measurement, while the expansion measurements at the peaks and valleys approach their true values. Although the strain profile is inaccurate, it effectively documents the development of the alternating-diamond buckling pattern.

A partial roll of toilet tissue with a radius of 3-1/2 in., height of 4-7/16 in., and an individual sheet thickness of 0.003 in. was randomly spotted with xerographic toner particles. The roll was then heated to thermoset the toner particles on the roll surface. The roll was subsequently humidified to raise the moisture content of the exterior. A series of video images of the roll surface recorded the development of the alternating-diamond buckling pattern. The area over which the images were taken was approximately 3/8 in. in the axial direction (CD) and 3/4 in. in the circumferential direction (MD). The images covered 1-1/2 buckles, as shown in **Fig. 2**. A final image

was recorded after the roll was returned to ambient conditions. The images were compared using image-correlation analysis, and the strain fields were recorded.

Results

Figures 3A-3F and Figs. 4A-4F are strain-intensity profiles over an imaginary grid of 9×21 points on the surface of the toilet tissue roll. **Figures 3A-3F** show the strains in the axial direction (CD), while **Figs. 4A-4F** show the corresponding strains in the radial direction (MD). Each figure depicts the strains over a matrix of 21 columns and 9 rows. Strain intensity is displayed on the vertical axis for each of the 189 points within the field of measurement. The first five figures in each series (**Figs. 3A-3E** and **Figs. 4A-4E**) depict the progressive change in shape for the humidified tissue roll over time. The final image in each series (**Fig. 3F** and **Fig. 4F**) shows the roll surface after ambient conditions had been restored.

