On the effect of centrifugal force on winding

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ABSTRACT: The centrifugal force is included in Hakiel's nonlinear model for wound roll stresses. As expected, an increase in centrifugal force has an increasing effect on the in-roll tangential stress and a decreasing effect on the radial pressure. Analyses carried out for newsprint show that the effect becomes significant for winding velocities above 500 m/min.

KEYWORDS: Centrifugal force, equations, newsprint, stresses, winding, wound rolls.

The stresses in a wound roll determine the quality of the roll. Thus, methods for calculating these stresses are important when trying to increase roll quality. Altmann started the analysis of wound roll stresses by finding an analytical solution to the problem based on the assumption of linear elasticity (*1*). For nonlinear elasticity, no solutions have been found analytically. However, Hakiel derived a numerical method for calculating the stresses for a roll with nonlinear elastic response in the radial direction (*2*).

By the methods of Altmann and Hakiel, it is possible to analyze the effects of parameters such as geometry, elastic properties of web and core, and winding tension. Based upon the work of Hakiel, I have found a method that also includes the effects of the centrifugal force. From a linear analytical solution, Yagoda concluded in 1980 that the effect of the centrifugal force was insignificant (*3, 4*). The winding velocity has been more than doubled since then, and my calculations show that the effect should not be neglected.

Basic equations

The roll is thought of as an orthotropic, elastic cylinder of concentric hoops of a web that is uniform in thickness, width, and length. We assume that the in-roll stresses are functions of the radius, not of axial or circumferential position. The linear orthotropic constitutive equations for the cylindrical roll can thus be written in the following form:

$$\varepsilon_r = (1/E_r)\,\sigma_r - (\nu_{rt}/E_t)\,\sigma_t \qquad (1)$$

$$\varepsilon_t = (1/E_t)\,\sigma_t - (\nu_{tr}/E_r)\,\sigma_r \qquad (2)$$

where

ε_r = strain in radial direction

ε_t = strain in tangential direction

σ_r = stress in radial direction

σ_t = stress in tangential direction

E = modulus of elasticity

E_t = elastic modulus of the web in length direction

E_r = elastic modulus of a stack of sheets of web

ν = Poisson ratio of the web.

Cauchy's equation for axisymmetrical plane stress in the presence of centrifugal forces is

$$r\,(\partial\sigma_r/\partial r) + \sigma_r - \sigma_t = -\rho\,\omega^2\,r^2\,R_c^{\,2} = -\rho(\upsilon/s)^2\,r^2 \qquad (3)$$

where, as depicted in **Fig. 1**,

r = radius to a point in the roll, made dimensionless by division by R_c

s = outside radius of a winding roll, made dimensionless by division by R_c

ρ = local density of the roll

ω = angular velocity

υ = winding speed

R_c = outer radius of the core by which all distances are normalized.

Because of the strain energy constraint:

$$\nu_{tr}/E_r = \nu_{rt}/E_t \qquad (4)$$

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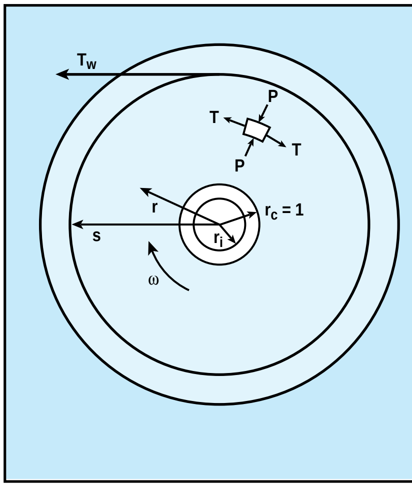
I. Web properties of newsprint

Thickness, µm	71
Poisson's ratio	0.01
E_t, GPa	3.37
C_0	0.0
C_1, kPa	50.6
C_2, kPa^{-1}	– 0.0964
C_3, kPa^{-2}	0.0001
Core radius, m	0.05
E_c, GPa	8.0
Roll density, kg/m^3	650

II. The decrease in radial pressure, P, at r = 2.0 due to the centrifugal force for winding tensions of 3000 kPa and 5000 kPa

