A limiting-consistency model for pulp dewatering and wet pressing

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ABSTRACT: Dewatering is a common and critical component of pulp and papermaking operations, with impacts on production capacity, product quality, and process economics. This paper presents a new model for dewatering and pressing based on the concept of "limiting consistency" and the Kozeny–Carman equation for permeable flow. The new model fits well with data from preliminary low-consistency drainage experiments and with data for high-consistency pressing from the literature. The advantages of the limiting-consistency model include mathematical simplicity, readily measurable parameters, and applicability over a wide range of consistency (1–50%). The theoretical development of the model and the initial results are presented.

KEYWORDS: Consistency, drainage, equations, fluid flow, mathematical models, measurement, models, parameters, theories, water removal, wet pressing.

Dewatering is a common and critical component in pulp and papermaking operations, with impacts on production capacity, product quality, and process economics *(1–3)*. The ability to predict the dewatering rates and final consistencies of pulp undergoing compression has been a goal of pulp and paper researchers for many years *(4)*.

Wet pressing on paper machines is the most obvious application of compressive dewatering in the industry. Compressive dewatering also occurs in brownstock washing and paper-machine wet-end operations. The differential pressure required to dewater the pulp pad can be obtained by applying pressure directly to the pad (wet pressing) or by pulling a vacuum under the pad (vacuum box). The main difference between the two dewatering methods is that air is likely to enter the pad upon application of vacuum to an open system. In any case, the ability to predict compressive dewatering rates is useful in optimizing the design and operation of equipment for pulp and paper mills.

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Overview

Limiting-consistency model

This paper presents a new model for compressive dewatering based on the concept of "limiting consistency" and the Kozeny–Carman equation *(5)* for permeable flow. The appendix at the end of this paper shows the derivation of the model.

The simple tenet of the limiting-consistency concept is that for any dewatering system, there is a maximum, or limiting, consistency that can be reached regardless of how long a dewatering force (p) is applied. The water that cannot be removed from the pulp pad is treated mathematically as if it were part of the fiber matrix.

The constant- p form of the limiting-consistency model is

$$\Delta p t = \frac{\mu K \Theta^2 B^2}{\rho^2}\left(\frac{1}{(c_L - c)^2} - \frac{1}{(c_L - c_S)^2}\right) \tag{1}$$

where

p = pressure drop

t = time

μ = viscosity (fluid)

K = Kozeny constant

θ = specific surface area

B = basis weight

ρ = density

c_L = limiting consistency

c = final consistency

c_S = initial consistency.

Solving for final consistency, c, the equation can be rewritten as

$$c = c_L - \frac{1}{\sqrt{\alpha \Delta pt + \frac{1}{(c_L - c_S)^2}}} \quad (2)$$

where

$\alpha = \rho^2/\mu K\theta^2 B^2$

This form of the equation allows the final consistency from a dewatering process to be calculated from readily measurable variables.

In the case where media resistance (e.g., drum washer or paper-machine wire) is significant, Eq. 1 can be modified to accommodate media permeability

$$p\, dt = ((\mu K\theta^2 B^2/\rho^2) \{[1/(c_L-c)^2]-[1/(c_L-c_S)^2]\}) + \{(B/K_M\rho)[(1/c_S)-(1/c)]\} \quad (3)$$

where K_M is a permeability constant for the filtering medium.

Consistency range

Basing the limiting-consistency model on the Kozeny–Carman equation results in a simple model but confines the model to a consistency range where pad permeability is the dominating resistance to flow. At some very low consistency value (<1%), the system is dominated by the resistance to flow through the pad's "retaining" device (e.g., paper-machine wire) *(6)*. At extremely high consistencies, the major resistance to extracting water from the pad is likely to be the compressive resistance of individual fibers.

The specific-surface-area term, θ, of the Kozeny–Carman equation is intended for impermeable solids. However, wood fibers are highly permeable. Thus, there are two specific surface areas of interest in a pulp pad:

1. Macro level—the specific surface area of the pad as if the fibers were impermeable
2. Micro level—the specific surface area of voids within the fiber.

