

Mechanisms of liquid sheet breakup and the resulting drop size distributions

Part 1: types of spray nozzles and mechanisms of sheet disintegration

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ABSTRACT *Both a wave mechanism and a perforation mechanism have been proposed as the first step in the breakup of fluid sheets. For black liquor sprays, the dominant mechanism is the formation and growth of perforations. According to either mechanism, cylindrical strands develop and subsequently break up to form drops. By combining the results of analyzing the breakup of both the sheet and strands, only a discrete number of drop sizes can be predicted from the wave mechanism. However, the perforation mechanism can be used to predict a distribution of drop sizes similar to that observed experimentally.*

KEYWORDS

Bibliographies
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Sheets
Sprayers
Spraying
Waves

The presence of undersized and oversized drops in a kraft recovery boiler contributes to reduced capacity and decreased stability. Experimental studies of black liquor sprays have been performed using various commercial and laboratory-scale nozzles operating over a wide range of conditions. A distribution of drop sizes was observed in every case (1, 2). In fact, almost all industrial atomizers

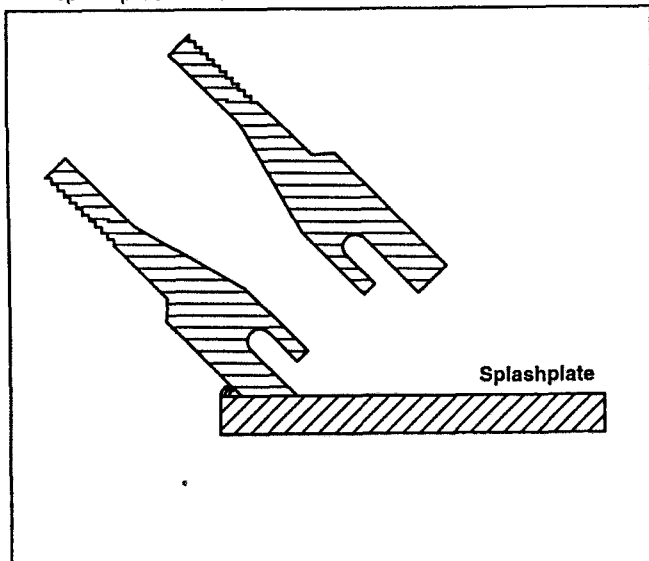
produce drops over a broad distribution of sizes.

In the black liquor spraying process, the bulk fluid is converted to drops through fundamental hydrodynamic mechanisms. In the first step of this process, the black liquor nozzle forms a fluid sheet. The pertinent literature describes how the wave and perforation mechanisms of sheet breakup lead to the formation of

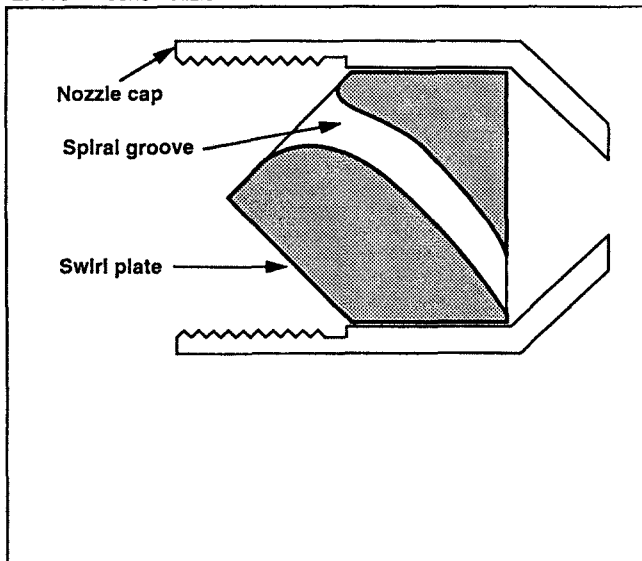
cylindrical strands of fluid.

The formation of drops from liquid strands is usually described using jet breakup theory. These theories are reviewed in Parts 1 and 2, with a focus on the conditions expected to dominate in black liquor sprays. Finally, the results of analyses of sheet and jet breakup can be combined to predict drop sizes. Part 1 begins with a description of the different types of

1. A splash plate nozzle



2. A swirl cone nozzle



black liquor spray nozzles and presents the mechanisms of sheet disintegration. Part 2 presents the theories of strand breakup and includes the experimental observations.

