Plane stress fracture toughness of paper

Rajinder S. Seth, A. Gordon Robertson, Yiu-Wing Mai, and John D. Hoffmann

ABSTRACT: Fracture toughness is a material's ability to resist crack propagation. In most end uses where fracture toughness can be an important service parameter for paper, stresses are applied in the plane of the sheet. Elmendorf or Brecht-Imset tear tests do not measure a plane stress fracture toughness, as the tearing forces are applied out of the sheet's plane. Methods currently available for plane stress measurements on tough papers are either inappropriate or are tedious and inaccurate. A method which is simple, practical and amenable to laboratory-size specimens has been explored. It is based on the "essential work of plane stress ductile fracture" concept developed by Cotterell and Mai. The method has been successfully applied previously to metallic and polymeric sheet materials, and the results, which are independent of the specimen geometry, have been identified with the fracture toughness J_c or R of the material. With measurements on a variety of samples, we demonstrate that the method is applicable to paper.

KEYWORDS: Bibliographies, fracture, paper properties, paper sheets, stresses, test methods, toughness.

The three fundamental mechanical properties of a material are tensile strength, elastic modulus, and fracture toughness. Fracture toughness is a material's ability to resist crack propagation. In most end uses where fracture toughness can be an important service parameter for paper, stresses are applied in the plane of the sheet. Although the tensile strength and elastic modulus of paper are measured for stresses in the plane of the sheet, resistance to crack propagation currently is measured by the Elmendorf or Brecht-Imset tear tests *(1, 2)*, which apply tearing forces out of the sheet's plane. The resistance of paper to fracture depends on the failure mode, and for a test to be relevant, it must relate clearly to the end-use failure mode. Therefore, the fracture toughness of paper should be measured under plane stress loading. Plane stress fracture toughness should prove to be more reliable than out-of-plane tear for evaluating pulp and paper strength for end use.

Although the mechanics of fracture is a complex subject, considerable understanding has been developed *(3, 4)*. During fracture, irreversible work is done to create new surfaces. Deformation, damage, and fracture processes are concentrated in the region just ahead of the crack tip when a sample containing a crack is strained. This localized region is called the fracture process zone (FPZ). The irreversible work consumed in this zone, per unit crack area, is the specific work of fracture (R) or the fracture toughness of the material. It is a material property. This work is supplied by the strain energy in the material and the external applied forces. The experimental techniques for measuring R depend on the size of the FPZ relative to the crack length, how loads are applied, specimen dimensions, and the magnitude of the strain in the material away from the crack faces *(3)*.

In materials for which the FPZ is small and all irreversible deformations associated with fracture are contained within this zone, and deformations elsewhere are below the yield strain of the material, fracture is called elastic. The mechanics of elastic fracture can be analyzed in terms of the elastic properties of the material. During fracture, the work consumed in the process zone is supplied by the strain energy U of the elastic material that surrounds the crack. At fracture, the critical strain energy release rate G_c (= $-dU/dA$) equals the specific work of fracture R, where A is the crack area. Although G_c can easily be determined from laboratory-size specimens using elastic fracture mechanics, for materials with large FPZ, it is not possible to obtain the fracture criterion G_c unless large specimens are used. Seth and Page *(5)*, who used elastic fracture mechanics to obtain G_c in plane stress for a variety of papers, found that only for brittle pa-

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pers such as newsprint was the FPZ small enough that G_c could be readily measured. For tough papers, G_c depended strongly on specimen width, and specimens wider than 50 cm were required to obtain the fracture toughness.

For tough ductile materials such as those with low yield strength, the FPZ is less clearly defined, as it is always surrounded and screened by an outer plastic zone whose shape and size depend on the specimen and crack geometry and the loading configuration. The assumptions of elastic fracture mechanics do not apply. In such cases, ductile fracture criteria such as J-integral (J_c) and the critical crack tip opening displacement (CTOD) are more appropriate *(3)*.