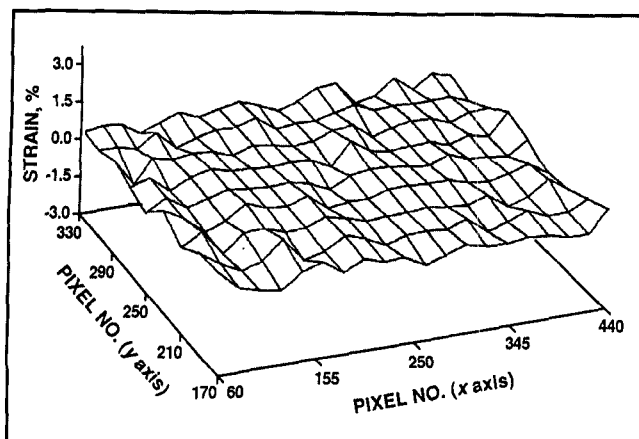
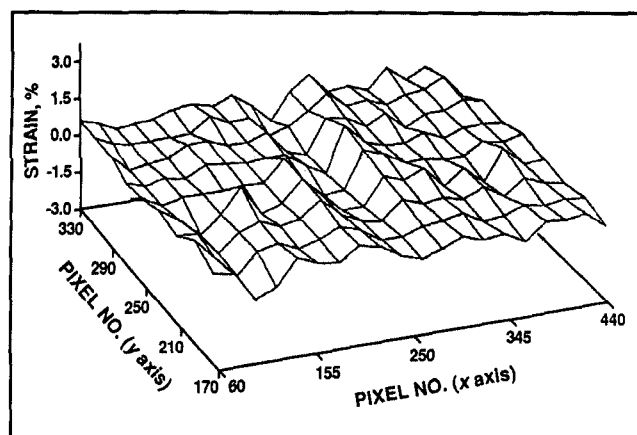
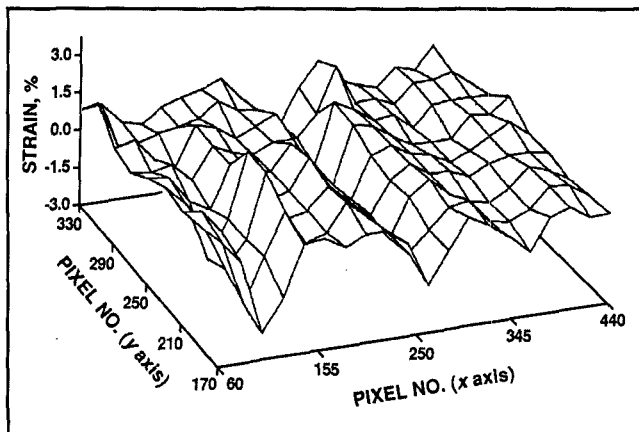
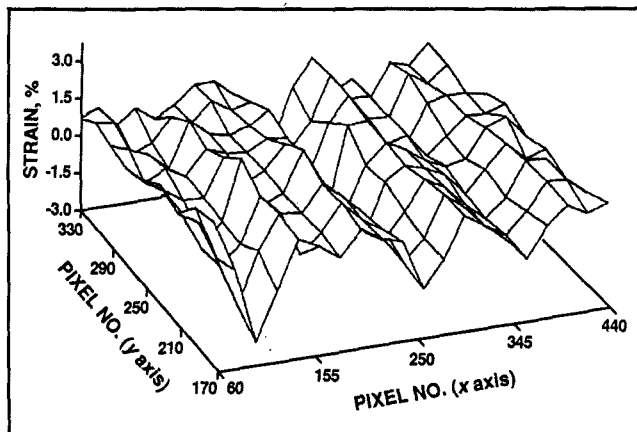
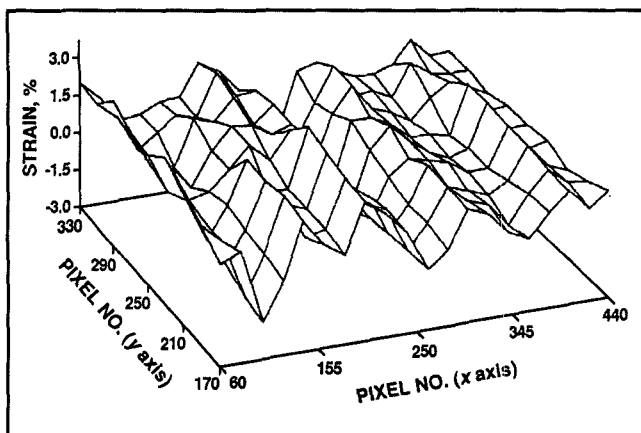
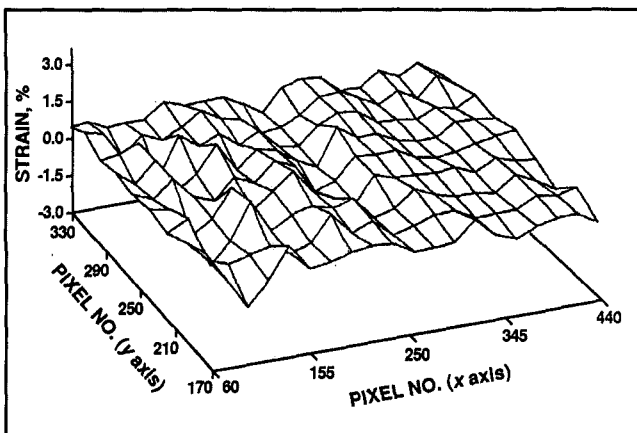
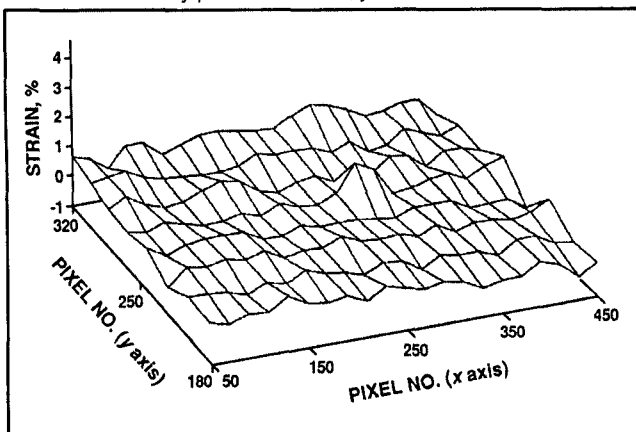
3A. Strain-intensity profile in the CD; $\bar{\epsilon} = 0.15\%$ **3B.** Strain-intensity profile in the CD; $\bar{\epsilon} = 0.22\%$ **3C.** Strain-intensity profile in the CD; $\bar{\epsilon} = 0.34\%$ **3D.** Strain-intensity profile in the CD; $\bar{\epsilon} = 0.35\%$ **3E.** Strain-intensity profile in the CD; $\bar{\epsilon} = 0.55\%$ **3F.** Strain-intensity profile in the CD; $\bar{\epsilon} = 0.26\%$ 

Figure 3A shows the first stage in hydroexpansion of the smooth cylindrical surface in the axial direction (CD). The average in-plane axial strain is 0.15%, while strain ranges from -0.41% to 0.84%. The profile does not suggest any trend. Figure 3B has