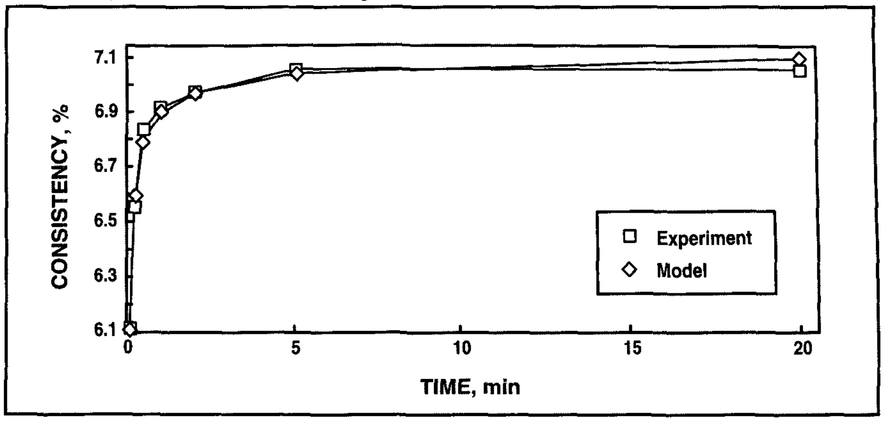
In dewatering systems that do not significantly compress individual fibers (e.g., rotary drum washers), the macro-surface-area term should dominate. Likewise, in higher consistency systems that compress fibers greatly (e.g., paper-machine presses), the micro-surface-area term should dominate. Additionally, the micro and macro voids have different size (effective radius) distributions. For example, in the micro-dominant case the fiber pores can be considered to be of uniform size, and θ_{mic}, the effective specific surface area, can be expressed as a function of θ_i, the dry fiber specific surface area, and c_{mic}, where c_{mic} is the internal consistency of the pulp fibers.

$$\theta_{mic} = \theta_i[(c_L - c_{mic})/(1 - c_{mic})] \quad (4)$$

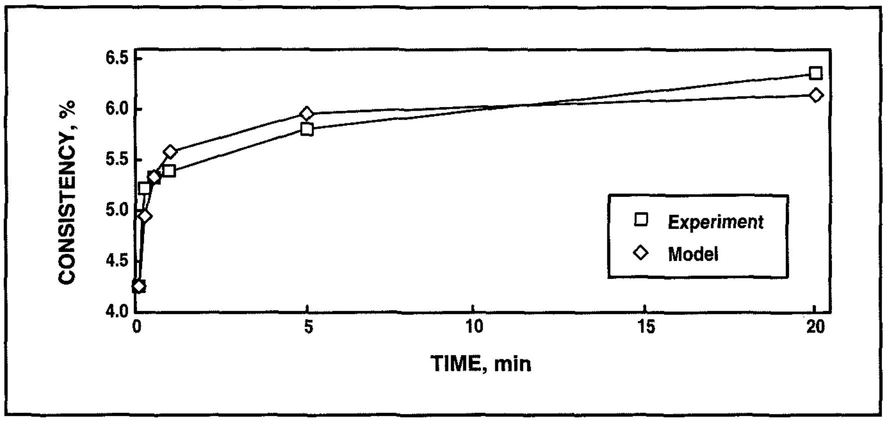
Initial results

Initial investigations into the limiting-consistency model are split along two lines. An experimental dewatering device was constructed to test the low-consistency range (0–20%), while curve fitting of published data on wet-pressing experiments *(7)* was used to test the high-consistency range (>20%).

1. Measured and predicted dewatering of oxygen-delignified southern pine kraft at low consistency; vacuum-assisted drainage



2. Measured and predicted dewatering of Douglas-fir brownstock at low consistency; vacuum-assisted drainage; no soap

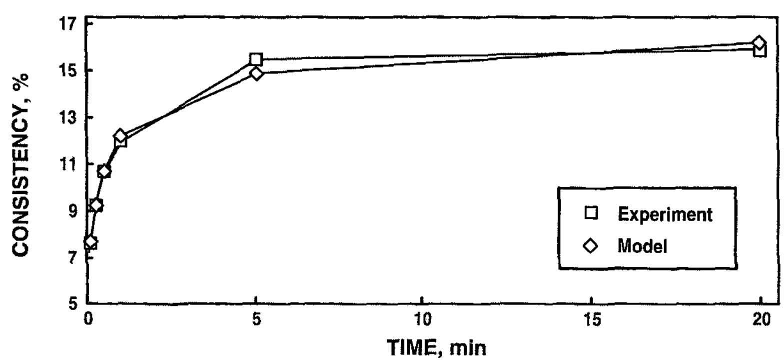


I. Experimental results for vacuum-assisted dewatering at low consistency

Kraft pulp	*Kappa no.*	*Conditions* [a]	*Pad weight, g*	*Limiting consistency (c_L), %*	*Coefficient α, (in. H_2O·min)$^{-1}$*	*r^2*	*No. of measurements* [b]
Southern pine	15	Air infiltrated	6.15	7.14	18.94	0.97	21
Douglas-fir	65	Air infiltrated No soap	3.58	6.24	2.22	0.77	18
Douglas-fir	65	Air infiltrated 0.05% soap[c]	4.27	17.26	0.032	0.98	18

[a]All tests were run at a p of 5.5 in. H_2O, 20°C, and initial slurry consistency of 1%.
[b]Each data point in Figs. 1–3 represents the average of three measurements.
[c]Soap added at 0.05% by liquid volume (5% on pulp).