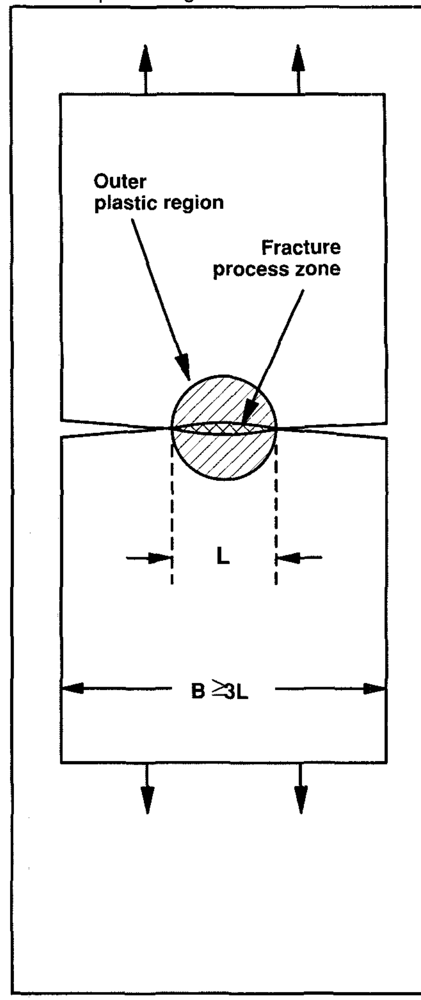
Attempts to promote brittle fracture in otherwise ductile materials have been made *(3, 4)*. By reducing plasticity in the outer zone, a ductile material can be made to fracture in a more constrained quasi-brittle fashion, and R can be measured by the Gurney method *(6)*. Seth and Page *(5)* used this approach to study the fracture of paper, but were criticized *(7)* because conditions for such measurements were not entirely met for tough papers.

Recognizing that elastic fracture mechanics cannot be used for all papers, several authors have attempted to determine fracture toughness by the J-integral approach. Defined along a contour around the crack tip, the J-integral is the change in potential energy with crack extension, and provides the energy release rate. As in the elastic case, $G_c = J_c = -dU/dA$, where U is now the potential energy of the strained ductile material. However, the experimental methods for determining J_c are based on graphical interpretation of the load-displacement curves of notched specimens, and are either complex or tedious *(3)*.

Uesaka *et al.* *(8)* used the method proposed by Begley and Landes (BL) *(9)* to measure J_c of handsheets of a softwood kraft pulp beaten to various levels. The BL method is cumbersome and requires a large number of specimens. Subsequently, Uesaka and Matoba *(10)* investigated the effect of crack length to specimen width ratio on J_c using the BL method and a simpler, single-specimen method proposed by Rice, Paris, and Merkel (RPM) *(11)*. Differences in J_c determined by the two methods for a filter paper were within the experimental error, although the results varied with the crack length to specimen width ratio. Steadman and Fellers *(12)* compared the above two methods for a tough sack paper, and found that the results obtained with the RPM method depended strongly on the crack length to specimen width ratio, and also were sensitive to the specimen geometry. They rejected the RPM method in favor of the BL method in subsequent work *(13)* as the latter gave more consistent results. Pouyet *et al.* *(14)* compared the two methods for sack and writing papers, and found that the BL method gave results higher than the RPM method. The RPM method has also been used by Chatterjee *et al.* *(15)* to determine J_c of handsheets of various pulps which had been subjected to recycling. However, these authors *(14, 15)* did not investigate the sensitivity of J_c to specimen dimensions for their samples. Westerlind, Carlsson, and Andersson *(16)* compared the BL, RPM, and another simpler approach suggested by Liebowitz *et al.* (LEP) *(17–19)* using linerboard. They showed that while the BL and LEP methods gave somewhat similar results, those obtained with the RPM method were significantly higher than the other two, and depended strongly on the crack length to specimen width ratio. Fellers *et al.* *(20)* compared the LEP method and a "modified" BL method *(12)* for a variety of papers. Although the two measurements correlated, the LEP results were approximately 70% of those obtained with the "modified" BL method.

Another significant experimental difficulty in determining J_c lies in establishing precisely the critical load or displacement on the load-displacement curve at which unstable crack growth begins *(3)*. Since crack growth in a fibrous sheet material is difficult to detect, errors in J_c result.

