Effect of deformable roll cover on roll coating

Marcio S. Carvalho and L.E. Scriven

ABSTRACT: Of the two rolls that make a forward or reverse roll coating gap or nip (negative gap), one is often covered by a layer of more-or-less deformable elastomer. Liquid carried into the converging side of the nip can develop high enough pressure to deform the elastomer cover, which changes the nip profile and alters the velocity and pressure fields in the nip. This elastohydrodynamic coupled action is not yet well understood. Theoretical modeling has to take into account both the viscous flow and the elastic deformation in order to predict the relation of coat weight to roll loading and the rheological properties of the coating liquid and of the roll cover. Theoretical modeling in the same way should predict ribbing behavior. In this work, the flow is described by the lubrication approximation, and the elastic deformation by three different models: Hookean and neo-Hookean (ideal elastomer elasticity) springs oriented radially, which constitute one-dimensional models; and a layer in Hookean plane strain, which is a more realistic volume-conserving, two-dimensional model. The predictions of the models are compared, and their relative merits are analyzed in the light of available experimental data.

KEYWORDS: Deformation, models, roll coating, viscous flow.

Roll coating is distinguished by the use of one or more gaps and nips between rotating cylinders to meter a continuous liquid coating and to apply it to a strip, web, or base. The liquid layer carried into the metering gap or nip by a rotating cylinder is commonly dipped from a pool, pond, or puddle in which the liquid fed is distributed crosswise by gravity flow. More and more often the liquid layer is instead formed by a blade-enclosed fountain backed by a chamber in which pressure flow distributes crosswise the liquid fed to it, or by a chamber-and-slot device (essentially a slot-die) in which pressure flow does the same.

The metering and application functions can be performed in gaps or nips between co-moving, i.e., counter-rotating cylinders, or rolls, which is called forward roll action; or between counter-moving, i.e., co-rotating ones, which is called reverse roll action. By *gap* here is meant the closest approach, or clearance, between two rolls that would not be in contact or interference were liquid not present; by *nip* is meant the closest approach, or clearance, between two rolls that would be in interference, i.e., a *negative gap*. In this latter circumstance, one or both rolls must be, to some degree, *soft*: by soft is meant a roll that deforms adequately in operation. This is generally achieved by covering a hard roll core with a layer of material that is elastomeric, like rubber, or elastic with low modulus, like certain composites. Metering can be carried out either in the gap between two hard rolls, or in the nip between a hard and a soft roll. Application is commonly carried out in the nip between a hard and a soft roll, one or the other bearing the substrate, that is, the strip, web, or base.

In coating, the various purposes of a deformable roll cover are (1)

Carvalho and Scriven are graduate student and Regents' Professor, respectively, Coating Process Fundamentals Program, Center for Interfacial Engineering and Department of Chemical Engineering and Material Sciences, University of Minnesota, Minneapolis, MN 55455.

metering at a cost lower than that of (a) the risk of clashing two hard (typically steel) rolls or (b) the surface finishing, the bearings, and the mountings of a pair of hard rolls; (2) metering thinner films than are ordinarily achievable with a pair of hard rolls; (3) reducing the ribbing pattern that though sometimes tolerable is often a constraining defect; and (4) transferring an already metered liquid layer virtually completely to a substrate. The last refers to what is called the reverse wipe transfer action in a reverse roll nip where the substrate, if a relatively stiff strip, is carried on the hard roll (as in coil coating) but, if a flexible web, is more often carried on the soft roll (as in film coating). The functions and other terminology here are as defined by Benjamin and Scriven *(1)*.

The deformation of the roll cover affects the shapes of the boundaries of the coating flow in the nip region. That flow generates pressures, viscous stresses, and in some cases viscoelastic ones that, in turn, affect the deformation of the roll cover. Hence the viscous liquid flow and the elastic deformation on the roll cover are coupled. This elastohydrodynamic behavior, its dependence on the thickness and properties of the roll cover, and its influence on wet coating thickness (which translates to "coat weight") versus roll loading, roll speeds, etc., as well as on crossweb and downweb uniformity, roll drag, and its influence on other quantities is not yet completely understood. This is the subject addressed in this paper.

Similar situations occur in the elastohydrodynamic lubrication of bearings, in the lamination of preformed films to strip or web and in certain calendering operations. The fundamentals of elastohydrodynamic lubrication have been established well, especially by Dowson and Higginson *(2, 3)* and their collaborators and successors, e.g., Bennet and Higginson *(4–6)*. Their greatest concern is the gap or nip. They regard the pressure distribution of the intervening liquid on the solid surfaces as a perturbation of that in dry contact, which is the Hertzian distribution from linear elasticity theory. The liquid flow rate does not concern them and they do not report it. Only a few studies have addressed cases of low elastic modulus or "soft" solids, e.g., the study by Baglin and Archard *(7)*. The effect of viscosity variation with pressure has been extensively studied by Berthe and Vergne *(8)*, and likewise of its variation with temperature by Wang *et al. (9)*. Dobbels and Mewis *(10)* put forward a model that included viscoelastic behavior of a rubber roll cover, but they imposed rather than measured experimentally or predicted theoretically the roll deformation in the nip region. Batra and co-workers *(11–14)*, in a series of papers, analyzed dry contact between co-moving rubber and solid rolls; they employed a nonlinear viscoelastic model of the elastomer and nonlinear finite strain theory.