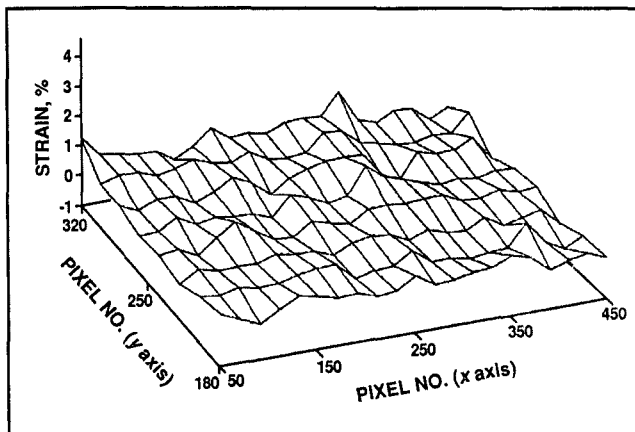
an average in-plane axial strain of 0.22%, with a range of -0.73% to 1.09%. The figure shows incipient ridging, with peaks of higher-than-average strain and valleys of lower-than-average strain. Figure 3C has an average axial strain of 0.34% with a

range between -2.13% and 2.28%. Four definite ridges and valleys of strain are evident. Figure 3D has an average axial strain of 0.35% and a range of -2.40% to 2.16%. Despite the small change in average strain from Fig. 3C to 3D, there is movement

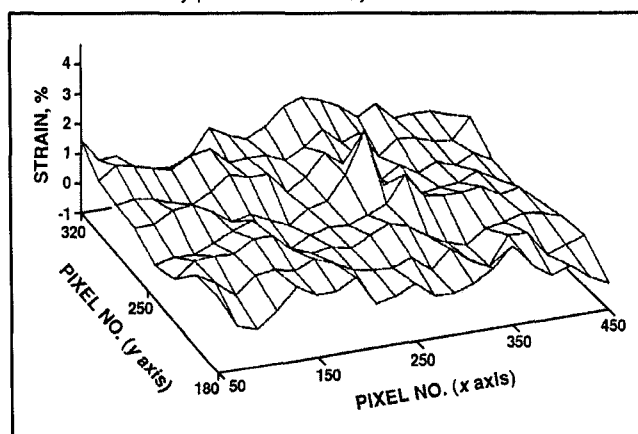
4A. Strain-intensity profile in the MD; $\bar{\gamma} = 0.21\%$



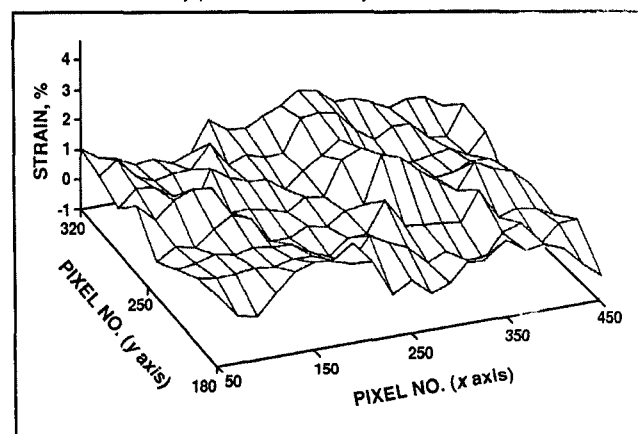
4B. Strain-intensity profile in the MD; $\bar{\gamma} = 0.35\%$



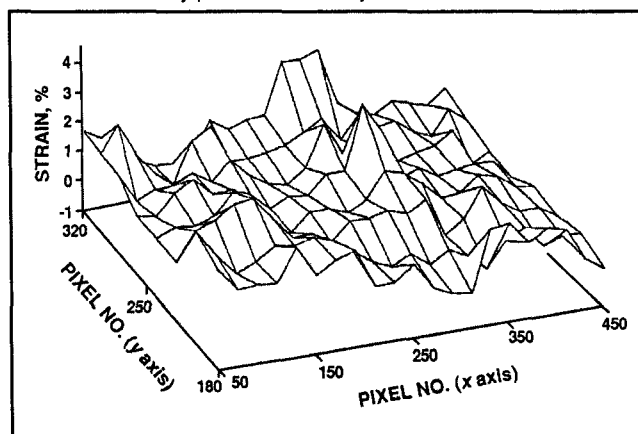
4C. Strain-intensity profile in the MD; $\bar{\gamma} = 0.67\%$



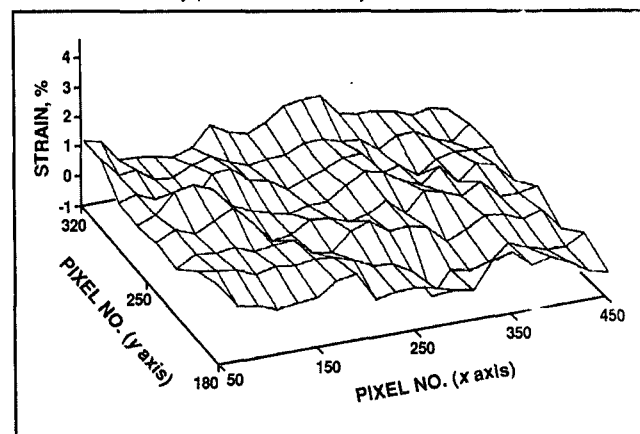
4D. Strain-intensity profile in the MD; $\bar{\gamma} = 0.88\%$



4E. Strain-intensity profile in the MD; $\bar{\gamma} = 1.16\%$



4F. Strain-intensity profile in the MD; $\bar{\gamma} = 0.49\%$



within the strain profile. Figure 3E has an average axial strain of 0.55% and a range of -1.84% to 2.77%. The movement shown is not so much in magnitude alone but in translation as well. Figure 3F is the relaxed or contracted strain profile after the humidity was allowed to return to ambient conditions. The average axial

strain is 0.26% and the range extends from -1.01% to 1.32%. The strain-intensity profile in Fig. 3F resembles the profile in Fig. 3B.