Black liquor spray nozzles

Splash plate nozzles

Figure 1 depicts a typical splash plate nozzle. In this nozzle, the fluid is accelerated through a tube with a short entrance length, and it exits from a circular orifice usually 1–3 cm in diameter. The resulting free jet travels a short distance before striking a flat plate at an oblique angle. The forces developed in the region of impact drive the fluid out radially, parallel to the plate surface (3). The cup region at the rear of the nozzle redirects the backward flowing portion of the fluid toward the front of the splash plate. The resulting fluid sheet has three distinct regions.

The edges of the sheet are thick rims formed as the fluid contracts as a result of surface tension. As these rims break up from the action of surface tension, inertial, and aerodynamic forces, new rims are rapidly formed (4). If this rim disintegration can be described using conventional theories of fluid jet breakup, then the resulting drop sizes can be predicted.

The redirected back flow impinges the forward-flowing liquid near the center line of the sheet. The complex interactions of the sheet and the impinging recirculation flow would be difficult to characterize analytical-

ly. Therefore, a description of the breakup of the center of the sheet would rely almost exclusively on experimental observations.

A radially expanding, thinning sheet constitutes the bulk of the spray. For the simple case in which an inviscid circular jet impinges on a plate at a 90° angle, the half thickness of the sheet h_x at any radial position x measured from the point of jet impact can be obtained from a mass balance.

$$h_x = \left[\frac{D_o^2}{16} \right] \left(\frac{1}{x} \right) \quad (1)$$

An estimate of the half thickness of the undisturbed sheet region of the spray produced by a typical splash plate nozzle is given by Eq. 2, where α , the angle through which the sheet spreads, is approximately π .

$$h_x = \left[\frac{U_o}{U_s} \frac{\pi D_o^2}{8\alpha} \right] \left(\frac{1}{x} \right) \quad (2)$$

In Eq. 2, the ratio of jet velocity to sheet velocity, (U_o/U_s) , is included to account for the decrease in fluid velocity and the corresponding increase in sheet thickness that occur as a result of viscous drag on the plate. In most circumstances, the sheet velocity cannot be measured directly, nor can it be predicted exactly unless the impact angle is 90° (3). However, Obuskovic and Adams (5) developed an expression, based on the principles of conservation of mass and momentum, which can be used to approximate the velocity of a sheet leaving the tip of a splash plate nozzle. The resulting expression, given as Eq. 3.

$$\frac{U_o}{U_s} = 2 + \frac{32}{\text{Re}_j} \left[\left(\frac{x_t}{D_o} \right)^3 - \left(\frac{x_v}{D_o} \right)^3 \right] \quad (3)$$

In this expression, x_t is the radial position of the edge of the plate as measured downstream from the point of jet impact. The radial position at which viscous forces are assumed to begin, x_v , is expected to be on the order of the jet diameter, D_o . Equation 3 can be used to estimate the half thickness of the sheet in the bulk portion of the splash plate spray.

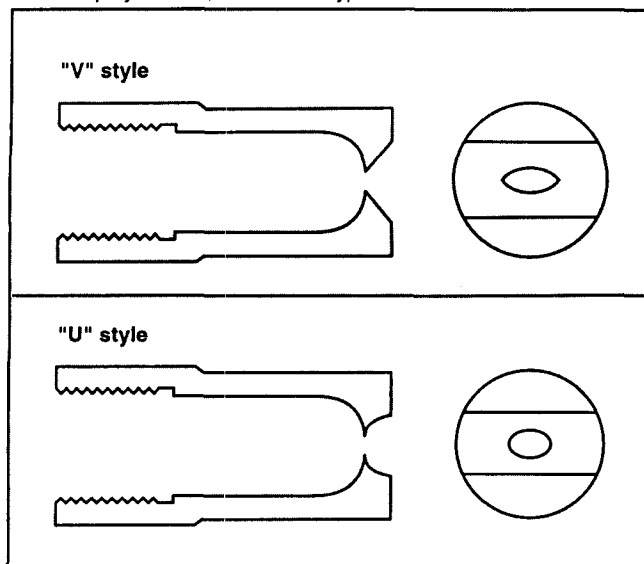
$$h_x = \left[\left\{ 2 + \frac{32}{\text{Re}_j} \left(\left(\frac{x_t}{D_o} \right)^3 - \left(\frac{x_v}{D_o} \right)^3 \right) \right\} \left(\frac{\pi D_o^2}{8\alpha} \right) \right] \left(\frac{1}{x} \right) \quad (4)$$

