A rational approach to formation analysis and spreadsheet schema for data interpretation

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ABSTRACT: This article provides a schema and spreadsheet format for interpreting measurements of paper formation in an appropriate way. The input data required consist of the fiber length and coarseness distributions, filler and fines fraction. The output is the expected random sheet variance of local grammage for square inspection zones of sides 1, 2, 3 and 4 mm, for comparison with measured values at any given mean grammage. Also given are some bounds on formation number for special cases and a means to analyze the smoothing effect of the forming process.

KEYWORDS: Analysis, basis weight, data processing, drainage, fiber length distribution, formation, on line measurement, paper, random processing.

Experimentally, it is now possible to measure appropriate scale statistics of flowing fiber suspensions *(1–3)* and in papermaking situations *(4,5)*. Models exist *(6,7)* to relate the flocculation state to the turbulent-like mixing prior to formation. Correlations between headbox furnish static conditions and paper structure have been demonstrated *(8)* and the effects of turbulent activity on a fourdrinier table on paper structure and properties have been measured *(4)*. Nanosecond pulsed lasers with dedicated chip imaging can capture real time transmission images calibrated to give two-dimensional bitmaps of grammage over a few square centimeters, and hence provide variances of local grammage about the millimeter scale. Fourier analysis can filter out periodic variations and provide diagnostic process information. New simulations of flocculation and dispersion mechanisms provide graphic representatives of different stochastic states *(9)*.

There is a real possibility in the near future that physical and computer simulation models could be used to allow engineering design of papers for specific purposes, and to control paper formation and its stability. In order to achieve this, it is necessary to have a stable, measurable formation index that is intrinsic, easily interpreted, and stochastically well defined. The appropriate candidate function seems to be $n_f(x)$, the ratio of actual grammage variance for inspection zones of size x to that for a random sheet from the same furnish; the range of interest for printing papers is typically $1 \le x \le 4$ mm. Indeed, for monitoring and control purposes, the value $n_f(1)$, for square inspection zones of side $x = 1$ mm and the gradient there with respect to x, has considerable merit and is easily measurable. The typical range for commercial fourdrinier papers is $1 \le n_f(1) \le 7$, *(8)*. Usually, the papermaker wishes to minimize n_f for any particular making, but sometimes it is maintained at a slightly higher level than necessary on purpose, for example, to counteract curling tendency in anisotropic copier paper or to make more fracture resistant strong sack paper.

The local grammage variance for arbitrary inspection zones in random fiber networks made from arbitrary fibers was actually derived and tabulated in *(10)*. With the advent of fast image capture and analysis, it has become feasible to use those results very effectively and several researchers have expressed interest in having a convenient spreadsheet format for computations in laboratory formation testing. The schema provided below will serve this purpose and could also be used in an automated on-line detector-controller system.

The computation of variance of local grammage for random sheets made

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from any given mixed furnish is straightforward; for, by definition, the different components arrive independently in the structure and hence their variances are simply summed to give the net variance. The fines and filler mass fractions in a random sheet are presumed to be distributed with horizontal uniformity and hence would make zero contribution to the variance. The fact that $n_f(x)$ for real paper is greater than 1 and an increasing function, puts scale and intensity to the manifest lack of independence in the forming process. We discuss this further below, in some special cases providing bounds for n_f.

Typical papermaking furnishes are often well approximated by log normal fiber length distributions. The effect of this on the formation of a sheet is to cause the variance of local grammage to be somewhat *lower* than that of a similar random sheet made from fibers all of the same length *(11)*. In commercial papers, which are less uniform than random at the fiber length scale, the effect is sometimes much larger.

The effect of anisotropy due to preferential fiber orientation is to cause the small scale variance of local grammage measured in cross machine strips to be higher than that in machine direction strips, as might be expected by observing that more fibers are cut by scanning lines in the cross direction. In fact, the fiber orientation distribution is approximately given by a $\pi/2$ rotation of the angular variance density distribution *(12)*.

I. Spreadsheet for computing the variance of local grammage Var_{Random} $(g/m^2)^2$, using 1mm² zones, in a random sheet made from the specified furnish. Fiber mass fractions m_i, lengths l_i mm and coarseness k_i g/mm. Units factor $g_i = 5 \times 10^7\, k_iB$, where mean grammage is B g/m². For example, a 100 g/m² random sheet made from 100% fibers of uniform length 1 mm, and coarseness 2×10^{-7} g/mm has variance given by $(12.5652 \times 10^{-3})(5 \times 10^7)(2 \times 10^{-7})(100) = 12.5652\ (g/m^2)^2$.