Coyle *(15)* launched a theoretical analysis of the elastohydrodynamics of roll coating. He approximated the behavior of a roll cover by a one-dimensional, linearly elastic model and the behavior of the coating liquid by the well-established lubrication flow theory. He computed predictions of roll surface deflection, pressure distribution and thus roll loading, and flow rate through the nip.

In this paper, we take up Coyle's *(15)* analysis and extend it to the important regime of reverse roll action, and examine the differences between using a linearly elastic model and a nonlinear one that better describes the response of rubber when deformation is appreciable. We also explore the use of a two-dimensional plane strain approximation that is a bit more complicated, but more realistic. The responses predicted in these different cases of both forward and reverse roll action are contrasted. The only comparable data that are published are from a few experiments on bearing models. We discuss the need for measurements of wet coating thicknesses (coat weight) as a function of load, speed, liquid viscosity, and roll cover properties.

Theoretical modeling

Usually, when a soft roll cover is used, the axes of the two rolls are kept parallel, one axis being fixed but the other movable in response to externally imposed loading (force per unit length). The gap or nip (negative gap) adjusts according to the hydrodynamic pressure in the liquid between the rolls. If all else is fixed, then to each loading there corresponds a gap or nip, a roll deformation, and a center-to-center distance between the rolls. In modeling, it is more convenient, however, to set the center-to-center distance between the rolls and evaluate the corresponding loading. This tactic is used here.

The liquid is taken to be Newtonian and incompressible. Because the roll radii R_A and R_B are taken to be much larger than the distances between the roll surfaces, the flow in the gap region is nearly rectilinear and the governing Navier-Stokes system is well approximated by lubrication theory *(16)*. Thus the pressure distribution $P(X)$, where X is the distance along the gap region ($X = 0$ is at the plane through the two roll centers), is given by:

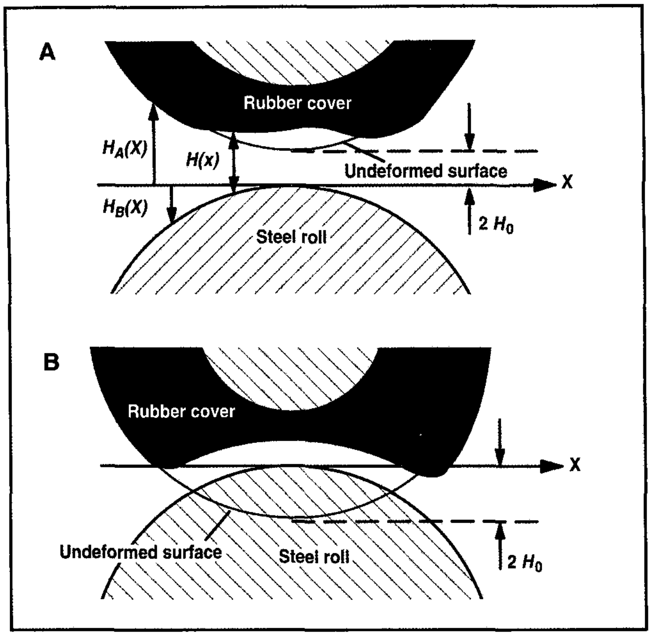
$$dP/dX = 12\mu/[H(X)^3][\overline{V}H(X) - Q] \quad (1)$$

$\overline{V} \equiv (V_A + V_B)/2$ is the average surface velocity of the rolls, V_A being the surface velocity of the roll above, which is taken to be deformable, and V_B being the surface velocity of the roll beneath, which is not deformable. The average surface velocity is related to speed ratio $S = V_A/V_B$ by:

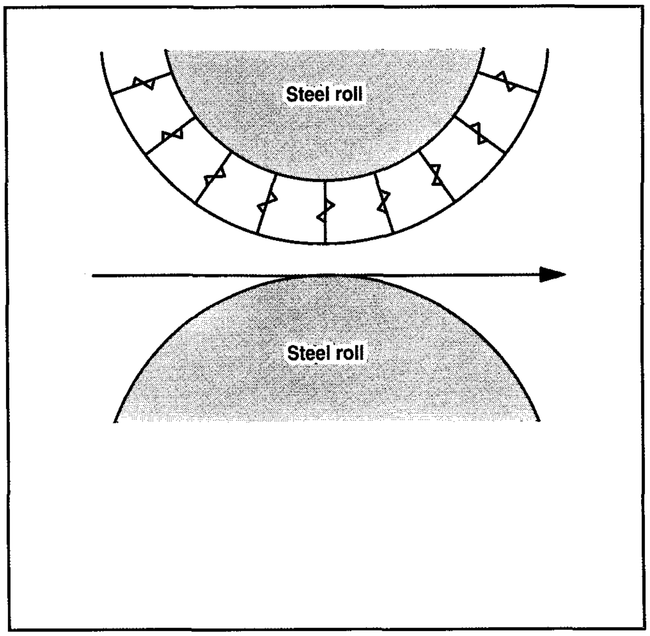
$$\overline{V} = 1/2(S + 1)V_B \quad (2)$$

$S > 0$ describes forward roll ac-

1. Deformable roll coating configuration. A: positive gap, B: negative gap.



2. Spring model sketch



tion and $S < 0$, reverse roll action. $S = 0$ amounts to a stationary but deformable wiper bar. $H(X) = H_A(X) - H_B(X)$ is the local clearance between the roll surfaces, H_A and H_B being the respective distances from a reference plane Y = 0 (which is perpendicular to the plane between roll centers). μ is the viscosity of the liquid. Q is the flow rate through the gap, per unit length of the rolls; the integral of the liquid velocity $U(Y)$ parallel to the reference plane, it is the sum of a drag flow contribution from the moving roll surfaces and a pressure flow contribution from the pressure gradient (as can be seen by solving Eq. 1 for Q). Here it is treated as a parameter.