Swirl cone nozzles

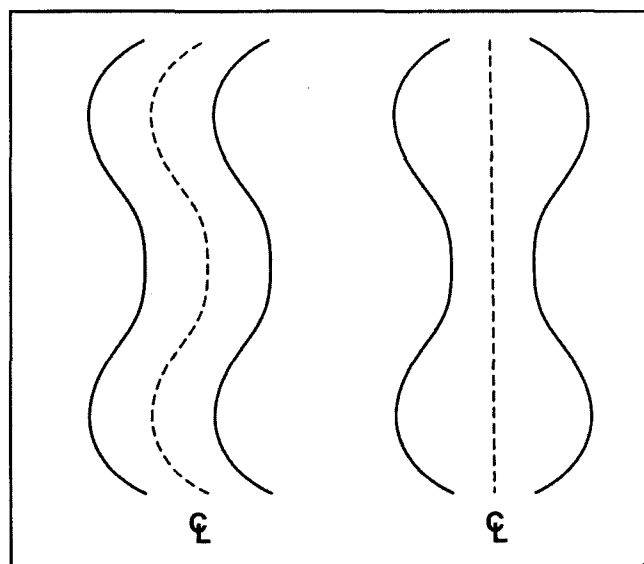
Figure 2 shows a sketch of a swirl cone nozzle. As the liquid enters the nozzle, the flow is split in two by the swirl plate. Each half is accelerated along a short curved path, thus increasing the average linear velocity of the fluid and introducing an angular velocity component. The fluid leaves the swirl plate and enters a short converging passage.

Here, by conservation of angular momentum, the angular velocity of the fluid increases as the radius of the passage through which it passes decreases. When the fluid exits the orifice, the centripetal force imparted to the fluid by the retaining walls is no longer active, and the liquid spreads out to form a conical, radially expanding sheet. The initial angle of the cone is determined by the relative magnitudes of the tangential and

3. Fan spray nozzles, "V" and "U" type



4. Sinuous and dilational wave forms



radial velocity components.

The half thickness of the sheet as a function of the radial distance from the nozzle and the cone angle, β , can be estimated from a mass balance.

$$h_x = \left[\frac{m_o}{4\pi\rho_{liq}U_s \tan\beta} \right] \left(\frac{1}{x} \right) \quad (5)$$

The local cone angle is fixed by a balance of inertial and surface tension forces. Since the magnitudes of these forces vary with the distance from the orifice, the cone angle will also be a function of the position downstream. Taylor developed a prediction for the shape of water bells produced by jets impinging on various conical diffusers (6). Using the equation developed for the case of no pressure difference between the inside and outside of the cone and negligible gravity effects, an expression for $\tan\beta$ as a function of the downstream distance and the initial cone angle, β_o , can be derived. The result is given as Eq. 6.

$$\tan\beta = \frac{1 - \cosh[\cosh^{-1}(\sec\beta_o) - X \sec\beta_o] \cos\beta_o}{x} \quad (6)$$

where

$$X = x(4\pi\sigma/m_o v_o) \\ = \text{dimensionless distance}$$

The breakup of the swirl cone sheet is expected to be similar to that of the bulk sheet portion of the splash plate spray. However, the curvature of the sheet may affect breakup, as we will consider.

Fan spray nozzles

A nozzle of the fan spray style consists of a smooth entrance pipe with a spherical cap. The orifice is formed by cutting a groove across the top of this cap. The nozzle discharge coefficient and the spray characteristics are functions of the shape of the groove. Though there are numerous groove configurations (7), two common cross sections used in black liquor sprayers are the "V" and "U" shapes. These two groove styles are shown in Fig. 3.