Winding velocity, m/min	*Decrease in P, %*	
	3000 kPa	*5000 kPa*
500	*2.7*	*1.5*
1000	*10.6*	*5.9*
2000	*39.5*	*23.1*

1. Model of calculation



only three of the four elastic constants in the constitutive equations are independent. We apply Eq. 4, and we define $\nu \equiv \nu_r$ and

$$g^2 \equiv E_t/E_r \tag{5}$$

so that the constitutive equations, Eqs. 1 and 2, can be rewritten as

$$\varepsilon_r = (1/E_t)\,(g^2\sigma_r - \nu\,\sigma_t) \tag{6}$$

$$\varepsilon_t = (1/E_t)\,(\sigma_t - \nu\,\sigma_r) \tag{7}$$

From the linear definitions of strain in cylindrical coordinates, the strain compatibility equation, Eq. 8, may be derived:

$$r\,(\partial\varepsilon_t/\partial r) + \varepsilon_t - \varepsilon_r = 0 \tag{8}$$

Now, Eqs. 3, 6, 7, and 8 can be combined and rearranged to yield the following differential equation in terms of the radial stress:

$$r^2\,[(\partial^2\sigma_r)/(\partial r^2)] + 3r\,[(\partial\sigma_r)/(\partial r)]$$

$$+ (1 - g^2)\,\sigma_r = -(3 + \nu)\,\rho\,(\upsilon^2/s^2)r^2 \tag{9}$$

Let δP, positive in compression, be the incremental radial pressure at any radius r caused by the winding on of a single lap of thickness h at outer radius s under the winding tension $T_w(s)$. ($P = -\sigma_r$ the total radial pressure, positive in compression, at a point in the roll.) Incrementing Eq. 9 for the incremental radial pressure gives:

$$r^2\,[\partial^2(\delta P)/(\partial r^2)] + 3r\,[\partial(\delta P)/(\partial r)]$$

$$+ (1 - g^2)\,(\delta P) = (3 + \nu)\,\rho\,\delta(\upsilon^2/s^2)r^2 \tag{10}$$

where $d(\upsilon^2/s^2)$ is the difference in the square of the angular velocity compared with that of the previous lap.

Boundary conditions

Since Eq. 10 is a second-order differential equation, two boundary conditions are needed if we are to pose the problem appropriately. At the interface between the core and the roll, the radial deflection of the core must equal that of the roll to assure continuity. For the core, we have by definition of the core modulus E_c:

$$\delta u(1) = -\,\delta P(1)/E_c \tag{11}$$

where

u = radial deflection, made dimensionless by division by R_c

R_c = outer radius of the core.

The core modulus, also known as the core stiffness, is discussed by Pfeiffer (5). Substitution of the definition of the tangential strain, $\varepsilon_t = u/r$, in Eq. 7, gives the radial deflection of the roll. At the interface (r = 1), we have:

$$\delta u(1) = (1/E_t)\,(\delta T(1) + \nu\,\delta P(1)) \tag{12}$$

for the roll. Here δT is the incremental in-roll tangential stress caused by the winding on of a single lap onto the roll. ($T = \sigma_t$ total in-roll tangential stress at a point in the roll.) The incremented version of Eq. 3 yields

$$\delta T = -\,\delta P - r\,[\partial(\delta P)/\partial r] + \rho\,\delta(\upsilon^2/s^2)\,r^2 \tag{13}$$

Thus, Eqs. 11, 12, and 13 can be combined to give the boundary condition at the interface between the core and the roll:

$$\left.\frac{d(\delta P)}{dr}\right|_{r=1} = \left[(E_t/E_c) - 1 + \nu\right]\delta P(1) + \rho\,\delta\left(\frac{\upsilon^2}{s^2}\right) \tag{14}$$