In the nomenclature of Coyle *(15)*, situations where the roll surfaces would not intersect if they were undeformed are called *positive gaps*; those where they would intersect, or nip, i.e., be in interference, are called *negative gaps*.

For convenience the clearance (positive gap) or the interference (negative gap) between undeformed rolls is called $2H_0$. **Figure 1** contrasts the geometry of the two cases. The circular profile of the roll beneath is conveniently and well approximated by a parabola, and the nearly circular profile of the roll above by a deformed parabola. For the positive gap case, the expressions are:

$$H_B(X) = -X^2/2R_B \qquad (3)$$

$$H(X) = 2|H_0| + (X^2/2R_A) + D(X) \qquad (4)$$

It follows that the local clearance is:

$$H(X) = H_A(X) - H_B(X) =$$
$$2|H_0| + (X^2/R) + D(X) \qquad (5)$$

For the negative gap case, the deformable roll profile changes to:

$$H_A(X) = -2|H_0| + (X^2/R_A) + D(X) \qquad (6)$$

and the local clearance is:

$$H_A(X) = H_T(X) - H_B(X) =$$
$$-2|H_0| + (X^2/R) + D(X) \qquad (7)$$

R is the effective radius of the two rolls:

$$1/R \equiv 1/2([(1/R_B) + (1/R_A)]$$

$D(X)$ is the local displacement of the deformable roll surface in the direction parallel to the plane between roll centers. It can be related to the pressure distribution $P(X)$ by a constitutive equation, i.e., an elastic model of the deformable roll cover. Three different models are considered and compared here.

One-dimensional Hookean model

One-dimensional models amount to a continuous distribution of parallel, independent springs. A discrete analog is diagrammed in **Fig. 2**. The deformation of each spring depends solely on the pressure at its terminus in the roll surface.

Though simple, such a model represents neither shear stresses in the roll cover nor its (lack of) compressibility. Consequently, it does not account for the contributions to deformation at one place on the surface by pressures acting at others. The Hookean version, which was proposed and used by Coyle *(15)*, makes the local displacement directly proportional to the local pressure:

$$D(X) = (L/E)P(X) \qquad (8)$$

E is an effective Young's Modulus and L is an equivalent spring length. They are roughly related to the moduli and thickness of the deformable roll cover, though just how is not clear without a more accurate model to compare with.

One dimensional Neo-Hookean model

Except at small strains, rubber and other elastomers do not follow Hooke's law. A better description is the neo-Hookean model, which for one-dimensional, spring-like elements reduces to the cubic equation:

$$[D(X)/L]^3 + [D(X)/L]^2 + [D(X)/L][1 - [2/3E](\nu + 1)P(X) = [2/3E](\nu + 1)P(X) \quad (9)$$

When displacements $D(X)$ are small enough, this simplifies to the Hookean model. Because an ideal elastomer is incompressible, its Poisson's Ratio ν is 1/2; in this case the neo-Hookean model becomes:

$$[D(X)/L]^3 + [D(X)/L]^2 + [D(X)/L][1 - P(X)/E] = P(X)/E \quad (10)$$

Two-dimensional Hookean model

In this model the deformable roll cover is regarded as a deep elastic slab ("semi-infinite Hookean body") in a plane strain condition *(17)*. The local displacement is a function of the whole pressure distribution, not just the local pressure:

$$D(X) = \int I(X,X')\, P(X')\, dX' \quad (11)$$

The influence function $I(X, X')$ follows from the displacement at X on the surface of a semi-infinite body caused by a concentrated line loading perpendicular to the surface, F per unit length, at X' on the surface *(17)*:

$$D(X) = [(1 - \nu)/\pi G]\, F \ln|X|/|X_0| \quad (12)$$

By superposition, the total displacement at X caused by the distribution of local loadings $P(X')dX'$ is:

$$D(x) = [(1 - \nu)/\pi G] \int_{-\infty}^{\infty} \ln(|X - X'|/|X_0 - X'|)P(X')\, dX' \quad (13)$$

Here G is the shear modulus, $G = E/2(\nu + 1)$.

Dimensionless formulation

The theoretical models consist of Eq. 8, 10, or 13 for the local displacement, Eq. 5 or 7 for the local clearance, and Eq. 1 for the pressure distribution. The last is a first-order ordinary differential equation with an unknown parameter Q, the flow rate per unit length of roll. Hence not one, but two, boundary conditions on pressure are needed. For present purposes, we disregard, as did Coyle *(15)*, the feed condition somewhere to one side of the gap and the film split or film delivery condition somewhere to the other side and, instead, suppose that both conditions are distant enough from the gap that the pressure is substantially atmospheric at both boundaries. Atmospheric pressure can conveniently be taken as the pressure datum, so that P becomes the departure from atmospheric, and the two needed boundary conditions are:

$$P(-|X_1|) = 0,\ P(X_2) = 0$$

With these, Eq. 1, with Eqs. 5 or 7 and 8, 10, or 13 could be integrated (numerically) if the appropriate value of Q were known a priori. It is not—the system is nonlinear, so the procedure is to iterate on Q until the integration satisfied both boundary conditions on P. Then the load W required to maintain the specified center-to-center separation can be found by simple integration:

$$W = \int_{-|x1}^{x2} P(X)\, dX \quad (14)$$

In order to solve the theoretical equations most efficiently, appropriate units should be chosen for the variables and the equations thereby put in dimensionless form. Accordingly:

$$x = X/(R|H_0|)^{1/2}$$

$$h(x) = H(X)/|H_0|$$

$$d(x) = D(X)/|H_0|$$

$$q = Q/\overline{V}|H_0| = Q/[V_B|H_0|(S + 1)] \quad (15)$$

$$p_v = (P|H_0|/\mu\overline{V})(|H_0|/R)^{1/2} = (P|H_0|/\mu V_B)[2/(S + 1)](|H_0|/R)^{1/2} \quad (16)$$

$$p_e = PL/[|H_0|E]$$

$$w = W/[ER]$$

The structure of the equations makes it convenient to use the dimensionless pressure p_v in the positive gap case and p_e in the negative gap case. In the former the equation system becomes:

$$dp_v/dx = \{12/[h(x)]^2\} - \{24q/[h(x)]^3\}$$
$$p_v(-|x_1|) = 0,$$
$$p_v(x_2) = 0 \quad (17)$$

$$h(x) = 2 + x^2 + d(x)$$

whereas the negative gap case, the equation system becomes:

$$dp_e/dx = N_e\{12/[h(x)]^2\} - \{24q/[h(x)]^3\}$$
$$p_v(-|x_1|) = 0,$$
$$p_v(x_2) = 0 \quad (18)$$

$$h(x) = -2 + x^2 + d(x)$$

The equation that relates dimensionless displacement distribution $d(x)$ and dimensionless pressure distribution $p(x)$ depends on the case and the model chosen.

Positive gap regime

- One-dimensional Hookean model:

$$d(x) = N_e\, p_v(x) \quad (19a)$$

- One-dimensional neo-Hookean model:

$$d^3(x) + d^2(x)L/H_0 + d(x)(L/H_0)^2\{[1 - N_e(H_0/L)]p_v(x)\} = N_e(L/H_0)^2 p_v(x) \quad (19b)$$

- Two-dimensional Hookean model:

$$d(x) = N_e \cdot \sum_{i=1}^{N} \ln[|x - x_i|/|x_0 - x_i|]p_v(x_i)\Delta x \quad (19c)$$

Negative gap regime

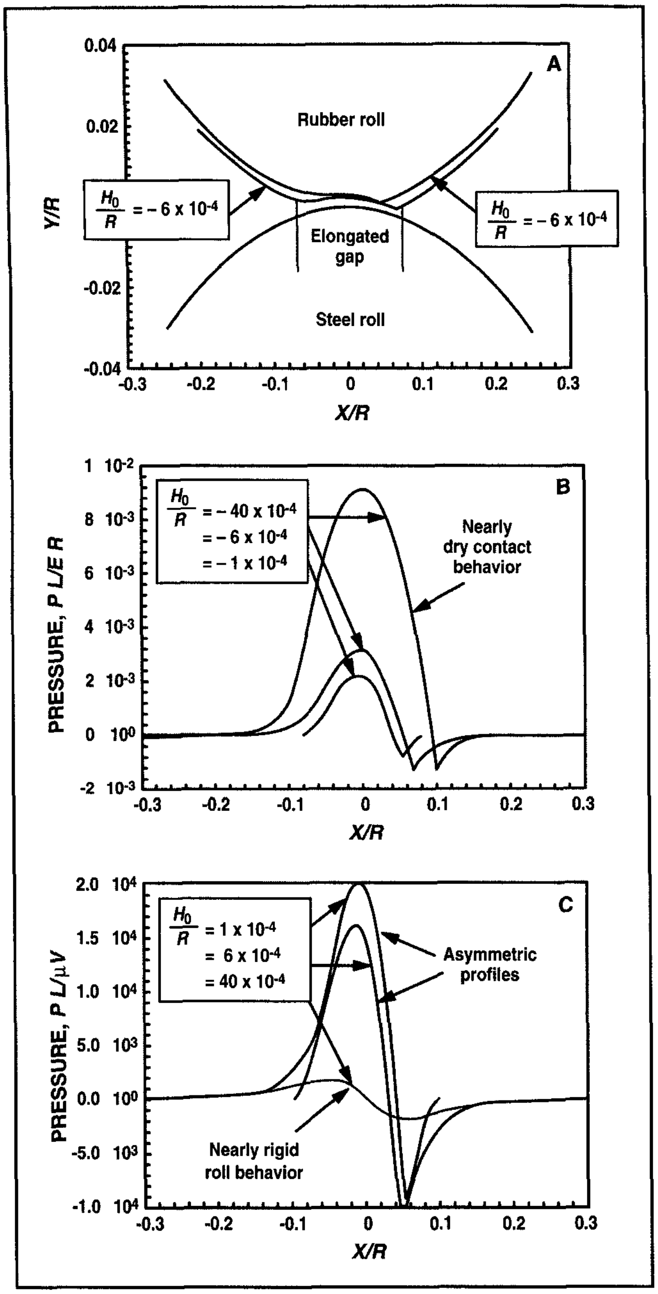
- One-dimensional Hookean model:

$$d(x) = p_e(x) \quad (20a)$$

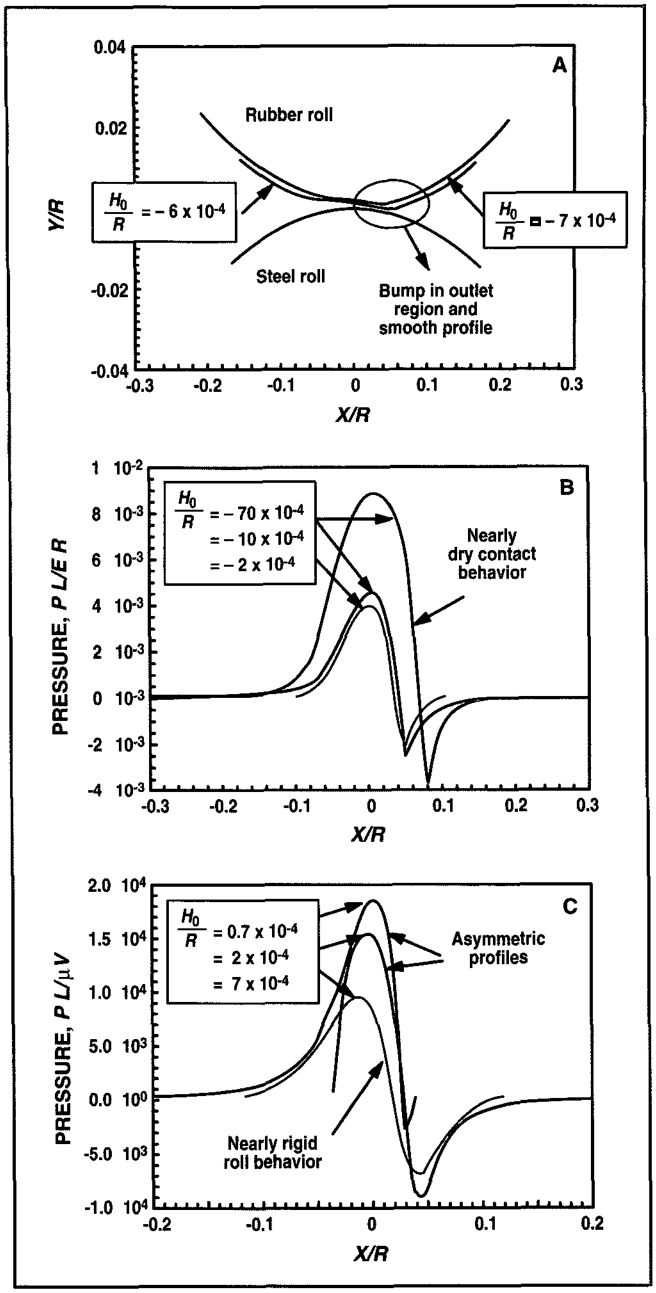
- One-dimensional neo-Hookean model:

$$d^3(x) + d^2(x)L/H_0 + d(x)(L/H_0)^2$$

3. Predictions of the one-dimensional model. A: roll profile, B: pressure distribution (negative gap), C: pressure distribution (positive gap). Elasticity number = 10^{-6}.



4. Predictions of the two-dimensional model. A: roll profile, B: pressure distribution (negative gap), C: pressure distribution (positive gap). Elasticity number = 10^{-6}.



$$[1 - (H_0/L)]p_e(x) = (L/H_0)^2 p_e(x) \tag{20b}$$

- Two-dimensional Hookean model:

$$d(x) = A\sum_{i=l}^{N}\ln[|x - x_i|/|L - x_i|]p_v(x_i)\Delta x \tag{20c}$$

Plainly, the solution of any of these equation systems depends on three parameters: dimensionless flow rate q; one or the other of the length ratios $A = 0.477\ (R/L)(|H_0|/R)^{1/2}$ and $L/|H_0|$; and either N_e^* or N_e. The dimensionless parameter N_e is a modified elasticity number that measures the relative pressure in the liquid, which is proportional to viscosity and average roll speed, and elastic stress in the roll cover, which is proportional to the Young's modulus:

$$N_e = (\mu\overline{V}/ER)(L/R)(R/|H_0|)^{5/2} =$$
$$(\mu V_B/ER)(L/R)(R/|H_0|)^{5/2}(S + 1)/2 \tag{21a}$$

Here, as above, $\overline{V} \equiv (V_A + V_B)/2$ is the average velocity of the two (undeformed) roll surfaces. A slightly different parameter N_e^* emerges from the two-dimensional Hookean model:

$$N_e^* \equiv 0.477(\mu\overline{V}/ER)(R/|H_0|)^2 =$$

$0.477(\mu V_b/ER)(R/|H_0|)^2(S + 1)/2$ (21b)

As Coyle emphasized about the forward roll regime, the elementary elasticity number $\mu\bar{V}/ER$ appears only in combination with geometric ratios L/R and $|H_0|/R$, and speed ratio is extraneous as a parameter, when average roll velocity is used in the elasticity numbers (as it therefore should be). It follows that there is no essential difference between the forward and reverse roll regimes, so far as the theoretical model is concerned. However, when in the reverse roll regime the average velocity is zero, the formulas break down.