Figures 4A-4F show the radial (MD) hygroexpansion changes in the toilet tissue roll that occurred simultaneously with the corresponding axial (CD) changes depicted in Figs.

3A-3F. Figure 4A has an average radial strain of 0.21% with a range of -0.41% to 1.19%. The average radial strain for Fig. 4B is 0.35% with a range of -0.19% to 1.32%. The strain profile has become irregular without suggesting any trend. In Fig. 4C, the average radial strain is 0.67% and the range falls between -0.21% and 2.64%.

Peaks and local troughs appear, but without a pattern. Figure 4D has an average radial strain of 0.88% and a range between -0.25% and 2.48%. The peaks appear to alternate. Figure 4E has an average radial strain of 1.16% and a range between -0.12% and 3.53%. The alternation of the peaks is reinforced. Figure 4F is the relaxed strain profile after returning to ambient conditions. The average radial strain is 0.49% with a range of -0.93% to 1.47%. Residual strains continue as diminished local peaks and troughs.

Discussion of results

The alternating-diamond buckling pattern is a result of the axial compression initiated in the outer cylinders of a paper roll by constrained hygroexpansion forces. The pattern can be observed visually during its development. In its pure form, the pattern of alternating-diamond buckles can only develop as a nonextensional mapping of a smooth cylindrical surface that has been translated into individual alternating-diamond patterns by straightening the corresponding parts of the cylinder. However, for geometric reasons, the cylinder surface can only acquire the alternating-diamond buckling pattern through both extensional deformation and large bending deformations. The bending deformations are necessary along the edges of the diamonds and develop only if the material has a very low bending rigidity.

The best first approximation of the local buckling deformation from the cylindrical shape that characterizes the alternating-diamond buckling pattern is given by the following deformation equation.

$$W = W_i \sin(\pi/l_x) \sin(\pi/l_y) \quad (2)$$

where

W_i = maximum buckle deformation

l_x = axial position on the cylinder

l_y = radial position on the cylinder

Measured strains will be in-plane strains that might be interpreted with the deformation equation.

The progression of strains in Figs. 3A-3E shows the early emergence of four ridges of high strain and three valleys of low strain. The ridges in Figs. 3D and 3E show a splitting or

broadening as the corner of the buckle is encountered. The accompanying strains in the MD shown in Figs. 4A-4E show the development of individual peaks of high strain that again alternate within the field of view.

It was anticipated that local hygroexpansion would initially be quite uniform in each direction. Axial (CD) and circumferential (MD) expansion were expected to differ. The uniform expansion was then expected to give way to nonuniform strains in the areas of bending at the buckling edges. It appears that the initial uniform expansion was short in duration. Figures 3A and 4A might be characterized that way. But Figs. 3B and 4B show the inception of a change in buckling plane that suggests that even though the strain may be uniform, the buckling has begun. This early buckling thwarted attempts to define the precise hygroexpansion of the cylinder surface.

Quantitative measurements appear to be beyond the data that were collected. Attempts were made to examine the change in local strain in an area where the end of a diamond buckle arises (pixels 60 to 250 in the x -direction and 170 to 250 in the y -direction). Resolution of the stretching that takes place locally can be seen to arise with the unaided eye but cannot be measured accurately. The in-plane measurements show the trends very well but do not allow a separation of the out-of-plane bending from the stretching.

Conclusion

Roll cockle—a pattern of alternating diamonds on the surface of a paper roll—is caused by the compression buckling that occurs when hygroexpansion is constrained. This is a significant source of economic loss, and a deeper understanding of the compression-buckling phenomenon may help reduce such losses.

The image-correlation strain-measurement technique was used to measure the complex, dynamic dimensional instability of a paper roll. This first attempt to experimentally define the development of roll cockle on the exterior of a paper roll met with limited success. The strains and deformations provided in this article are in-plane (two-dimensional) measurements. However, the actual defor-

mations in the buckling pattern are interesting because of their alternating out-of-plane (three-dimensional) oscillations.

Precise measurements at the areas of combined high bending and stretching could not be made because of the out-of-plane deformation. These measurements should prove to be most interesting in the future. Nevertheless, the work presented here provides an important first step by illustrating the axial and radial expansions that lead to compression buckling of the paper roll. □

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