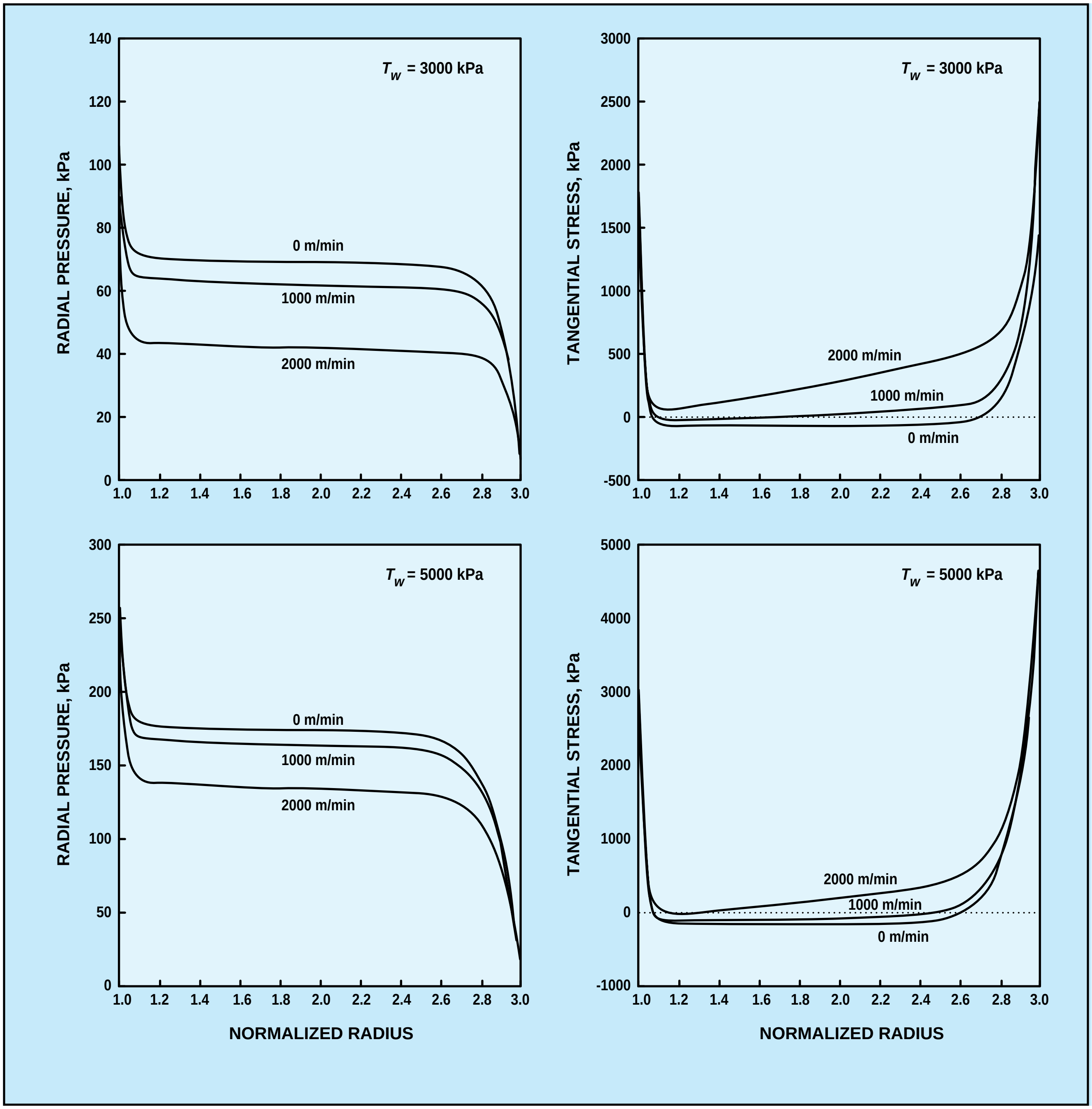
At the outside of the winding roll, the incremental radial pressure caused by the winding on of the last lap

$$\delta P(s) = T_w\,(h/s) - \rho\,\upsilon^2\,(h/s) \tag{15}$$

can be verified directly from Eq. 3 by setting r equal to the outside radius s.