3. Measured and predicted dewatering of Douglas-fir brownstock at low consistency; vacuum-assisted drainage; soap added



Low consistency

The three dewatering experiments at low consistency yielded results that fit well with the limiting-consistency model. Results are presented in **Figs. 1–3**. Dewatering data were generated by running experiments lasting between 0.1 min and 20 min. The conditions and results for each trial run are shown in **Table I**. The parameters α and c_L were determined by using a nonlinear solver program *(8)* (Newton's method and central difference) to minimize least-squares error (experiment vs. model). The r^2 value for the second experiment (Fig. 2) is much lower than the other two (Figs. 1 and 3). This is the result of two outlier points, a low one in the 5-min tests and a high one in the 20-min tests. Removal of these two points increases the r^2 value to >0.9 for the experiment in Fig. 2.

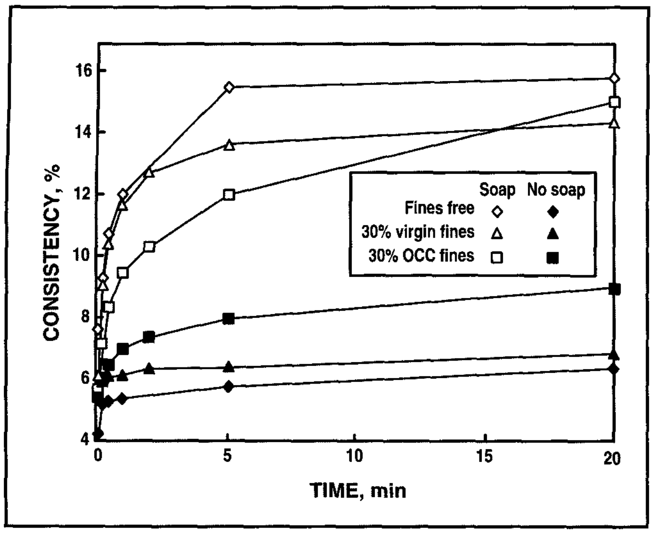
Experimental data points were obtained by introducing a pulp slurry of approximately 1% consistency to a vacuum-assisted drainage cell with a bottom constructed from a brownstock washer wire. The wire was designed for a horizontal washer and featured a double-layer nylon weave with a permeability of 600–850 ft³/min/ft². Pulp pads formed on the wire were extracted after drainage times ranging from 0.1 min to 20 min. Each experiment was repeated three times, and the resulting consistency values were averaged. The average variance was 0.067 in units of percent consistency.

The effect of soap (surfactant) addition on dewatering is remarkable, as seen in Figs. 2 and 3 and in Table I. The limiting consistency, c_L, jumped from a base of 6.2% with no soap up to 17.3% with soap. The coefficient A decreased by two orders of magnitude, from 2.22 (in. H_2O·min)$^{-1}$ to 0.032 (in. H_2O·min)$^{-1}$. The large decrease in A indicates that the effective specific surface area increased by one order of magnitude, with a corresponding increase in c_L. Similarly, Britt and Unbehend *(6)* have found that retention and drainage aids significantly increase pulp freeness, demonstrating that the effective specific surface area of pulp can be changed chemically.

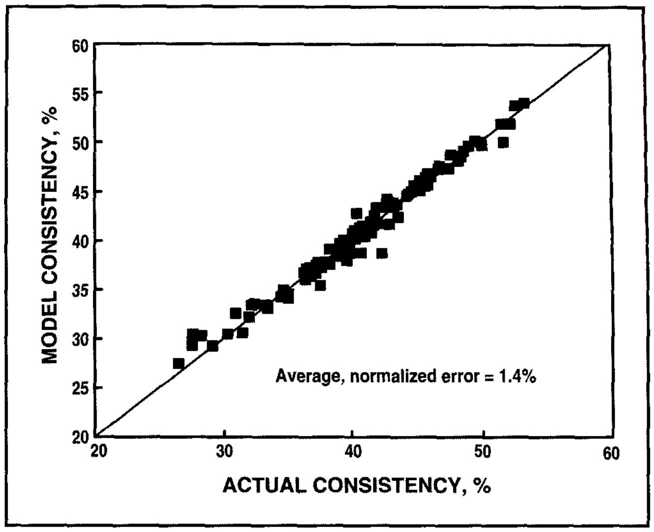
Additional experiments were conducted to test the effect of a surfactant (soap) and fines on dewatering. The OCC (old corrugated container) fines used in the experiments were recovered from the reject system of an OCC repulping operation. Virgin fiber fines were produced with a Waring blender. All other conditions were the same as for the experiments described in Table I. The experimental results are shown in **Table II** and **Fig. 4**.