The fluid exits the fan spray nozzle as a thick sheet. Though the angle through which this sheet spreads varies with nozzle design, it is commonly much less than the spread angle for a B&W splash plate nozzle. The resulting sheet thickness is generally much greater than that formed by a splash plate or a swirl cone nozzle.

The actual sheet thickness at a given distance from the nozzle will depend on the nozzle geometry and the initial angular distribution of mass at the orifice. If a uniform sheet thickness is assumed to exist at the point of discharge, Eq. 7 can be used to estimate the half thickness of the sheet as a function of the distance downstream.

$$h_x = \left[\frac{A_o}{2\alpha} \right] \left(\frac{1}{x} \right) \quad (7)$$

Mechanisms of sheet disintegration

The wave mechanism

A liquid sheet moving relative to a second fluid is subject to Kelvin-

Helmholtz type instabilities (8). The position of the air-liquid interfaces will fluctuate as a result of the random disturbances present in all real systems. When the position of the interface is perturbed locally, aerodynamic, inertial, viscous, and surface tension forces will be active, and the stability of the boundary will be determined by the balance among them. If these disturbances can be decomposed into their normal modes, the stability of each frequency component can be examined separately.

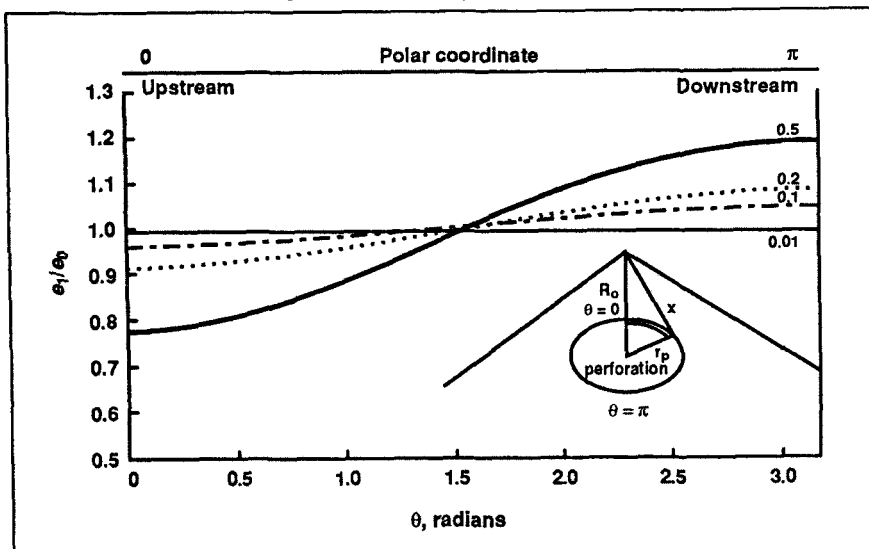
According to the wave mechanism, sheet breakup results from the temporal or spatial growth of the unstable modes. When a disturbance reaches some critical amplitude, the sheet breaks at half wavelength intervals and forms bands of fluid. Surface tension forces rapidly contract this fluid into cylindrical strands that ultimately break up into drops (9).

Planar inviscid sheets

Numerous analytical studies on the growth of disturbances in fluid sheets have been performed using the ideal flow assumption (10-12). (An ideal flow is inviscid, incompressible, and irrotational.) In these studies, the stability of planar fluid sheets was modeled in Cartesian coordinates using a first-order perturbation scheme. Here, we will look at some of the more important results from these analyses.

A semi-infinite liquid sheet of thickness $2h_o$ travels at a velocity U_s relative to the surrounding air. The

5. Velocity variation vs. the angle θ in radians for $r_p/R_0 = 0.5, 0.2, 0.1$, and 0.01



positions of the upper and lower surfaces are perturbed from their equilibrium positions, $+h_0$ and $-h_0$, respectively, as shown in Eqs. 8 and 9. In a perturbation analysis, H is assumed to be much less than one, and terms of order H^2 and higher are neglected to obtain a solution valid to first order in the perturbation parameter. In Eq. 9, ζ is the phase angle between the disturbances to the upper and lower surfaces.

$$h_+ = +h_0 + H_{\exp}[i(kx - st)] \quad (8)$$

$$h_- = -h_0 + H_{\exp}[i(kx - st) + \zeta] \quad (9)$$

In these expressions, the nature of the radial perturbation wave number, k , governs the spatial behavior of the disturbance, while s , the growth rate parameter, governs the temporal behavior. If k is real only, the disturbance will oscillate sinusoidally with position downstream. If k has a positive imaginary component, the magnitude of the disturbance will decay exponentially with position downstream. If k has a negative imaginary component, the magnitude will grow exponentially. A disturbance which grows exponentially is said to be unstable in an absolute sense.