The differential equation, Eq. 10, and the boundary conditions, Eqs. 14 and 15, represent a mathematically well-posed problem. We assume that the nonlinear elastic modulus in the radial direction can be written in the following form:

2. In-roll stresses for newsprint wound with tensions of 3000 kPa and 5000 kPa at different velocities



$$E_r = C_0 + C_1P + C_2P^2 + C_3P^3 \quad (16)$$

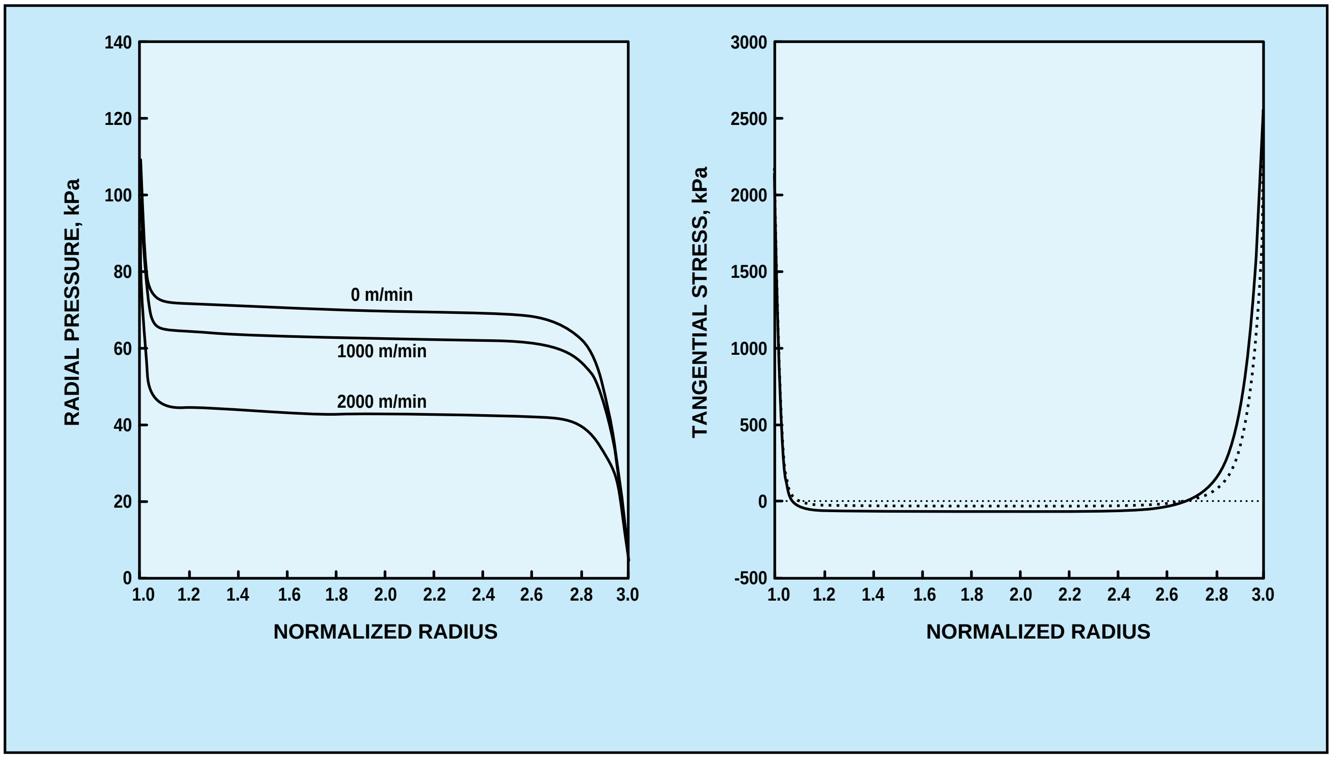
where C_1 is roughly equal to the K_2 factor of Pfeiffer (*6*).

Now, g^2, or E_t/E_r, will depend on the radius r, and thus the solution cannot be found analytically. At a specific radius, the elastic modulus in the radial direction is considered to be a constant during the winding on of any single lap. When the next lap is wound on, the modulus is recalculated from Eq. 16. This makes the use of linear constitutive equations valid. Note that the problem is equal to that of Hakiel if the centrifugal effect is neglected by setting $\upsilon = 0$.