Adding OCC fines to the virgin fiber resulted in a large increase in limiting consistency (9.2%) relative to the fines-free base case (6.1%) and virgin fiber fines (6.7%). This result may be attributed to the OCC fines containing entrapped air and/or OCC fines carrying surfactants and retention aids from a previous papermaking process. Because of the high

4. Effect of fiber fines and surfactant on dewatering of Douglas-fir base stock at low consistency



5. Measured [data from Caulfield *et al.* *(7)*] and predicted dewatering of southern red oak kraft at high consistency; compressive dewatering



surface area to volume ratio of fines and the high percentage of fines in the OCC furnish (10–20%), most of the surface-active chemicals introduced to a paper machine from OCC would be on the fines. Variability in the volume and type of chemicals carried by the OCC fines has the potential to upset paper-machine wet-end chemistry.

Soap (surfactant) was added to the three furnishes to act as a drainage aid. The OCC fines retarded the dewatering rate at low consistency, indicating that they had a higher effective specific surface area than the virgin fiber fines. The virgin fiber fines resulted in a slightly retarded dewatering rate and a lower limiting consistency (14.9%) relative to the fines-free base case (17.3%).

II. Effect of fiber fines and surfactant on vacuum-assisted dewatering at low consistency

Trial	*Conditions* [a]	*Limiting consistency* *(c_L), %*	*Coefficient* α, *(in.* H_2O*·min)* $^{-1}$	Θ/Θ_I^b
		No soap		
1	Fines-free base pulp	6.1	3.63	1
2	30% virgin fines	6.7	5.88	0.8
3	30% OCC fines	9.2	0.15	4.9
		With soap[c]		
4	Fines-free base pulp	17.3	0.032	10.4
5	30% virgin fines	14.9	0.097	6.1
6	30% OCC fines	15.5	0.019	13.4

[a]The base pulp for all experiments was a fines-free Douglas-fir kraft of kappa no. 65.
[b]Ratio of effective specific surface area to base case *(1)*.
[c]Soap added at 0.05% by liquid volume (5% on pulp).

High consistency

The high-consistency range of compressive dewatering was tested against the limiting-consistency model using the data of Young, Wegner, and Caulfield *(7)*. The data were generated in an experimental paper-machine wet-press section using handsheets of kraft pulp from southern red oak of kappa no. 22. Handsheets were 205 g/m² (42 lb/1000 ft²). Although the data were obtained from a traditional roll press (variable *p* during impulse), the data fitting was performed as if the applied pressure was constant over the impulse duration. The parameters c_L and α were determined using the nonlinear solver cited previously *(8)*. The model matched the published data very closely, with an average error of ±1.4% of the measured consistency. The data-fitting exercise used 114 data points from 24 press trials, and the results are shown in **Fig. 5.** This compared very favorably with an empirical log-impulse relationship suggested by Busker and Cronin *(4)*, below, which yields an average error of ±6.6% of the measured consistency for the same pressing data.

$$c = c_S + [M \ln(P/V)] \qquad (5)$$

where
P = pli (pounds per lineal inch)
V = web velocity.
M = constant

Table III contains the parameter values and conditions for each trial.

III. Data-fitting results for high-consistency dewatering*

Freeness, mL CSF	*Initial consistency (c_S), %*	*p, MPa*	*Limiting consistency (c_L), %*	*Coefficient α, (MPa·ms)* $^{-1}$
350	24.4	2.2	39.6	4.28
		2.8	44.4	2.14
		3.5	46.3	1.45
		3.8	47.8	0.87
	35.7	2.2	45.2	2.22
		2.8	48.5	0.66
		3.5	48.6	0.51
		3.8	50.1	0.30
500	24.4	2.2	47.9	1.91
		2.8	49.1	2.16
		3.5	50.7	1.43
		3.8	52.6	0.88
	35.7	2.2	51.0	1.51
		2.8	56.5	0.38
		3.5	58.2	0.20
		3.8	56.4	0.25
620	24.4	2.2	52.1	1.46
		2.8	56.5	0.38
		3.5	52.8	1.37
		3.8	56.5	0.51
	35.7	2.2	53.5	2.07
		2.8	56.7	0.84
		3.5	59.5	0.41
		3.8	60.0	0.29

*Data from Young, Wegner, and Caulfield *(7)*

As predicted in Eq. 4, the effective specific surface area was found to be a function of absolute specific surface area (freeness or hydrodynamic specific surface area) and limiting consistency. This is illustrated by the results presented in **Fig. 6**.

