Mathematical formulations for thermodynamic properties of moist air at high temperatures

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This paper provides a mathematical model for the accurate computation of absolute humidity given a wet-bulb and a dry-bulb temperature reading. In several industrial applications, such as paper or cement, it is often required to calculate absolute humidity accurately. A common method is to take the wet-bulb and dry-bulb readings and look up the humidity in a suitable reference.

However, with the advent of microprocessor systems in the industrial workplace, it is possible to program the calculation of process air variables such as volume, enthalpy, and viscosity. These variables are generally given as a combination of the air and water combinations. Therefore, absolute humidity must be calculated in order to calculate other variables. While not the only available methodology, this paper presents a sound analytical formulation to achieve this task.

Background

When the exit gas is saturated, the steady state leaving temperature of the gas is that of the feed water. This is known as the adiabatic saturation temperature (Ts). For calculation purposes, the wet-bulb temperature and adiabatic saturation temperature are taken as equal.

Performing a heat-balance on the system shown in Fig. 1:

$$\int_{T_s}^{T_d} C_{pm} dT + W\Lambda_s = W_s\Lambda_s \tag{1}$$

where

C_{pm} = heat capacity of the inlet (unsaturated air/water) stream, BTU/lbm °F

W = absolute humidity, lbm water/lbm dry air

Λs = latent heat of vaporization, Btu/lbm water

T_s = adiabatic saturation temperature, °F

T_d = dry bulb temperature, °F

W_s = saturation humidity, lbm water/lbm dry air

Equation (1) states that the sum of the latent and sensible heat terms remains constant if there is no heat loss to the surroundings.

To evaluate the absolute humidity W, the heat capacity of the mixture must be expressed as a combination of the air and water vapor components. This form is typically linear and can be expressed as follows:

$$C_{pm} = C_{pa} + WC_{pwv} \tag{2}$$

where

C_{pa} = heat capacity of dry air, Btu/lbm °F

C_{pwv} = heat capacity of water vapor, Btu/lbm °F

Substituting Eq. (1) in (2) yields Eq. (3):

$$\int C_{pa}\, dT + W\int C_{pwv}\, dT + W\Lambda_s = W_s\Lambda_s \tag{3}$$

From basic thermodynamics,

$$\Delta h = \int C_p\, dT \tag{4}$$

where

$$\Delta h = \text{enthalpy change, Btu/lbm} \tag{5}$$

It follows then that,

$$\Delta h_a = C_p \int dT \tag{6}$$

and

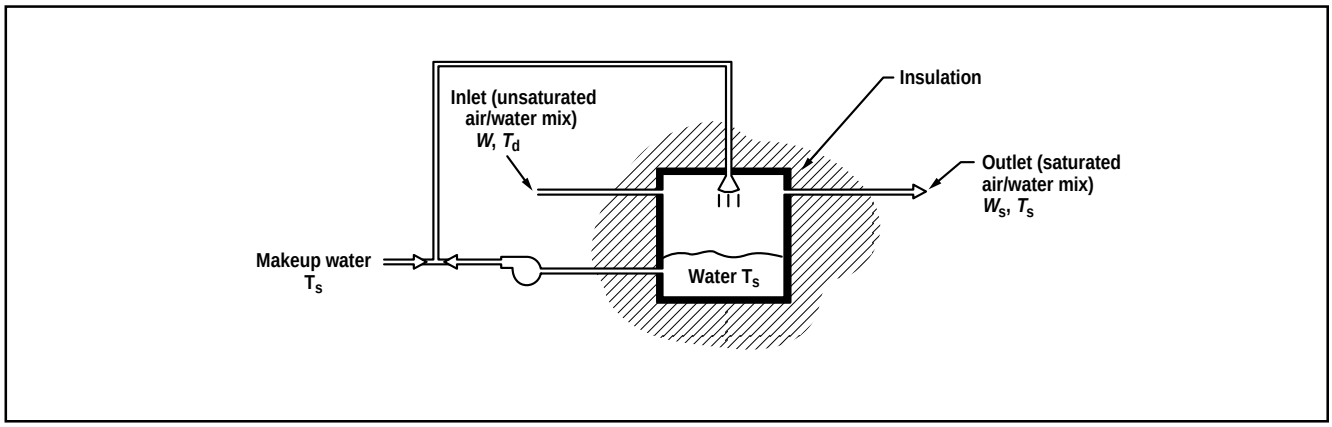
$$\Delta h_{wv} = C_p \int dT \tag{7}$$

where

Δh_a = enthalpy change of air, Btu/lbm

Δh_{wv} = enthalpy change of water, Btu/lbm

1. Adiabatic air-water vapor saturation contactor



To evaluate Δh_a and Δh_w, C_{pa} and C_{pwv} must be known as functions of temperature. Using the data in Ref. *(2)*, the following relationships are used:

$$C_{pa} = A_1 + A_2T + A_3T^{1.5} + A_4T^{2.5} \quad (8)$$

$$C_{pwv} = B_1 + B_2T + B_3T^{1.5} + B_4T^2 \quad (9)$$

where

A_1 = 0.2400959

A_2 = -2.0445841 x 10^{-5}

A_3 = 1.7933932 x 10^{-6}

A_4 = -4.2735761 x 10^{-10}

and

B_1 = 0.443695101

B_2 = -2.2795 x 10^{-5}

B_3 = 4.74358 x 10^{-6}

B_4 = -5.5531 x 10^{-8}

In equations (8) and (9) C_{pa} and C_{pwv} are in (Btu/lbm °F), T in °F. Equations (8) and (9) are valid for temperatures between 40.3°F and 1040.3°F.