Solution of the equation systems

One-dimensional models

The chosen equation for $d(x)$ and the equation for $h(x)$ were substituted in the differential equation for p_v or p_e, which was then integrated using a forward difference scheme. The integration was performed backward in space (from x_2 to $-|x_1|$). An initial flow rate was set and Newton iteration were made until the inlet pressure boundary condition was satisfied; the Newton updates of flow rate were calculated with:

$$q_{n=1} = q_n - [p_{-|x1|}/(\partial p_{-|x1|}/\partial q)]_n \quad (22)$$

The sensitivity of the pressure at $-|x_1|$ to flow rate was evaluated with a finite difference approximation:

$$\partial p_{-|x1|}/\partial q \approx [p_{-|x1|}(q_n + \Delta q) - p_{-|x1|}(q_n)]/\Delta q$$

The step in flow rate was set to $\Delta q = 0.01q_n$.

The initial guess for flow rate was determined by a first-order continuation in $|H_0|/R$, i.e., it was found from the previous result and its sensitivity to flow rate.

Two-dimensional model

With this model, the equation system is more complex, because the displacement at each x depends on the pressure at all values of x. One simple strategy would be Picard iteration: evaluate the pressure profile using the displacement distribution (roll profile) from the previous iteration; with the new pressure profile, evaluate the new displacement distribution; and so on. As the literature about this procedure in elastohydrodynamics testifies *(3)*, it is generally slow to converge if it converges at all.

What was done was to divide the domain into n equal intervals by means of $n + 1$ uniformly spaced nodes, and to adopt as unknowns the values of pressures at the $n - 1$ interior nodes, the values of the displacement at the n midpoints of the intervals, and the flow rate q. Suitably unequal intervals could be generated adaptively and would be more efficient. The unknowns had to satisfy the correspondingly discretized version of Eqs. 17 and 19c or 18 and 20c, with derivatives approximated by forward differences. The equations, $2n + 1$ in number, are nonlinear and were solved by Newton iteration with the full Jacobian matrix and initialization by first-order continuation in the geometric parameter H_0/R. By controlling the step sizes by which these parameters were changed, convergence was achieved within 3–5 iterations and was generally at the expected, efficient, quadratic rate. With all the parameters values used, $n = 300$ intervals sufficed; increasing n to 400 produced no more than 1% change in q, the flow rate through the gap.

As for the one-dimensional model, the positions x_1 and x_2 were chosen as the minimum values that left the flow rate no more than 1% different from the value when x_1 and x_2 were very large.

Results and discussion

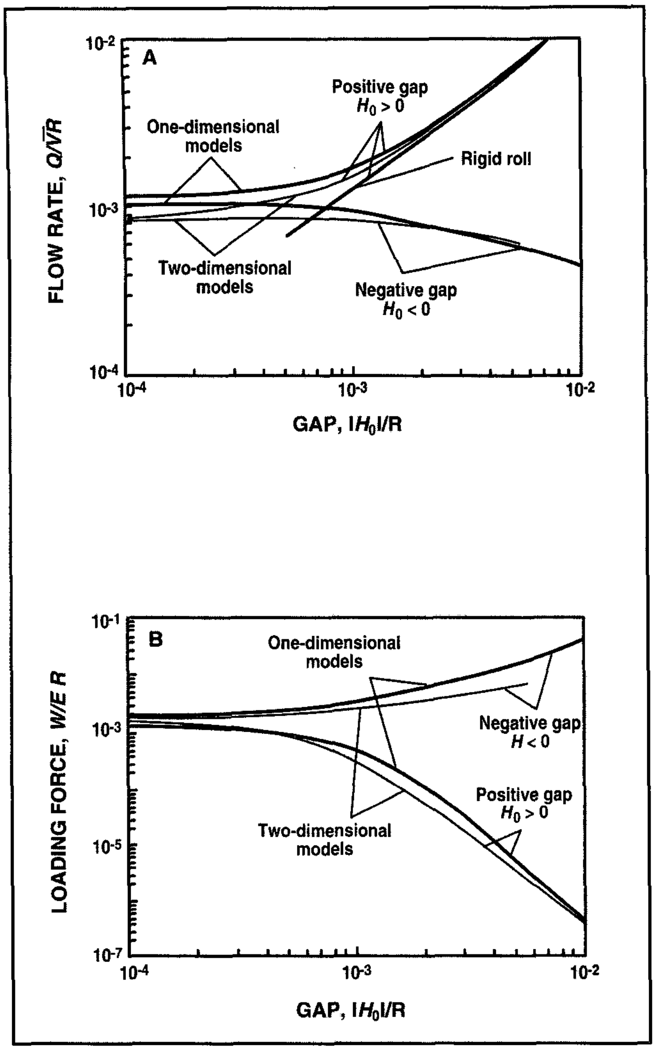
It is important to recognize that the prediction for given values of N_e (or N_e^*) and $L/|H_0|$ (or A) pertains to an entire range of undeformed gap or nip setting, i.e., $2|H_0|$, given the ratio of the thickness L of the deformable roll cover to the outer radius R of the covered roll, and the basic elasticity number $\mu\bar{V}/ER$. In the positive gap case, as the rolls are brought together, $2|H_0|$ diminishes and N_e or N_e^* rises. In the negative gap case, the same quantity falls.