Freeness changes are expected to have a greater effect on macro pad drainage than on micro drainage because of fiber compression. More variability in α is observed in the lower initial-consistency data. The limiting consistency, c_L, is clearly a function of inlet moisture and pulp freeness in this set of experimental data. While the association with freeness is expected, the relationship between c_L and inlet consistency may be due to saturation of press felts or rewetting of the pulp pads at the nip exit. Experimental studies are planned to investigate this effect within the 20–50-psi range. **Figure 7** shows two press dewatering curves taken from the data of Young, Wegner, and Caulfield *(7)*, with the only variation being the initial consistency. Comparison of the two curves illustrates the apparent effect of initial consistency on limiting consistency.

Parameters that determine limiting consistency

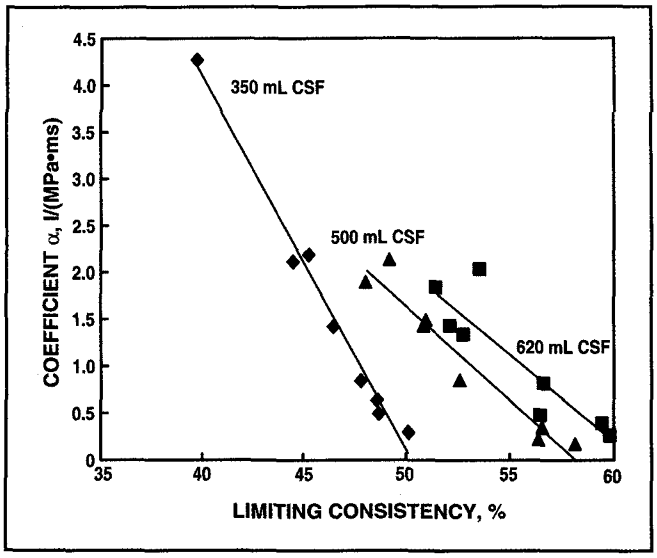
Based on the initial results, c_L is largely a function of applied pressure and effective specific surface area. In the case of wet pressing, the initial consistency, the basis weight, and the rewetting potential within the press are also important. In the case of drainage, the effective surface area can be greatly affected by fines content (high surface area to volume ratio) and slurry chemistry (drainage/retention aids)

Conclusion

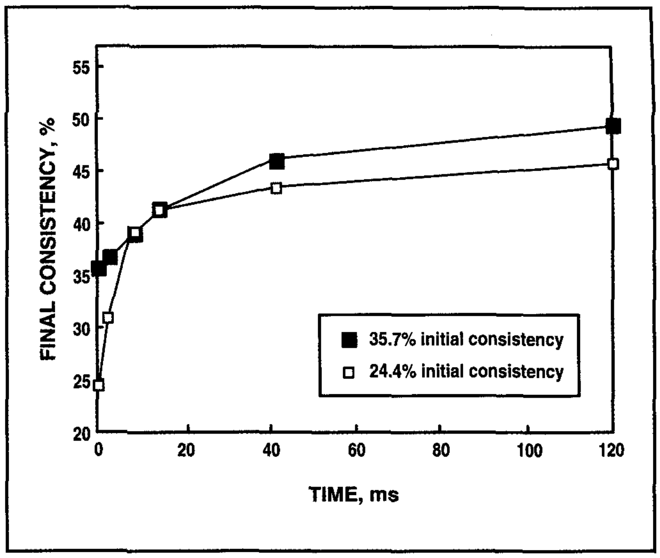
The limiting-consistency model fits well with experimental dewatering data over a wide consistency range. The concepts of limiting consistency and effective specific surface area can serve as the basis for evaluating both the dewatering characteristics of various furnishes and the effectiveness of drainage-enhancing chemicals. Also, the base model has been modified to account for media resistance.

An ongoing experimental program is aimed at applying the model to mill operations such as brownstock washing, paper-machine forming, and wet pressing. While it is unlikely that any single model can predict the exact dewatering behavior in these complex operations, the limiting-consistency model does provide a new framework for attempting to explain observable dewatering phenomena.