Similarly for s , a real value indicates an oscillatory disturbance in time, while an imaginary component indicates exponential growth or decay. Because of the negative sign in front of the temporal exponent, the imaginary part of s must be positive for absolute temporal instability to occur.

Expressions for the wave number and the growth rate parameter as functions of the fluid physical properties and the values for the operating conditions are obtained by simultaneously solving the gas- and liquid-phase continuity and momentum equations. Using this procedure, Hagerty and Shea (10) demonstrated that only two wave forms could exist in the sheet. These are: antisymmetric (sinuous) waves, for which the disturbances in the positions of the upper and lower surfaces are in phase ($\zeta = 0$), and symmetric (dilatational or varicose) waves, for which the disturbances are 180° out of phase ($\zeta = \pi$). These wave forms are shown in Fig. 4. At all conditions under which these disturbances were temporally unstable, the calculated growth rate of the antisymmetric wave was greater than that of the corresponding symmetric wave (10, 11). This theoretical result has been supported by experimental observations (10, 13).

For the case of antisymmetric waves, the temporal growth rate of a disturbance is given by Eq. 10.

$$s = \frac{2kU_s \tanh(kh_0) \pm \{[2U_s k \tanh(kh_0)]^2 - 4[\tanh(kh_0) + \gamma][k^2 U_s \tanh(kh_0) - \sigma k^3 / \rho_{\text{liq}}]\}^{1/2}}{2[\tanh(kh_0) + \gamma]} \quad (10)$$

In general, $\gamma \equiv \rho_{\text{gas}}/\rho_{\text{liq}} \ll \tanh(kh_0)$, and the expression for the growth rate parameter reduces to:

$$s = kU_s \pm \left(\frac{\sigma k^3 / \rho_{\text{liq}} - \gamma k^2 U_s^2}{\tanh(kh_0)} \right)^{1/2} \quad (11)$$

In Eqs. 10 and 11, s can be expressed as the sum of a real and an imaginary part.

$$s = s_{\text{real}} + s_{\text{imag}} \quad (12)$$

For the imaginary component to exist, the term in the parentheses in Eq. 11 must be negative. This condition is met when

$$k < \frac{\rho_{\text{gas}} U_s^2}{\sigma} \quad (13)$$

This result was presented by both Hagerty and Shea (10) and Fraser *et al.* (12) as the minimum or critical wavelength of instability. This wavelength, λ_c , is given by Eq. 14.

$$\lambda_c = \frac{2\pi\sigma}{\rho_{\text{gas}} U_s^2} \quad (14)$$

where the definition of the wave number has been used.

$$k \equiv \frac{2\pi}{\lambda} \quad (15)$$

The wavelength of the disturbance of maximum growth was predicted by Squire (11) and Fraser *et al.* (12) and is given by Eq. 16.

$$\lambda_{\text{opt}} = \frac{4\pi\sigma}{\rho_{\text{gas}} U_s^2} \quad (16)$$

Because first-order perturbation methods were used to obtain these results, the expressions are valid only when the magnitude of the disturbance is small compared to the sheet thickness. However, it is generally assumed that the optimally growing wave at this condition will also dominate at breakup when the amplitude is large (14).

Planar viscous sheets

The results obtained for inviscid planar sheets have been extended by Dombrowski and Johns to include the effects of a finite liquid-phase viscosity and a decreasing sheet thickness (9). They assumed that the rate of change in sheet thickness with position downstream was small, such that the sheet could be treated as planar at any given location. They found that for an inviscid sheet the wave of maximum growth was not affected by sheet thickness and could be calculated using Eq. 16.