Figure 3 shows the roll profiles and the pressure distributions predicted by the two one-dimensional models. At all settings of the undeformed gap, the pressure rises steeply in the converging part of the gap or nip and then falls precipitously. The negative (i.e., subambient) pressure predicted to occur in the diverging part raises the possibility of cavitation on the one hand, but on the other hand points to the desirability of improving the theory by accounting for the actual boundary conditions on the feed and delivery sides of the gap or nip. When the undeformed gap is large, the profiles approach those of rigid rolls, as expected. As the undeformed gap is narrowed and made negative by pushing the rolls together, the profiles become more and more asymmetric, finally approaching the Hertzian profiles which belong to dry contact. But as that limiting case is approached, the liquid-filled clearance persist, elongates, and is virtually constant over its length.

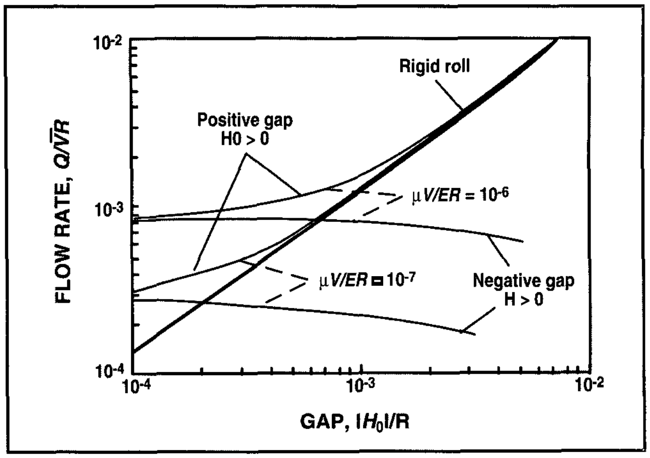
Figure 4 shows that the same features are predicted when the two-dimensional model is used, except that the profile of the deformable roll cover is generally smoother yet has a bump in what would be the outlet zone of forward roll action (flow from left to right in Fig. 4). This protuberance can be traced to the tendency of the model to conserve volume, a tendency that becomes a requirement when the Poisson ratio ν is one-half, as it is for the ideal elastomer. Such a bump, protuberance, or standing wave has been observed in experiments on the elastohydrodynamics of bearings *(3)*.

Figure 5 shows, for the case of elasticity number $\mu\bar{V}/ER = 10^{-6}$, two quantities that are important to coating applications: flow rate and loading per unit length. In the range of undeformed gaps explored, the predictions of the Hookean and neo-

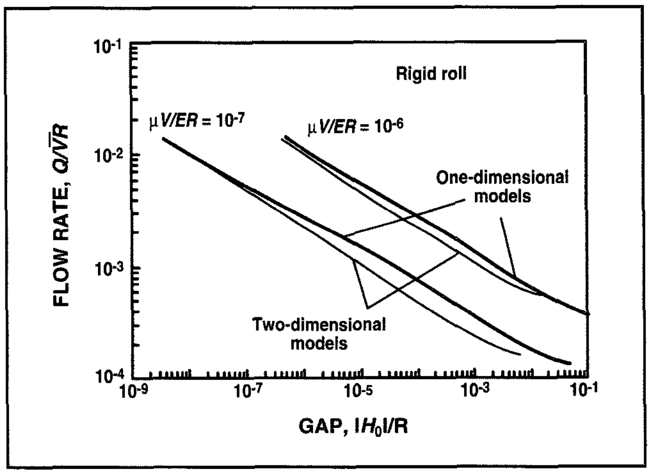
5. Predictions of one-dimensional and two-dimensional models. A: flow rate, B: loading force. Elasticity number = 10^{-6}.



6. Flow rate predictions at two different elasticity numbers, 10^{-6} and 10^{-7}



7. Flow rate–loading force relation predicted by one-dimensional and two-dimensional models



Hookean models do not differ appreciably. The reason is that the loads and deformations are small enough that the neo-Hookean response is linear and so matches the Hookean response. Figure 5 shows that flow rate and loading each converges to the same value whether the undeformed gap approaches zero from positive or negative values—as must be expected of physically correct modeling.

Figure 5A shows further that as the undeformed gap is widened (greater $2|H_0|$, for positive gap) its influence on flow rate becomes more and more nearly linear, as it should according to the lubrication theory of flows between two hard rolls. On the other hand, as the undeformed gap is narrowed (greater $2|H_0|$, for negative gap), its influence on flow rate diminishes and nearly disappears, but not completely. These findings are in accord with Coyle *(15)*. The predictions of the two-dimensional model show the same behavior as do the one-dimensional results, but they differ in detail. The two-dimensional model shows a weaker dependence of flow rate on gap setting in the negative gap regime.

Figure 5B shows that the loading required to maintain a given undeformed gap approaches zero as the undeformed gap is widened, again in accord with the lubrication theory of flow between hard rolls *(18)*. Again the two-dimensional model displays the same tendency as the one-dimensional one, but with differences in detail.

The behavior at great enough loading (negative gap case) is fairly well described by a power-law dependence:

- One-dimensional Hookean model

  $Q/[V_B R(S+1)] \approx (H_0/R)^{-0.3}$, $W/ER \approx (H_0/R)^{1.5}$

- Two-dimensional Hookean model

  $Q/[V_B R(S+1)] \approx (H_0/R)^{-0.2}$, $W/ER \approx (H_0/R)^{0.7}$

Figure 6 reflects the fact that, as the elasticity number falls, the roll cover is deformed less, the local clearances diminish, and so does the flow rate. Of the curves for two values of elasticity number, $\mu\overline{V}/ER$ = 10^{-6} and 10^{-7}, the predictions for the lower value, corresponding to the more rigid cover, are closer to the rigid roll limit.