1. Deep double edge notched tension (DENT) specimen showing fracture process zone and the outer plastic region



As an alternative approach to measuring R, Steadman and Sloane *(21)* recently attempted to evaluate the fracture toughness by measuring the critical crack tip opening displacement (δ_c) *(3)*. This is the maximum plastic deformation at the crack tip before crack propagation. At failure, $R \approx \sigma_y \delta_c$, where σ_y is the yield stress of the material. However, their results were preliminary and inconclusive. Moreover, σ_y is arbitrarily defined, and δ_c is difficult to measure for a fibrous material like paper.

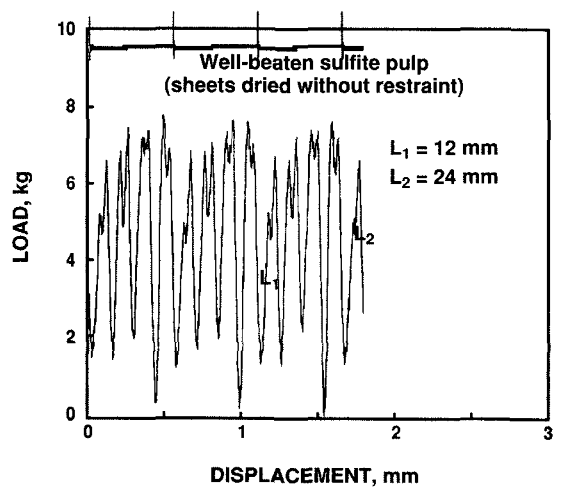
To this point, then, it appears that neither the J-integral approach nor the CTOD method has provided a simple and unambiguous measure of the plane stress fracture toughness of paper with laboratory-size specimens. For a test

I. Description and physical properties of test materials

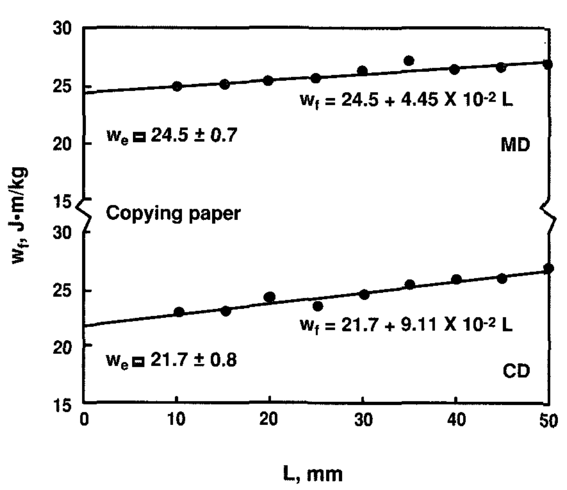
	Airdry [a] grammage, g/m^2	Apparent thickness, μm	Tensile strength, N·m/g	Extension at break, %	Elastic modulus, kN·m/g	0.2% offset yield stress, N·m/g
1. Copying paper, machine direction	77.0	104	62.8	1.64	6.08	43.1
2. Copying paper, cross-machine direction	77.0	104	26.0	3.57	2.25	15.3
3. Handsheets of unbeaten softwood bleached kraft pulp	66.0	111	31.9	2.01	4.62	22.3
4. Handsheets of well-beaten[b] softwood bleached kraft pulp	61.4	90.3	96.3	3.55	7.43	49.3
5. Handsheets of lightly beaten[c] hardwood bleached kraft pulp	65.5	112	38.3	1.95	5.54	27.6
6. Handsheets of well-beaten[d] hardwood bleached kraft pulp	64.8	82.6	93.0	4.03	8.01	51.0
7. Handsheets of well-beaten[b] softwood bleached sulfite pulp	61.6	82.2	64.3	2.92	5.62	43.8
8. Handsheets of well-beaten[b] softwood bleached sulfite pulp, dried without restraint	68.6	119	49.7	7.63	2.61	22.7
9. Softwood thermomechanical pulp	67.3	178	41.0	1.82	3.67	31.6

[a] At 23°C and 50% RH
[b] 6000 revs. in PFI mill
[c] 500 revs. in PFI mill
[d] 12,000 revs. in PFI mill