6. Coefficient α vs. limiting consistency for high-consistency dewatering [based on data from Caulfield *et al.* *(7)*]



7. Final consistency as a function of pressing time for pulp at two levels of initial consistency [500 mL CSF, 2.8 MPa, based on data from Caulfield *et al.* *(7)*]



Appendix: Theoretical development of the limiting-consistency model for compressive dewatering

Introduction

Development of models to predict dewatering rates of pulp pads during compression has been an area of pulp and paper research for many years. Traditional pressing models view the applied force as being divided between a hydraulic component (water removal) and a compressive component (fiber network compression) *(9)*.

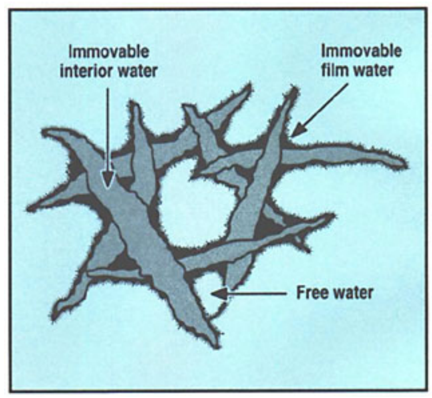
In a recent paper, McDonald and Kerekes *(10)* applied the Kozeny–Carman equation (permeable flow) to wet pressing. The key assumptions they employ are that (a) all of the force applied to the pulp pad is used to drive out water and (b) the parameters in the Kozeny-Carman equation are nonlinear functions of the moisture ratio. The result is a decreasing-permeability model, in which the terms in the Kozeny–Carman equation are combined into a single permeability factor and the nonlinear dependence on moisture ratio is expressed with a single compressibility factor. These two terms greatly simplify the model but are difficult to relate to measurable parameters within dewatering systems.

In the present work, a new compressive dewatering model is presented. It is based on

- The Kozeny–Carman equation
- The assumption that all of the applied force drives water from the pad
- The concept of limiting consistency.

The simple tenet of this new assumption is that for any dewatering system, there is a maximum consistency that can be reached regardless of how long a constant dewatering force (p) is applied. The water that cannot be removed from the pulp pad is treated mathematically as if it were part of the fiber matrix. **Figure** 8 depicts the matrix of fiber and immovable liquid. The approach is similar to that employed by Jewett and Ceckler *(11)*, where water-retention ratio (WRR) was used to reduce the effective voidage in the pad. By comparison, WRR is a single point for a given furnish, while the limiting consistency is a continuous function of some combination of dewatering system and furnish parameters such as pressure, felt moisture, pad elasticity, film thickness, etc.

8. Depiction of fiber–immovable-liquid matrix



Thus the limiting consistency, which is represented as

$$c_L = \text{pulp}/(\text{pulp} + \text{immovable water}) \qquad (A1)$$

defines the amount of immovable water in the pulp pad and, by difference, the amount of water that is free to drain from the pad.

The resulting model fits well with data from preliminary low-consistency (1–20%) experiments and high-consistency press (20–50%) data from the literature. In addition, the parameters in the model lend themselves to measurement in both experimental and commercial dewatering systems.

Theory: constant *p*

The Kozeny–Carman equation for permeable flow is intended for modeling fluid flow through a packed bed of nonporous particles. Adapting the Kozeny–Carman equation to compressive dewatering of pulp requires dealing with a compressing "packed bed" of porous pulp fibers. The limiting-consistency model is based on the assumption that for any dewatering system with a constant pressure drop, there is a maximum attainable consistency regardless of dewatering time. Therefore, the portion of the fluid that will not exit the mat is included as part of the solid matrix. The hypothesis applied to the Kozeny–Carman equation leads to the following derivation. The terms used in the derivation are as follows:

Nomenclature.

p = pressure drop

μ = viscosity (fluid)

U = velocity

k = permeability coefficient

ε = voidage

θ = specific surface area

K = Kozeny constant

c = final consistency

c_L = limiting consistency

c_S = initial consistency

l = flow path length

W = fiber weight

A = pad surface area

ρ = pad density

V = pad volume

t = time

B = basis weight

Base model. Starting with Darcy's law

$$dp/dx = -\mu U/k \qquad \text{(A2)}$$

where k, the permeability coefficient, is defined by the Kozeny–Carman equation as

$$k = \varepsilon^3/[(1-\varepsilon^2)\theta^2\mathrm{K}] \qquad \text{(A3)}$$

where K is the Kozeny constant.

From the limiting-consistency assumption, it follows that the voidage, ε, excludes not only the fiber but also the water that cannot be removed from the pad, even at infinite dewatering time. Therefore,

$$\varepsilon = 1 - (c/c_L) \qquad \text{(A4)}$$

where c_L is the limiting consistency.