x	Length l_i	1000 u_i	Fraction m_i	k_i	g_i	Variance contribution $v_i = u_i m_i g_i$
1	Fines/filler	0				0
1	0.2	3.6636				
1	0.4	6.7035				
1	0.6	9.1665				
1	0.8	11.1035				
1	1.0	12.5652				
1	1.2	13.6154				
1	1.4	14.3728				
1	1.6	14.9411				
1	1.8	15.3831				
1	2.0	15.7367				
1	2.2	16.0260				
1	2.4	16.2671				
1	2.6	16.4711				
1	2.8	16.6460				
1	3.0	16.7975				
			$\Sigma m_i = 1$			Net variance $\Sigma v_i = Var_{Random}$

Formation spreadsheet

The derivation of the variance of local grammage in random sheets has been discussed in detail elsewhere *(13,14,8)*. The variance depends on fiber length and coarseness, but is practically independent of fiber width. The following tables give the variance contributions for fibers of different lengths in the range 0.2 mm to 3 mm, and for square zones of inspection of side lengths 1, 2, 3, and 4 mm. Square zones are used because these will be most convenient for on-line applications when local grammage data is extracted from two-dimensional gray level bitmaps.

The use of the spreadsheet format in **Table I** should be clear: fiber length mass fractions m_i are given class centers l_i in steps of 0.2 mm from 0.2 mm to 3 mm; the fines and filler mass fraction is assumed to be uniformly distributed and so contributes zero to the variance. By completing Table I, the required variance is arrived at in the lower right corner; repeating for the other choices of inspection zone size x = 2, 3, 4 using **Tables II–IV** gives the corresponding variances at those scales.

For those who are interested, the units factor $g_i = 5 \times 10^7\, k_iB$ is arrived at as follows. The tabulated values of u_i are actually the fractional between-zones variances of coverage by fibers of width 0.02 mm. Here, of course, *fractional* means in ratio to the variance at points. Now, to a satisfactory degree of approximation, [*cf. (15, 13)* for more discussion] the variances are proportional to fiber width w_i. Hence

$$u_i(w_a) = (w_a/w_b)u_i(w_b) \qquad (1)$$

so

$$u_i(w_a) = (w_a/0.02)u_i(0.02) \qquad (2)$$

for each width w_a.

The coverage in a given choice of zone is the number of fibers thick on average in that zone and coverage is simply the ratio of local grammage to the grammage of one fiber. The grammage of one fiber is the ratio of coarseness k_i to width w_i, so the variance of grammage is $(k_i/w_i)^2$ times the variance of coverage. The variance of coverage at points is equal to the mean coverage at points because it is a Poisson variable. Hence, the variance of grammage, for each length fraction l_i, is given by

$$v_i = (w_i/0.02)u_i(0.02)(k_i/w_i)^2 m_i(Bw_i/k_i) = (u_im_i)k_iB/0.02 \qquad (3)$$

in consistent units. For grammage in g/m², width in mm and coarseness in g/mm, we find

$$v_i = u_im_ig_i \qquad (4)$$

where

$$g_i = 5 \times 10^7 k_iB$$

Interpreting formation data

It is suggested that formation analyses of paper be represented in the form

$$n_f(x) = Var_{Real}(x)/Var_{Random}(x) \quad (5)$$

for square inspection zones of side length, say, $x = 1, 2, 3, 4$ mm. Typical findings are

$$n_f(x) \approx n_f(1)[1 + K(x - 1)] \quad (6)$$

for $1 \le x \le 4$ mm

where K is in the range from 0.75 to 2. Hence, the gradient of n_f with respect to x at $x = 1$ mm is

$$dn_f/dx \approx Kn_f(1) \quad (7)$$

In *(8)* there was reported a regression equation for $n_f(1)$:

$$n_f = 3.05 + 0.09n_{crowd} - 0.03(°SR) \quad (8)$$

Here, n_{crowd} is the number of fibers expected on average in a spherical volume of diameter equal to the mean fiber length in the headbox, and °SR is Schopper Riegler. So, n_{crowd} is the ratio of the expected mass of fiber in such a sphere to the average mass of a fiber:

$$n_{crowd} = 4/3\pi(l/2)^3c/lk = \pi l^2c/6k \quad (9)$$

where c is the headbox concentration, l is the mean fiber length, and k is the mean coarseness. The intrinsic concentration n_{crowd} is called the crowding number of the suspension and has been studied in detail by Kerekes *et al. (16-18)*. Some of the variability unexplained by the headbox furnish variables in the above regression equation is presumably due to process dynamic variables. This has been illustrated now by Kiviranta and Paulapuro *(4,19)*, who devised an image-analytic method to measure the intensity of turbulence on the forming table of a fast fourdrinier paper machine. Stelwagen *et al. (3)* showed that a one-dimensional reflection detector, with a sensing region of below 1 mm^3, correlated very well with a two-dimensional transmission device in the analysis of flocculation in flowing suspensions; it should be applicable to paper machine wet ends. Hot wire anemometry, and laser Doppler anemometry as described by Loewen and Buttler *(5)*, for example, provide yet more noninteractive detector and potential controller instruments for process dynamics.