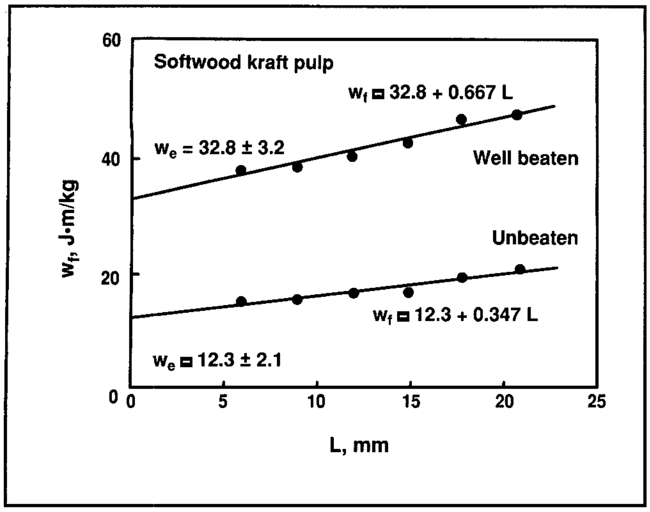
2. Load-displacement curves for DENT specimens with ligaments 12 and 24 mm. The test material is No. 8 in Table I.



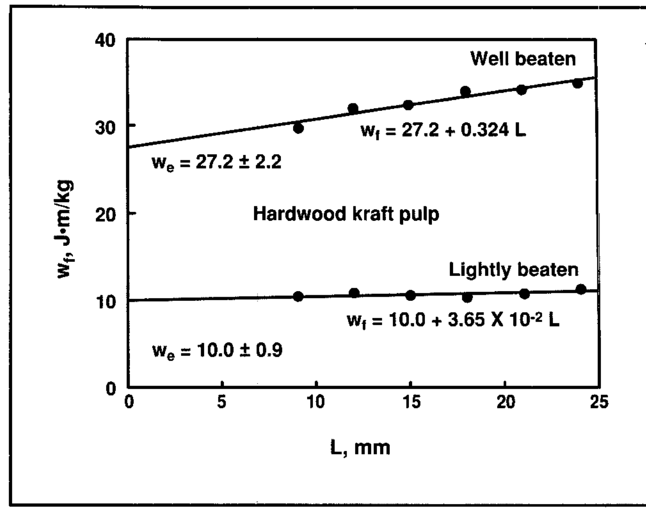
3. Work of fracture against ligament length for copying paper (materials No. 1 and 2). The specimen length was 150 mm.



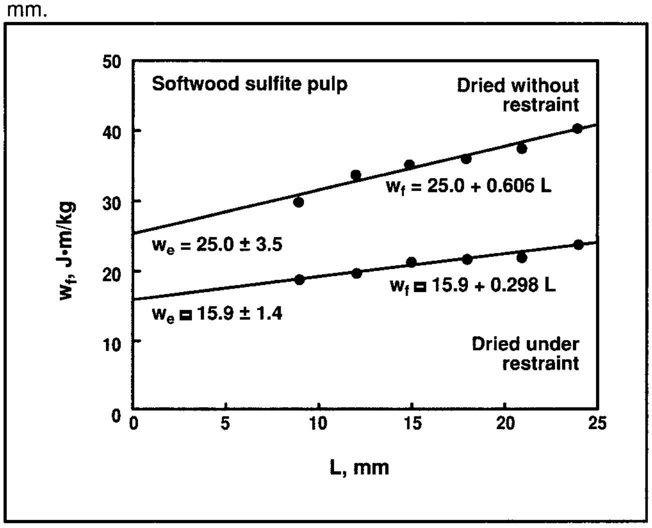
4. Work of fracture against ligament length for softwood kraft handsheets (materials No. 3 and 4). The specimen length was 100 mm.