In a viscous sheet, the growth rate of a disturbance was predicted to be a function of both wavelength and sheet thickness. Thus, the wave with the maximum growth rate at one

distance from the nozzle may not be the optimally growing wave farther from the nozzle where the sheet is thinner. However, the results of this analysis were used to demonstrate the existence of a wave of maximum amplitude at all positions downstream (9). Therefore, the breakup of the sheet is expected to be dominated by a disturbance of a particular wavelength.

Dombrowski and Johns also found that, as expected, viscous forces retard the growth of perturbations (9). Therefore, the growth rate of a disturbance in a viscous sheet is less than the growth rate for the corresponding disturbance in an inviscid sheet.

Radially expanding viscous sheets

The liquid sheet formed by a typical black liquor spray nozzle expands radially from the nozzle orifice. The resulting cylindrical geometry of the sheet leads to the prediction of waves that are spatially unstable in addition to the temporally unstable waves.

Weihs performed a theoretical analysis on the stability of a viscous, radially thinning liquid sheet (15). He found that the temporal instability limit was identical to that calculated by Fraser *et al.* (12) and Hagerty and Shea (10) and that the wave of maximum growth was a function of sheet velocity and thickness. The magnitude of the change in the wavelength of maximum growth per unit change in sheet thickness was predicted to increase with increasing fluid viscosity.

In addition to temporal instability, Weihs reported a form of spatial instability resulting from the presence of a sinusoidally varying disturbance with increasing successive maxima (15). This instability could occur even when the sheet was temporally stable.

Curved sheets

The fluid sheet formed by a swirl cone nozzle is curved. Crapper, Dombrowski, and Pyott (16) investigated the effects of this geometry by performing a first-order perturbation analysis on the growth rate of wave disturbances in an annular fluid film. The results of this analysis can be extended qualitatively to the case of a swirl cone spray in which the radius of the annulus increases with distance from the nozzle.

Crapper and co-workers predicted both the growth rate of disturbances and the wave of maximum growth to be functions of the inner radius of the annular film (16). However, the effect was negligible when the inner radius of the annular spray was greater than about 1 cm. The inner radius of a typical black liquor swirl cone spray is much greater than this critical value. Thus it appears that the curvature of the swirl cone sheet can be neglected when analyzing its stability.

Several features of wave breakup can be inferred from these theories. First, while the cause of the wave disturbances is unknown, if all waves result from random fluctuations in the sheet and the surrounding air, then all wavelengths are equally likely to occur, and all initial amplitudes will be of the same order of magnitude (9). Second, all disturbances for which the imaginary part of the temporal growth rate parameter (s_{imag}) is positive will grow exponentially. Furthermore, a disturbance of a particular wavelength will dominate in the region of breakup.

When the amplitude of this wave reaches a critical value, the liquid will break into ribbons, which will collapse into cylindrical strands and ultimately form drops. Since a single wave is predicted to cause the formation of strands from the sheet, all strands so produced will have the same diameter. As will be shown later, following this mechanism leads to the prediction of a discrete number of drop sizes.

The perforation mechanism

A second mechanism of sheet breakup is the initiation and growth of perforations. Once formed, a hole in a fluid sheet is rapidly expanded by surface tension forces, and the fluid is drawn up into a thick rim about the hole (7). The expansion of a perforation is halted when this rim reaches the edge of the sheet or another perforation rim. The result is a web of interconnected strands of fluid that will break up under surface tension and aerodynamic forces to form drops.

Growth rate of perforations

The growth rate of a perforation is rapid, provided its initial diameter is larger than the thickness of the fluid sheet. Perforations smaller than this

will close back into the sheet (17). Fraser *et al.* derived an expression for the velocity of the rim of a perforation growing in a planar liquid sheet (12). This analysis was based on the balance between the surface tension force driving the expansion and the increase in inertia of the growing rim. The resulting equation is given as:

$$e = \sqrt{\frac{\sigma}{h\rho_{\text{liq}}}} \quad (17)$$

If the sheet is expanding radially, the half thickness will be proportional to the reciprocal of the distance from the nozzle, as shown in Eqs. 2, 4, 5, and 7. In each case, a general proportionality constant, K_0 , can be used to replace the term in square brackets, reducing these expressions to Eq. 18. In the case of Eq. 4, K_0 is not a true constant because $\tan(\beta)$ varies with downstream position.