An additional assumption is that the flow path can be treated as a constant even though the pad is being compressed. An analogy to the compressing flow path is water running down a coil spring. The flow path is of constant length whether the spring is stretched out into a rod or the spring is fully coiled. Thus,

$$\int_0^l dx = \frac{W}{A\rho c_L} \qquad \text{(A5)}$$

where W is fiber weight, A is pad surface area, and ρ is pad density. Here again, the water that cannot be removed from the pad is considered to be part of the solid matrix. Further, ρ is treated as a constant, although in actuality it varies in the range 1–1.5 g/cm^3.

The velocity term, U, in Darcy's law is redefined in terms of consistency as follows:

$$U = dV/(A\ dt) \qquad \text{(A6)}$$

$$V = W/\rho c \qquad \text{(A7)}$$

$$dV = -(W/\rho c^2)\ dc \qquad \text{(A8)}$$

The specific surface area, θ, is also dependent on the limiting-consistency concept. If θ_i is the specific surface area of the dry fibers, then

$$\theta = \theta_i - \theta_S \qquad \text{(A9)}$$

where θ_S is the surface area that is "saturated" with immovable water. A detailed discussion of the surface-area aspects of the limiting-consistency model is presented under the heading "Specific surface area."

Finally, dp is p across the fiber pad, and combining all of the terms back into Darcy's Law gives

$$p\ dt = (-\mu W^2\theta^2\mathrm{K}\ dc)/[\rho^2 A^2 (c_L - c)^3] \qquad \text{(A10)}$$

The equation can then be integrated over the press time at constant p to yield

$$p\ t = (\mu\mathrm{K}\theta^2 B^2/\rho^2)\{[1/(c_L-c)^2]-[1/(c_L-c_S)^2]\} \qquad \text{(A11)}$$

where B is pad basis weight (W/A), c_S is initial consistency at time zero, and c is final consistency at time t.

Rearranging to solve for final consistency, the equation can be rewritten as

$$c = c_L - \left(1/\{[(\rho^2\ p\ t)/(\mu\mathrm{K}\theta^2 B^2)] + [1/(c_L-c_S)^2]\}^{1/2}\right) \qquad \text{(A12)}$$

This form of the equation allows the final consistency from a dewatering process to be calculated from readily measurable variables.

The limiting-consistency model has an advantage over the decreasing-permeability model in that the parameters used in the model are more easily measured in a laboratory. The model is also amenable to enhancement by adding felt or wire resistance to flow, as is the case in many low-consistency dewatering situations.

Specific surface area. The Kozeny–Carman equation specific-surface-area term, θ, is based on the idea that most, if not all, of the solid's surface area is exposed to the flowing liquid, thus acting as a resistance to flow. In the limiting-consistency model, much of the surface area must be considered as being out of contact with flowing liquid. The specific surface area that is "saturated" or surrounded with unmoving liquid plays no role in determining dewatering rates. Thus a pulp that drains to a higher limiting consistency under the same conditions as another pulp may have the *higher* effective specific surface—a notion that at first seems counterintuitive. However, preliminary analysis of both low-consistency and high-consistency data bears out this idea. Additional support comes from Britt and Unbehend *(6)*, who found that retention and drainage aids significantly increased pulp freeness, demonstrating that the effective specific surface area of pulp can be changed chemically.

Based on the concept of an effective specific surface area that is a function of both the fiber specific surface area and the immovable water film thickness, the following scenario for retention and drainage aids can be developed. Retention aids flocculate fines and fibers, greatly reducing the specific surface area of the pulp mat. However, the water film thickness remains unchanged. Thus a retention aid is expected to reduce the freeness of a fines-laden system, such as in a paper-machine headbox. On the other hand, retention aids are expected to have little impact on brownstock washers, where the fines content is negligible, or on paper-machine presses, where fines are already consolidated in the paper web. Drainage aids that reduce the water film thickness layer but do not promote web bonding should be more useful in brownstock washing, where fines retention and bonding are not critical.

Another aspect of the Kozeny–Carman specific-surface-area term, θ, is that it is based on an impermeable solid. However, wood fibers are highly permeable. Thus, there are two specific surface areas of interest in a pulp pad:

1. Macro level—the specific surface area of the pad as if the fibers were impermeable
2. Micro level—the specific surface area of voids within the fiber.

The macro voidage is composed of flow paths around fibers, fibrils and fines. The size (effective radius) distribution of macro voidage is wide, depending heavily on fines content and fiber development (freeness). Conversely, the micro voidage (internal fiber pores) should have a much narrower size distribution.