Numerical solution

Since it is not possible to solve the problem analytically, we need to find a numerical solution. We assume that the roll consists of $(j - 1)$ laps, each wound lap of thickness h. The radius to the inside of the ith lap is then:

3. In-roll stresses for finished rolls of newsprint wound with a tension of 3000 kPa at different velocities



$$r_i = 1 + (i + 1)h \quad i < j \tag{17}$$

where

j = number of laps in the winding roll

i = number of laps within the winding roll.

The increase in pressure at r_i caused by the winding on of lap j is denoted $\delta P_{i,j}$. Note that $s = r_j$.

The derivatives in the boundary conditions and Eq. 10 can be substituted by the central difference approximation. If we define

$$A_{i,j} \equiv 1 + (3h/2r_i) \tag{18}$$

$$B_{i,j} \equiv (h^2/r_i^2)\,(1 - g_{i,j}^2) - 2 \tag{19}$$

$$C_{i,j} \equiv 1 - (3h/2r_i) \tag{20}$$

and

$$D_{i,j} \equiv (3 + \nu)\,\rho\,\delta(\upsilon_j^2/r_j^2)\,h^2 \tag{21}$$

where

$$\delta(\upsilon_j^2/r_j^2) = (\upsilon_j^2/r_j^2) - (\upsilon_{j-1}^2/r_{j-1}^2) \tag{22}$$

we can write the numerical representation of Eq. 10 in the following form:

$$\delta P_{i+1,j}\,A_{i,j} + \delta P_{i,j}\,B_{i,j} + \delta P_{i-1,j}\,C_{i,j} = D_{i,j} \tag{23}$$

The boundary condition at the outside of the roll can be written as

$$\delta P_{j,j} = T_{w,j}\,(h/r_j)\,[1 - (\rho\,\upsilon_j^2)/(T_{w,j})] \tag{24}$$

From the boundary condition at the core–roll interface, Eq. 14, and a forward difference approximation, we find

$$\partial P_{2,j}\,(1/h) + \delta P_{1,j}\,[1 - (E_t/E_c) - \nu - (1/h)] = \rho\,\delta(\upsilon_j^2/r_j^2) \tag{25}$$

Together, Eqs. 23, 24, and 25 constitute a tridiagonal system of j linear equations in j unknowns. This set can be solved generally by Gaussian elimination or more efficiently by the *tridag* routine (*7*). Once we have found the solution of δP, we can calculate δT from Eq. 13.

The total in-roll stress distribution is found by calculating δP and δT for each lap wound on. Then all contributions are added to the solution.

$$P_i = \sum_{j=i+1}^{N} \delta P_{i,j} \tag{26}$$

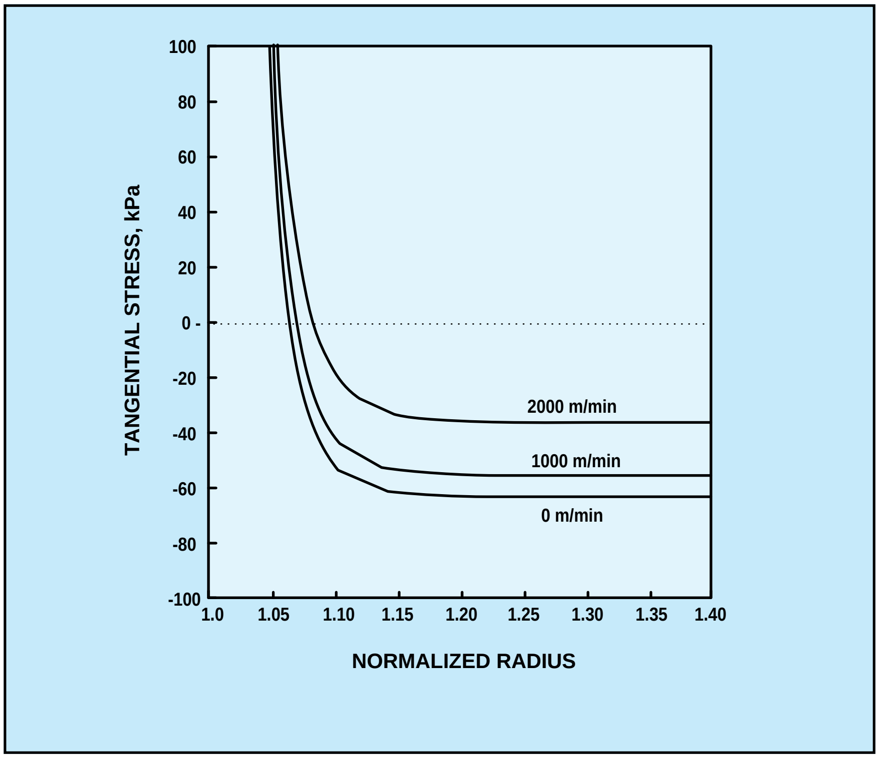
$$T_i = T_{w,i} + \sum_{j=i+1}^{N} \delta T_{ij} \tag{27}$$