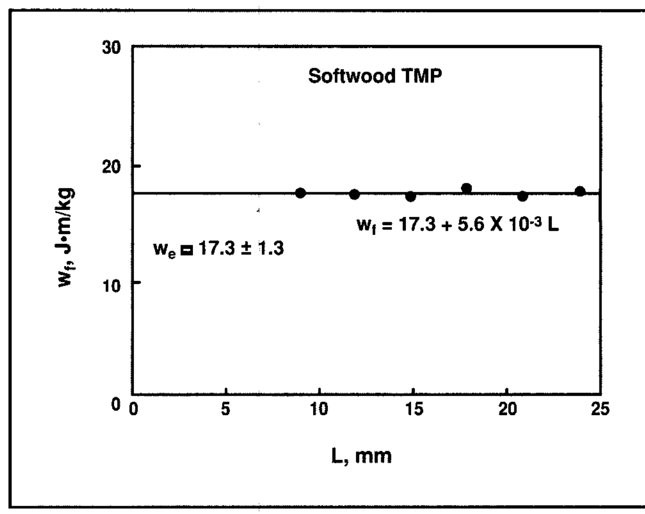
5. Work of fracture against ligament length for hardwood kraft handsheets (materials No. 5 and 6). The specimen length was 100 mm.



6. Work of fracture against ligament length for softwood sulfite handsheets (materials No. 7 and 8). The specimen length was 100 mm.



7. Work of fracture against ligament length for softwood TMP handsheets (material No. 9). The specimen length was 100 mm.



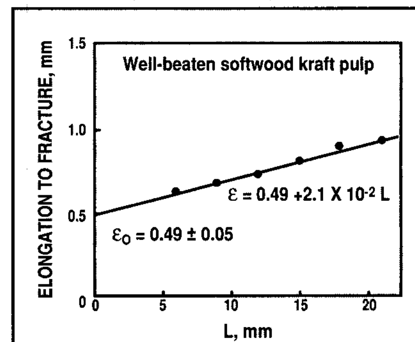
method to be useful, it should measure a material property. Otherwise, results for different furnishes and papermaking conditions cannot be compared. The need to find a simpler and workable method to measure the plane stress fracture toughness of paper thus remained.

Essential work of plane stress ductile fracture

In a ductile material which contains a crack and is under plane stress, the plastic zone ahead of the advancing crack tip is large, particularly in thin sheets. However, much of the plastic flow in the specimen is not directly involved in the fracture process, and the total work of fracture (W_f) in laboratory-size specimens is not a material constant. W_f is the area under the load-displacement curve obtained during the tensile failure of the notched specimen.

Broberg *(22)* suggested, and Cotterell and Reddel *(23, 24)* showed that this work can be partitioned into two components: (a) the essential work of fracture (W_e) consumed in the inner fracture process zone where actual failure occurs, and (b) the nonessential

8. Elongation to fracture against ligament length for DENT specimens of softwood kraft handsheets (material No. 4).



work (W_p) dissipated in the outer screening plastic region. Experimentally, the two work components can be separated using deep double-edge notched tension (DENT) specimens containing varying ligament lengths (L), as shown in **Fig. 1**. When such a specimen yields completely before fracture, the plastic region is confined to a circular area centered on the ligament. On dimensional grounds, one can write:

$$W_f = W_e + W_p$$

$$= Ltw_e + \beta L^2 tw_p \qquad (1)$$

In Eq. 1, w_e is the work consumed per unit crack area in the inner FPZ, and is called the specific essential work of fracture. It is a material property. w_p is the non-essential work dissipated per unit volume of the material, β is a shape factor for the outer plastic zone, which depends on the specimen and crack geometry, and t is the specimen thickness. βw_p is not a material property. Rewriting Eq. 1 gives:

$$w_f = W_f/Lt = w_e + \beta L w_p \qquad (2)$$

If w_f is now plotted against L, a straight line should be obtained whose positive intercept gives w_e and whose slope is proportional to w_p. It can be inferred that when w_p approaches zero, the plastic work dissipated outside the FPZ is negligible, and all the work consumed is the essential work of fracture. One can expect w_p to increase with ductility of the material and to vanish for brittleness.

For Eq. 1 to hold, the conditions are that the ligament remain in plane stress, that the fracture initiation occur after the ligament has fully yielded, that geometric similarity be maintained (i.e., all specimens are of one type, e.g., DENT), and that the fracture propagation be stable. This imposes upper and lower limits on the ligament length L. If L is too small, the fracture can be partly plane strain. If L is too large, the plastic zones at crack tips do not overlap, the ligament is not fully yielded, and W_p is not proportional to L^2. For yielding not to spread to the outer edges of the specimen, the specimen width (B) should be much larger than the ligament. The DENT geometry and ($B/3$, $2r_p$) > L > $5t$ satisfy practical requirements *(23, 24)*; r_p is the radius of the plastic zone at a crack tip in a large specimen. The zone radius, r_p ($=(1/2\pi)[ER/\sigma_y^2]$), is estimated from elastic fracture mechanics *(3)*; E is the elastic modulus of the material. Fracture propagation is stable if the test is performed slowly, and the test system's capacity for strain energy storage is small. A short specimen and stiff testing equipment help.