In dewatering systems that do not significantly compress individual fibers (e.g., brownstock washer drums), the macro-surface-area term should dominate. Likewise, in higher consistency systems that compress fibers greatly (e.g., paper-machine presses), the micro-surface-area term should dominate. Modified forms of the basic limiting-consistency model can account for changing dominating resistances to flow encountered in a dewatering system that spans a very wide consistency range. In the macro-dominant case, the specific surface area, θ, can be modified to account for the portion of the specific surface area that is tied up in the fiber–immovable-liquid matrix. For example, if the flow path volume is distributed uniformly over a wide range of effective radii, then the ratio of surface area to volume varies as the reciprocal of the radius. The effective specific surface area, assuming the smaller voids are preferentially filled with immovable liquid first, can then be defined as follows:

$$\theta_{\mathrm{mic}} = \theta_{\mathrm{i}}\, \mathrm{K} \ln[1/(1-c_{\mathrm{L}})] \qquad \text{(A13)}$$

where

c_{L} 1

For example, in the micro-dominant case, the fiber pores can be considered to be of uniform size, and θ_{mic} can be expressed as the following function of θ_{i}, the dry-fiber specific surface area, and c_{mic}, the internal consistency of the pulp fibers.

$$\theta_{\mathrm{mic}} = \theta_{\mathrm{i}}[(c_{\mathrm{L}}-c_{\mathrm{mic}})(1-c_{\mathrm{mic}})] \qquad \text{(A14)}$$

Many models for effective specific surface area can be developed based on different distributions for pore size within the pad (macro) and the individual fibers (micro).

Media resistance. In low-consistency dewatering systems, the filter medium can represent a major resistance to flow. The medium can be a paper-machine wire, filter cloth, etc. Media resistance is included in the limiting-consistency model by defining the system pressure drop as

$$p_{\mathrm{system}} = p_{\mathrm{media}} + p_{\mathrm{pulp}} \qquad \text{(A15)}$$

The medium is assumed to be effectively incompressible, which implies constant voidage. (However, the modeling of press felts may require that the medium be treated as compressible.) The incompressible medium allows all of the basic Kozeny–Carman assumptions to apply, and the following simplification can be made:

$$\mathrm{K_m} = \varepsilon^3/[\mu \mathrm{K}\theta^2(1-\varepsilon)^2] \qquad \text{(A16)}$$

Transforming the velocity, U, as before, the resulting term for media pressure drop is

$$\int_0^t \Delta p_{media}\, dt = \int_{c_s}^{c} \frac{B}{\mathrm{K_m}\rho} \frac{dc}{c^2} \qquad \text{(A17)}$$

Integration of the right-hand side of the equation yields

$$\int_0^t \Delta p_{media}\, dt = \frac{B}{\mathrm{K_m}\rho}\left(\frac{1}{c_s} - \frac{1}{c}\right) \qquad \text{(A18)}$$

Combining the two pressure-drop terms, the final equation for constant p is

$$p\, dt = \Big((\mu \mathrm{K}\theta^2 B^2/\rho^2)\,\{[1/(c_{\mathrm{L}}-c)^2]-[1/(c_{\mathrm{L}}-c_{\mathrm{S}})^2]\}\Big) + \{[B/(\mathrm{K_m}\rho)]\,[(1/c_{\mathrm{S}})-(1/c)]\} \qquad \text{(A19)}$$

Where medium resistance dominates, drainage rates should vary linearly with basis weight. Pulp-dominated drainage should vary with the square of basis weight.

Theory: variable *p*

While many systems can be modeled adequately using the constant-p equations derived here, some systems with highly variable pressures will require a more rigorous treatment. An example of a variable-p system is an extended-nip press. The differences between the variable-p case and the constant-p case are that c_L and θ are functions of p, and p is a function of time. A similar derivation to that for constant p can be made up to the point of integration. At that point, the solution can only be generated numerically, because c_L and c cannot be separated in the equation. One of the goals of the experimental program is to define the relationship between p and c_L and θ for a variety of furnishes. The resulting equations will be used to replace c_L and θ in the limiting-consistency model. Since p is a function of time, for a given furnish and pressing conditions, the limiting-consistency model contains only time and consistency as variables, and the numerical integration can proceed.

Summary

The concept of "limiting consistency" provides a basis for many avenues of experimentation and the development of modifications, e.g., accounting for a filter medium's resistance to flow. The individual parameters in the model should be readily measurable in the lab and are, hopefully, transportable to actual mill operations.

The assumptions required to apply the Kozeny–Carman equation (an equation for flow through a packed bed of incompressible, impermeable solids) to flow through a compressing, permeable pulp pad are sizable. Nevertheless, the resulting model and preliminary results are promising. At a minimum, the limiting-consistency model provides a new framework for attempting to explain observable dewatering phenomena.

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