where N is the total number of laps in the wound roll.

Numerical examples

Calculations of in-roll stresses for wound newsprint have been carried out. For newsprint wound at a tension of 3000 kPa with winding properties as listed in **Table I**, the effect of the centrifugal force is not measurable at the winding velocity of 15 m/min, the speed at which many tests

4. Tangential stress for finished rolls of newsprint wound with a tension of 3000 kPa at different velocities



of Hakiel's model are carried out.

However, the effect becomes significant for winding velocities greater than approximately 500 m/min. At 500 m/min, the centrifugal force reduces the radial pressure with more than 2%. For velocities of 1000 m/min and 2000 m/min, the centrifugal force has the effect shown in the upper two graphs of **Fig. 2**. The winding tension T_w and velocity v is constant during the winding of each roll.

As indicated by Eq. 15, the centrifugal force becomes less significant as the winding tension increases. With a winding tension of 5000 kPa, the effect of the centrifugal force is less than for the winding tension of 3000 kPa. But the effect is significant for this higher winding tension also.

So far we have discussed rolls which are still rotating at the specified winding velocities. Rolls that are brought to a stop will be discussed below.

The results illustrated in Fig. 2 indicate that the centrifugal force causes an increase in tangential stress and a decrease in radial pressure. This outcome is as expected. The terms that include the centrifugal force are proportional to the square of the winding velocity. Thus we would also expect the effect of the centrifugal force to be quadrupled every time the velocity is doubled. The results listed in **Table II** correspond quite well with this expectation. They also verify that the centrifugal force should not be neglected.

When the roll has reached its final diameter, the roll is brought to a stop. One might ask if the stresses in finished rolls are influenced by the centrifugal force at work during the winding procedure.

Figure 3 displays radial pressure and tangential stress in a roll of newsprint wound at different winding velocities and then brought to a stop with an instant shutdown. We see that the centrifugal force still is significant for the radial pressure. For the tangential stress, it appears that the curves representing different winding velocities coincide and that the centrifugal force is insignificant for finished rolls. This misunderstanding is caused by the unfortunate scaling in Fig. 3. If a more convenient scaling is applied, as shown in **Fig. 4**, we see that the tangential stress is significantly increased by the centrifugal force. Thus, the centrifugal force is significant both during and after winding.

So far, this model has not been confirmed by experiments. However, certain experiences from the paper mill at Skogn in Norway support the model. Rolls of newsprint wound at rewinders with winding velocities around 800 m/min are harder and denser than those wound on the high-velocity winders. This outcome is consistent with the model that yields higher radial pressure for lower winding velocities.

In summary, the centrifugal force is significant in winding. It decreases the radial pressure and increases the tangential stress in the rolls. The result of increased winding velocities is softer rolls. TJ

Literature cited

1. Altmann, H. C., *Tappi J.* 51(4): 176(1968).
2. Hakiel, Z., *Tappi J.* 70(5): 113(1987).
3. Yagoda, H. P., *Mechanics Research Communications* 7(3): 181(1980).
4. Yagoda, H. P., *Mechanics Research Communications* 7(4): 233(1980).
5. Pfeiffer, J. D., *TAPPI 1991 Finishing and Converting Conference Proceedings*, TAPPI PRESS, Atlanta, p. 25.
6. Pfeiffer, J. D., *Tappi J.* 64(4): 105(1981).
7. Press, W. H., Teukolsky, S. A., Vettering, W. T., and Flannery, B. P., *Numerical Recipes in C*, Cambridge University Press, 1992.

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