The specific essential work of fracture w_e has been shown to be a fundamental material property independent of specimen geometry *(25, 26)*. Furthermore, it has been identified as the fracture toughness J_c or R of the material *(26–30)*. This approach has been used extensively to evaluate the fracture toughness of metallic and polymeric sheet materials *(23–35)*. The objective of our investigation was to determine whether this technique could be used to measure the plane stress fracture toughness of laboratory-size specimens of paper.

Results and discussion

Samples used in this investigation included both commercial papers and laboratory handsheets. Their description and physical properties are given in **Table I**. **Figure 2** shows typical load-displacement curves for specimen sets for two ligament lengths. The area under each curve is the total work of fracture W_f. **Figures 3–7** show plots of w_f against L for nine samples. Since paper is rough and porous, its thickness (t) is uncertain. Therefore, thickness was replaced by sheet grammage (G) in calculating w_f ($=W_f/LG$). This is equivalent to expressing results as work of fracture per unit area divided by the material density. Each data point in Figs. 3–7 is a mean of at least seven tests. The lines are best linear fits through the points. The regression equations and the 95% confidence limits of the intercepts are indicated. In all cases, $L \leq B/3$; typically, $L/B = 0.3$. The smallest ligament chosen was not only several times the sheet thickness, but was also larger than the average floc in the sheet.

When experimental conditions prescribed for measuring the specific essential work of fracture (w_e) are met, Eq. 2 predicts that plot of W_f/LG against L should be linear. Our plots in Figs. 3–7 were invariably linear, as predicted. The intercepts which gave w_e were obtained with relatively small uncertainties.

We do not have independent estimates of the fracture toughnesses of our samples. The data in the literature are sparse, and are flawed for the reasons given in the beginning of the paper. Moreover, they are highly variable, as different fiber compositions and different manufacturing or specimen preparation and testing conditions were used. However, a general comparison of our results with published data was made *(12–16, 20, 36, 37)*. In no case were our results unreasonable.

Furthermore, within themselves, our results ranked in a plausible order. For instance, one expects the fracture toughness of pulps to increase several fold upon beating, which happened for the hardwood and softwood chemical pulps. One also expects the fracture toughness of sheets dried without restraint to be higher than that for normal sheets, as restraint-free dried sheets are considerably more extensible. This also occurred.

In Eq. 2, it was assumed that the plastic region around the ligament was circular. While performing the fracture tests, we observed the changes in the opacity of the region around the ligament and noticed that this region indeed appeared circular. The opacity of paper increases upon plastic straining. We further confirmed these observations by testing wax-impregnated samples in which the increase in the opacity upon straining is much higher.

For specimens of similar geometry, the slopes of the (W_f/LG) vs. L plots should relate to the relative resistance of the sheet to crack growth for that geometry, and to the sheet's ductility. It is interesting to note in Figs. 3–6 that, within each set, the sheets which had higher slopes were also more extensible (Table I). It seems that, for TMP sheets (Fig. 7) for which the slope was nearly zero, the entire work of fracture was the essential work, and none was dissipated outside the fracture process zone. We found a similar pattern for newsprint, which also had a nearly zero slope. The intercept w_e for newsprint was similar to the fracture toughness obtained by the quasistatic method *(38)*.

If the elongation to fracture is plotted against the ligament length, a linear relationship is obtained with a positive intercept ε_0 at zero ligament. This intercept ε_0 has been identified with the critical crack tip opening displacement *(23, 24)*. A typical plot of elongation to fracture against ligament length for paper is shown in **Fig. 8**. Our results on ε_0 shown in **Table II** were of similar magnitude as those reported on CTOD by Steadman and Sloane *(21)*. Within each set of sheets, ε_0 was nearly of the same magnitude as the elongation at which the initial part of the load-displacement curve of the unflawed sheet began to deviate from linearity.