$$h_x = \frac{K_0}{x} \quad (18)$$

For a perforation of radius r_p centered at distance R_0 from the nozzle, the distance from the orifice to a point on the rim is given by Eq. 19.

$$x = \sqrt{R_0^2 + r_p^2 - 2R_0r_p \cos(\theta)} \quad (19)$$

In some cases, the distance from the nozzle to a point on the rim can be approximated by the distance to the center of the perforation (zeroth order in r_p). When the radius of the perforation is of the same order as the distance from the nozzle, a first-order correction in r_p must be applied. The resulting expressions are given as Eqs. 20 and 21, respectively.

$$x_0 = R_0 \quad (20)$$

$$x_1 = R_0 - r_p \cos(\theta) \quad (21)$$

Fraser *et al.* used the zeroth-order approximation to estimate the growth rate of a perforation in a thinning sheet (12). Their result is given as Eq. 22. In this work, the effects of applying the first-order approximation are demonstrated. This expression for rim velocity is given as Eq. 23.

$$e_0 = \left(\frac{2\sigma}{K_0\rho_{\text{liq}}} \right)^{1/2} R_0^{1/2} \quad (22)$$

$$e_1 = \left(\frac{2\sigma}{K_0\rho_{\text{liq}}} \right)^{1/2} [R_0 - 0.8 r_p \cos(\theta)]^{1/2} \quad (23)$$

The zeroth-order equation predicts a uniformly expanding perforation, while the first-order solution predicts

that the perforation will grow faster in the downstream direction ($\theta = \pi$), where the sheet is thinner, and slower in the upstream direction ($\theta = 0, 2\pi$), where the sheet is thicker. The magnitude of this variation is a function of the ratio of the perforation radius to the distance from the nozzle, as shown in Fig. 5.

The radius of a strand formed via the perforation mechanism will depend on a number of parameters, including the location downstream at which it was formed, the time for growth, and interactions with other strands. Therefore, unlike the wave mechanism, strands are expected to form over a range of sizes. As will be shown in Part 2, this appears to be a prerequisite for the formation of the wide distribution of drop sizes observed in spray experiments.□

Literature cited

1. Spielbauer, T. M., Adams, T. N., Monacelli, J. E., and Bailey, R. T., 1989 CPPA Intl. Chemical Recovery Conference, CPPA, Montreal, p. 119.
2. Bennington, C. P. J. and Kerekes, R. J., *J. Pulp Paper Sci.* 12(6): J181(Nov. 1986).
3. Taylor, Sir G., *Phil. Trans. Royal Soc. London A*(260): 96(1966).
4. Clark, C. J. and Dombrowski, N., *Chem. Eng. Sci.* 26: 1949(1971).
5. Adams, T. N., Empie, H. L., Obuskovic, N., and Spielbauer, T. M. Kraft Black Liquor Delivery Systems, DOE contract no. FC02-88CE40839. Report No. 1, :38-59 (Feb., 1990).
6. Taylor, Sir G., *Proc. Royal Soc. London A* 253: 289(Dec. 1959).
7. Dombrowski, N. and Fraser, R. P., *Phil. Trans. Royal Soc. London A* 247: 101(1954).
8. Panton, R. L., *Incompressible Flow*, Wiley, New York, 1984, p. 675.
9. Dombrowski, N. and Johns, W. R., *Chem. Eng. Sci.* 18: 203(1963).
10. Hagerty, W. W. and Shea, J. F., *J. Applied Mechanics* (Dec.): 509(1955).
11. Squire, H. B., *Brit. J. of Applied Physics* 4: 167(June 1953).
12. Fraser, R. P., Eisenklam, P., Dombrowski, N., and Hasson, D., *AIChE J.* 8(5): 672(Nov. 1962).
13. Crapper, G. D., Dombrowski, N., and Jepson, W. P., *Proc. Royal Soc. London A* 342: 225(1975).
14. York, J. L., Stubbs, H. E., and Tek, M. R., *Trans. ASME* 1279(Oct. 1953).
15. Weihs, D., *J. Fluid Mech.* 87(2): 289(1978).
16. Crapper, G. D., Dombrowski, N., Pyott, G. A. D., *J. Fluid Mech.* 68(3): 497(1975).
17. Taylor, G. I. and Michael, D. H., *J. Fluid Mech.* 58(4): 625(1973).