Because metering and transferring gaps and nips are often controlled by loading rather than setting the undeformed gap, it is useful to eliminate the latter from the foregoing results, and relate flow rate and loading directly. Curves are shown in **Fig. 7** for two values of elasticity number, $\mu\overline{V}/ER = 10^{-6}$ and 10^{-7}. In the range explored, the results can be fitted fairly well to power law dependence:

- One-dimensional Hookean model

$Q/[V_B R(S + 1)] \approx (W/ER)^{-0.29}$, at small deflection limit

$Q/[V_B R(S + 1)] \approx (W/ER)^{-0.21}$, at large deflection limit

- Two-dimensional Hookean model

$Q/[V_B R(S + 1)] \approx (W/ER)^{-0.33}$, at small deflection limit

$Q/[V_B R(S + 1)] \approx (W/ER)^{-0.26}$, at large deflection limit

The small deflection limits are close to the predictions of the flow rate–loading relation of rigid roll coaters *(15)*. There are no published data available, so far as we know, with which these predictions can be compared. Presumably the two-dimensional Hookean model, which makes use of the plane strain approximation, is the more accurate—particularly because its prediction of roll surface profile seems more realistic.

Closing

The flow through gaps between rotating cylinders is used to perform the metering function and application function of a coating process. The cylinders can be operated either in a forward mode (counter-rotating rolls) or reverse mode (co-rotating rolls). There are advantages to carrying out the application in a nip between a hard and soft roll, but in this case the gap is not pre-set by geometry; rather it is a result of a coupled action of the flow and the roll cover deformation. In order to control better the process, it is important to understand this type of flow and know the effects of operational parameters relations, such as how coating weight is affected by the external loading.

Coyle *(15)* initiated study of elastohydrodynamic action in the roll coating context. He used a one-dimensional model to describe the roll deformation. We confirmed results of that model and its generalization to neo-Hookean response. We went on to model the relation of roll cover deflection to pressure distribution in the liquid by means of a plane strain Hookean model. We solved the governing equations by superposing the surface deformations caused by a single force. This approach avoided the use of a complete two-dimensional formulation for the whole elastomer layer. The results yielded by the plane strain model proved slightly different from those from the one-dimensional model. Presumably they are more accurate, because the predicted roll surface profile seems more realistic and describes the behavior observed in bearing lubrication experiments *(3)*.

To validate and improve the theoretical modeling, experimental measurements are needed of flow rate and coat weight versus liquid viscosity, roll cover properties and gap setting. We have embarked on such measurements.

It should be pointed out that the lubrication approximation to flow used here does not give directly the coating weight because, as it stands, it does not cover the film split flow. A next step is to model this part of the flow field by means of a viscocapillary model. This is under way, but until it is done, the speed-ratio dependence of the film split could be assumed to be the same as in rigid-roll coating *(18)*. **TJ**

Literature cited

1. Benjamin, D. F. and Scriven, L. E., *Industrial Coating Research* 2: 1 (1993).
2. Dowson, D. and Higginson, G. R., *J. Mech. Eng. Sci.* 1: 6(1959).
3. Dowson, D. and Higginson, G. R., *Elasto-Hydrodynamic Lubrication,* Pergamon Press, Oxford, 1966.
4. Bennet, A. and Higginson, G. R., *J. Mech. Eng. Sci.* 12: 218(1970).
5. Dowson, D. and Longfield, M. D., *Nature* 197: 586(1963).
6. Higginson, G. R., *Int. J. Mech. Sci.* 4: 205(1962).
7. Baglin, K. P. and Archard, J. F., *1972 Elastohydrodynamic Lubrication Symposium Notes,* The Institute of Mechanical Engineers, p. 13
8. Berthe, D. and Vergne, Ph., *J. Rheology* 34: 1387(1990).
9. Wang, S., Cusano, C., and Conry, T. F., *J. Tribology* 113: 232(1991).
10. Dobbels, F. and Mewis, J., *Chem. Eng. Sci.* 33: 493(1978).
11. Bapat, C. N. and Batra, R. C., *Int. J. Numerical Methods in Eng.* 20: 1911(1984).
12. Batra, R. C., *J. Applied Mechanics* 47: 82(1980).
13. Batra, R. C., *Int. J. Numerical Methods in Eng.* 17: 1823(1981).
14. Batra, R. C., Levinson, M., and Betz, E., *Int. J. Numerical Methods in Eng.* 10: 767(1976).
15. Coyle, D. J., *Chem. Eng. Sci.* 43: 2673(1988).
16. Cameron, A., *Basic Lubrication Theory,* Wiley, New York, 1966.
17. Love, A.E.H., *A Treatise on the Mathematical Theory of Elasticity,* Dover Publications, New York, 1944.
18. Coyle, D. J., Scriven, L. E., and Macosko, C. W., *J. Fluid Mechanics* 171: 183(1986).
19. Macosko, C. W., Rheology: Principles, Measurements and Applications, manuscript to be published.

M. S. Carvalho was supported by a fellowship from CAPES (Brazilian Government). The research was supported through the Center for Interfacial Engineering, with funds from the National Science Foundation and from cooperating corporations.

Received for review May 21, 1993.

Accepted Oct. 15, 1993.

Presented at the TAPPI 1993 Polymers, Laminations and Coatings Conference.