If we assume that the crack propagates when elongation at the crack tip reaches a critical value ε_0, and the stress at the crack tip is either the yield stress σ_y or some higher value—for example, the average of σ_y and the ultimate tensile stress σ_u—fracture toughness can be estimated as $\varepsilon_0\sigma_y$ or $1/2\,\varepsilon_0(\sigma_y + \sigma_u)$. We have compared our w_e measurements against such estimates in **Figs. 9 and 10**. The yield stress was obtained from the stress-strain curves for unflawed specimens and corresponds to the stress at which the strain deviates from linearity by 0.2%. Considering that our ε_0 measurements are approximate (as they were obtained from elongations for the entire specimens, and not by using clip gauges across the outer plastic zones), and the 0.2% offset criterion for yield stress is arbitrary, the agreement between our w_e measurements and the estimated fracture toughnesses is good.

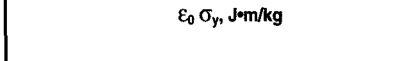
For metals, the plane stress work of fracture w_e depends on sheet thickness, as the specimen necks before fracture *(30)*. However, paper fails by different mechanisms (bond and fiber breakage, and fiber pull-out), and we do not expect w_e to depend significantly on sheet thickness. Indeed, this is what we have observed for handsheets with thicknesses in the range 86 μm to 160 μm. These sheets were made at different grammages from slurries of similar fiber concentrations, to avoid flocculation effects.

II. Results for w_e, βw_p and ε_0

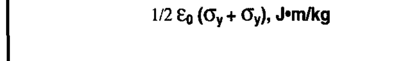
	w_e, J • m/kg	βw_p, 10^{-2} J/g	ε_0, mm
1. Copying paper, MD[a]	24.5	4.45	0.650
2. Copying paper, CD	21.7	9.11	0.918
3. Handsheets of unbeaten SW BK pulp	12.3	34.7	0.386
4. Handsheets of well-beaten SW BK pulp	32.8	66.7	0.489
5. Handsheets of lightly beaten HW BK pulp	10.0	3.65	0.355
6. Handsheets of well-beaten HW BK pulp	27.2	32.4	0.478
7. Handsheets of well-beaten SW BS pulp	15.9	29.8	0.394
8. Handsheets of well-beaten SW BS pulp, dried without restraint	25.0	60.6	0.794
9. SW thermomechanical pulp	17.3	0.56	0.559

[a] Crack propagating along CD

9. Essential work of fracture against an estimate of fracture toughness, $\varepsilon_0\sigma_y$



10. Essential work of fracture against an estimate of fracture toughness, $1/2\varepsilon_0\,(\sigma_y + \sigma_u)$



Concluding remarks

Plane stress fracture toughness is an important sheet property. For tough ductile sheets such as handsheets of softwood kraft pulps, it has been difficult to determine with laboratory-size specimens. The "essential work of fracture" concept offers a simple and practical way to measure the plane stress fracture toughness of a sheet, independent of specimen geometry. We have demonstrated that the method can be

11. Jig for making edge notches in paper specimens

12. Clamps for testing fracture specimens

used for paper. We will use this technique to determine the effect of fiber properties, pulping, and papermaking treatments on fracture toughness. In place of out-of-plane tear, we will use plane stress fracture toughness to understand better pulp properties and sheet reinforcement.

Experimental

Standard procedures of the CPPA (Technical Section) were followed for handling pulps and preparing specimens. The specimens for fracture tests were notched by using a jig shown in **Fig. 11.** It consisted of two sharp tool blades clamped between steel plates. The distance between the blades was adjusted by putting spacers of known length between them before clamping. The jig was placed vertically over a specimen backed by smooth cardboard and was hit gently with a hammer. In this way, sharp clean notches of known length could be introduced reproducibly. The fracture tests were made with a pair of line-type clamps (**Fig. 12**) mounted on the Instron universal testing instrument. The extension rate was 2 or 5 mm/min. All tests were carried out at 23°C and 50% RH after conditioning the samples. The grammages used in all calculations were air-dry.

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