Equations (6) and (7) can be integrated from the wet-bulb to the dry-bulb temperature to yield the following results:

$$\ddot{A}h_a = A_1 (T_d - T_s) + 0.5A_2(T_d^2 - T_s^2) + 0.4A_3(T_d^{2.5} - T_s^{2.5}) + 2/7A_4(T_d^{7/2} - T_s^{7/2}) \quad (10)$$

$$\ddot{A}h_{wv} = B_1 (T_d - T_s) + 0.5B_2 (T_d^2 - T_s^2) + 0.4B_3 (T_d^{2.5} - T_s^{2.5}) + 1/3B_4 (T_d^3 - T_s^3) \quad (11)$$

In order to solve for W, W_s and Λ_s must be known as functions of temperature. Using Refs. *(3 and 4)*, the following explicit equations are used:

$$W_s = (C_1 + C_2T_s + C_3T_s^2 + C_4T_s^3)/(1 + C_5T_s + C_6T_s^2 + C_7T_s^3) \quad (12)$$

where

C_1 = 7.6936453 x 10^{-6}

C_2 = 1.398434 x 10^{-4}

C_3 = -1.727916 x 10^{-6}

C_4 = 3.676279 x 10-8

C_5 = -1.009524 x 10^{-6}

C_6 = -2.2102366 x 10^{-5}

C_7 = -6.6994895 x 10^{-10}

and

(T_s in \F, 41<T_s 211)

To calculate Λ_s, the following equation is employed:

$$\Lambda_s = 1072.7 - 0.3T_s - 0.0008T_s^2 \quad (13)$$

Λ_s in Btu/lbm; T_s in °F)

Substituting Eq. (10), (11), (12) and (13) into (3) and isolating W, yields Eq. (14):

$$W = (W_s \Lambda_s - \ddot{A}h_a)/(\ddot{A}h_w + \Lambda_s) \quad (14)$$

Results

To evaluate the accuracy of the model, it is necessary to compare the results with an established standard. Ref. *(3)* has been selected for this purpose.Table I shows some results and the deviation from accepted values.

Conclusion

Table I illustrates the model is accurate, and that generally less than 5% deviation can be achieved. In addition, it is evident that absolute difference is generally in the order of less than 5 ×

I. Results of model calculations

Wet bulb, °F	Dry bulb, °F	W, (calculate)	W (actual)	% Diff.	Absolute *difference*
60	100	0.001847772	0.001968	6.30	0.000120
66	110	0.003474435	0.003652	4.98	0.000178
86	115	0.020147289	0.020370	1.10	0.000223
91	91	0.032004885	0.032190	0.58	0.000185
86	124	0.018029651	0.018260	1.27	0.000230
89	129	0.020364622	0.020570	1.00	0.000205
90	164	0.013439747	0.013660	1.63	0.000220
50	54	0.006782424	0.006742	0.60	0.000040
48	75	0.001033107	0.000983	4.94	0.000050
49	81	0.000166442	0.000151	9.47	0.000015
80	124	0.011761139	0.012000	2.01	0.000239
79	150	0.004886551	0.005125	4.76	0.000238
95	134	0.027052549	0.027230	0.65	0.000177
88	205	0.001842913	0.002053	10.78	0.000210
100	100	0.043016486	0.043120	0.24	0.000104
103	134	0.039678580	0.039770	0.23	0.000091
103	174	0.029987980	0.030110	0.41	0.000122
103	214	0.020597941	0.020720	0.59	0.000122
103	265	0.009034747	0.009089	0.60	0.000054
103	305	0.000267890	0.000309	14.25	0.000041
115	149	0.060721944	0.060650	0.12	0.000072
113	360	0.006562507	0.006434	1.98	0.000129
113	385	0.001196445	0.001063	11.81	0.000133
150	150	0.212240357	0.212000	0.11	0.000240
150	920	0.016360047	0.015660	4.37	0.000700
161	194	0.296941414	0.297300	0.12	0.000359
200	200	2.292488638	2.292000	0.02	0.000489
200	1000	1.499727406	1.495000	0.32	0.004727
140	400	0.081757109	0.081300	0.56	0.000457
160	180	0.291094121	0.291400	0.11	0.000306

10^{-4}, even when the percent difference is high. The percentage difference is greater than 5% when the humidity approaches zero. Therefore, this will not significantly affect most engineering calculations as this is a reflection of a negligible water contribution. Consequently, this model is considered appropriate for a wide range of applications, especially in the high wet-bulb temperature areas. Moreover, this model is easy to program. The author's program is 10 lines of code.

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