Modeling forming processes

What then of physical models to predict n_f? Deterministic models are naturally impossible. Since $n_f(x)$ increases with x, we conclude that there must be a cooperative process during forming. This slows the decay with scale of real paper variability relative to that of random paper, which has by definition no interactions among fibers. We do know that all paper has an essentially layered structure *(20)*, and this allows us to obtain some bounds on n_f. Consider the simplest case, where a paper has just two layers, P and Q, and suppose that they have the same mean grammage, so the final paper is a combination of the two layers. Let p be the variable representing local grammage in P and q the corresponding variable for Q, at some chosen scale— say 1-mm squares. Then we are interested in the variance of the combination, which is by definition

$$Var(p + q) = Var(p) + Var(q) + 2Cov(pq) \quad (10)$$

where

$$Cov(pq) = \overline{pq} - \bar{p}\bar{q}$$

is the covariance between the two layers. If $Var_{Random}(p)$ is the variance of a random layer of the grammage of P, then

$$n_f(p + q) = Var(p + q)/2Var_{Random}(p) \quad (11)$$

There are some special cases:

Independent layers:

$Cov(pq) = 0$ so $Var(p + q) = Var(p) + Var(q)$

and hence, for statistically similar layers we conclude that

$$Var(p + q) = 2Var(p) \quad (12)$$

$$n_f(p + q) = n_f(p) \quad (13)$$

Reinforcing layers:

$q = p$ so $Cov(pq) = Var(p)$

and hence

$$Var(p + q) = Var(2p) = 4Var(p) \quad (14)$$

$$n_f(p + q) = 2n_f(p) \quad (15)$$

Smoothing layers:

$Cov(pq) < 0$. Say, $Cov(pq) = -sVar(p)$, for $0 \le s < 1$.

Then

$$Var(p + q) = 2(1 - s)Var(p) \quad (16)$$

$$n_f(p + q) = (1 - s)n_f(p) \quad (17)$$

Presumably, the case of smoothing layers is most realistic because we expect that lower density earlier regions will attract more material from later depositions, giving a negative correlation and hence a negative covariance. For sufficiently thin layers, we might suppose that both layers are random before formation but that the second layer redistributes itself after the first is deposited. Then there is the possibility to investigate the smoothing factor s by measuring

$$Var(p + q)/2Var(p) = (1 - s) \quad (18)$$

Conclusions

Instrumentation now available makes it feasible to put fine-scale formation analysis on a rational basis using well-defined stochastic variables. The appropriate index is n_f, the ratio of real paper variance of local grammage to the corresponding variance of random paper made from the same furnish. A spreadsheet schema is provided from which the necessary computations can be automated. A method is described for investigating directly the smoothing effect of the forming process on the local grammage distribution. TJ

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II. Random variance data for square zones of side 2 mm

x	Length, l_i	$1000\ u_i$
2	Fines/filler	0
2	0.2	0.9574
2	0.4	1.8335
2	0.6	2.6314
2	0.8	3.3539
2	1.0	4.0042
2	1.2	4.5857
2	1.4	5.1013
2	1.6	5.5543
2	1.8	5.9479
2	2.0	6.2853
2	2.2	6.5703
2	2.4	6.8105
2	2.6	7.0144
2	2.8	7.1892
2	3.0	7.3408

III. Random variance data for square zones of side 3 mm

x	Length l_i	$1000\ u_i$
3	Fines/filler	0
3	0.2	0.4318
3	0.4	0.8392
3	0.6	1.2225
3	0.8	1.5827
3	1.0	1.9202
3	1.2	2.2361
3	1.4	2.5305
3	1.6	2.8041
3	1.8	3.0575
3	2.0	3.2914
3	2.2	3.5064
3	2.4	3.7033
3	2.6	3.8825
3	2.8	4.0447
3	3.0	4.1906

IV. Random variance data for square zones of side 4mm

x	Length l_i	$1000\ u_i$
4	Fines/filler	0
4	0.2	0.2446
4	0.4	0.4789
4	0.6	0.7029
4	0.8	0.9169
4	1.0	1.1211
4	1.2	1.3157
4	1.4	1.5008
4	1.6	1.6767
4	1.8	1.8436
4	2.0	2.0017
4	2.2	2.1511
4	2.4	2.2922
4	2.6	2.4250
4	2.8	2.5498
4	3.0	2.6668

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Received for review July 6, 1992.

Accepted Oct. 4, 1992.