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Nomenclature

A	= surface area, m^2
A_o	= cross-section of the area of a fan spray nozzle, m^2
D_o	= nozzle orifice diameter, m
e	= rim velocity, m/s = $\partial r_p / \partial t$
e_o	= rim velocity corrected to zeroth order for sheet thickness, m/s
e_1	= rim velocity corrected to first order in sheet thickness, m/s
h_o	= unperturbed half sheet thickness, m
h_x	= half sheet thickness at a position x downstream, m
H	= magnitude of a perturbation to the sheet interface position, m
h	= perturbed sheet surface position, m
k	= spatial perturbation wave number = $2\pi/\lambda$, m^{-1}
k_{max}	= wave number of the optimum disturbance at breakup, m^{-1}
k_o	= geometric nozzle parameter, m^2
L	= length of a perforation rim, m
L_{total}	= total length of all group q perforation rims, m
m_o	= mass flow rate through a swirl cone nozzle, kg/s
N_p	= number of perforations
N_d	= number of drops
n	= tangential perturbation wave number
P	= pressure, Pa
r	= strand radius, m
r_o	= initial strand radius, m
r_p	= perforation radius, m
r_x	= perforation radius at position x , m

R	= radius of curvature, m^{-1}
R_o	= distance from nozzle to center of perforation, m
s	= growth rate parameter, s^{-1}
t	= time, s
U_o	= fluid velocity at orifice exit, m/s
U_s	= mean free fluid sheet velocity, m/s
V	= gas velocity relative to the liquid surface, m/s
v_j	= velocity vector for phase j , m/s
v_x	= velocity component normal to the center line of the sheet, m/s
v_z	= velocity component tangent to the center line of the sheet, m/s
x	= radial position measured from the point of sheet formation, m
x_t	= radial position of the tip of a splash plate, m
x_v	= radial position at which viscous forces begin on a plate, m
z	= radial distance in a strand, m = out-of-plane distance in a sheet, m
I_n	= modified Bessel function of the first kind, of order n
Re_j	= Reynolds number, based on liquid properties, jet diameter = $\rho_{liq} U D_o / \mu_{liq}$
Re_r	= Reynolds number, based on liquid properties, strand radius = $\rho_{liq} U r_o / \mu_{liq}$
We_h	= Weber number, based on liquid properties, sheet thickness = $\rho_{liq} U^2 h_s / 2\sigma$
We_r	= Weber number, based on liquid properties, strand radius = $\rho_{liq} U^2 r_o / \sigma$

Greek symbols

α	= spread angle of a fluid sheet, radians
β	= swirl cone spray angle, radians
ϕ_j	= velocity potential for phase j , m^2/s
Φ_j	= magnitude of a disturbance to the velocity potential, m^2/s
γ	= density ratio = ρ_{gas} / ρ_{liq}
η	= disturbance magnitude, m
λ	= wavelength of a fluid sheet disturbance, m
λ_c	= minimum wavelength of an unstable sheet disturbance, m
λ_{opt}	= optimum sheet disturbance wavelength, m
λ_{max}	= wavelength of the optimum disturbance at breakup, m
μ	= viscosity, cp
θ	= angle, in radians or azimuthal polar coordinates
ρ	= density of the phase, kg/m
σ	= surface tension, dynes/cm
ξ	= dimensionless wave number
ψ	= dimensionless growth rate parameter
ζ	= phase angle between the disturbances to the upper and lower surfaces of the fluid sheet

Subscripts

c	= critical
d	= drops
gas	= gas phase
$imag$	= imaginary component
j	= phase, liquid or gas
liq	= liquid phase
max	= maximum
o	= nozzle orifice
opt	= optimum
$real$	= real component
p	= perforations
s	= sheet
x	= position downstream
$+$	= upper, in regard to the fluid sheet
$-$	= lower, in regard to the fluid sheet