

ABSTRACT

A dynamic model describing the transfer of mass, momentum, and energy for a continuous digester improves the understanding of the process and the complex interactions between the vertical motion of the mass and the chemical reactions. The model can potentially predict kappa number, the compaction of chips, and the chip level. It might therefore have use in predictive simulation to aid process operators. Simulating operating conditions of an industrial continuous digester have verified the model.

MATHEMATICAL MODELING OF the residence time for the chips in continuous digesters has had scant attention in the literature. Besides the influence from the wood chip quality, the temperature history of the chips, and the liquor concentration of alkali, the kappa number is strongly influenced by the residence time of the chips in the reactor as well. In addition, the compaction or packing of the chips closely relates to the kappa number, the production rate, the chip level, and the dilution factor. To a minor degree, there is also a relation to the liquor-to-wood ratio. There is therefore a close relationship between the compaction of chips and their residence time in the vessel.

From an operational point of view, one may argue that control of the residence time is the most difficult factor for pulp quality control. In two-vessel systems, chip level control is a typical control problem—often a key problem to control the residence time. The interactions between these factors and the kappa number are complex particularly during nonstationary process conditions. To explain these interactions, we present a novel, dynamic, first-principles model using mass, momentum, and energy balances.

A dynamic model of the interaction between the chemical reactions and the residence time in a continuous digester

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The proposed model is an integration of the ideas in earlier model work and designed for a steam/liquor phase digester. This is the digester part of a two-vessel system shown in Fig. 1. The concept of two-phase flow first proposed by Härkönen (1) describes the vertical motion. In this description, the incompressible free liquor flows through the bed of chips constructing a porous compressible medium. The kinetic model is simple with wood solids divided into one lignin and carbohydrate component. Liquor consists of an alkali component, a dissolved solids component, and water. The liquor is entrapped liquor within the chips and free liquor around the chips. In this way, the kinetic model is a simplified version of the model by Christensen *et al.* (2). The liquor may partially penetrate the chips, i.e., the chips may also entrap air.

This paper will first describe the model briefly and then show and discuss some simulation results. The steady-state profiles chosen resemble the operating conditions of an industrial digester. Finally there is a discussion of some uses of the model. This paper derives from an earlier published treatment giving a broader examination of the issues (3).

MATHEMATICAL MODEL

Balance equations come from the fundamental conservation laws of

mass, momentum, and energy. The boundary conditions are ordinary differential equations and empirical algebraic relations. These cover the top part of the mass including the steam zone, the extraction section, and the outlet section. Since the mass mainly moves vertically, the equations only apply to this direction.

Mass balances

A set of balance equations for the components mentioned in the introduction uses the density definitions of Michelsen (3). The lignin and the carbohydrate balances include convection, reaction, and dispersion, i.e., vertical mixing. The reactive terms describing the overall kinetics are functions of the residual content of the solid material, the concentration of alkali, and temperature. In addition, the balances for the entrapped alkali and dissolved solids also contain a term for interphase diffusion. The diffusion coefficient varies with the residual solid lignin content, pH, and the temperature.

One can derive an equation for the volume fraction of chips from the mass balance for the chip plug. This includes convection and dispersion. In addition, the mass balance equations for the two phases give a continuity equation on two-phase form to solve the liquor velocity. The chip and liquor levels are computed from mass balances of the two phases in the top of the vessel.

The dispersion coefficients are small so that the dispersion terms in

most of the equations only constitute 5–10% of the convection terms at steady-state. In dynamic transients, however, the dispersion may be substantial.

Momentum balances

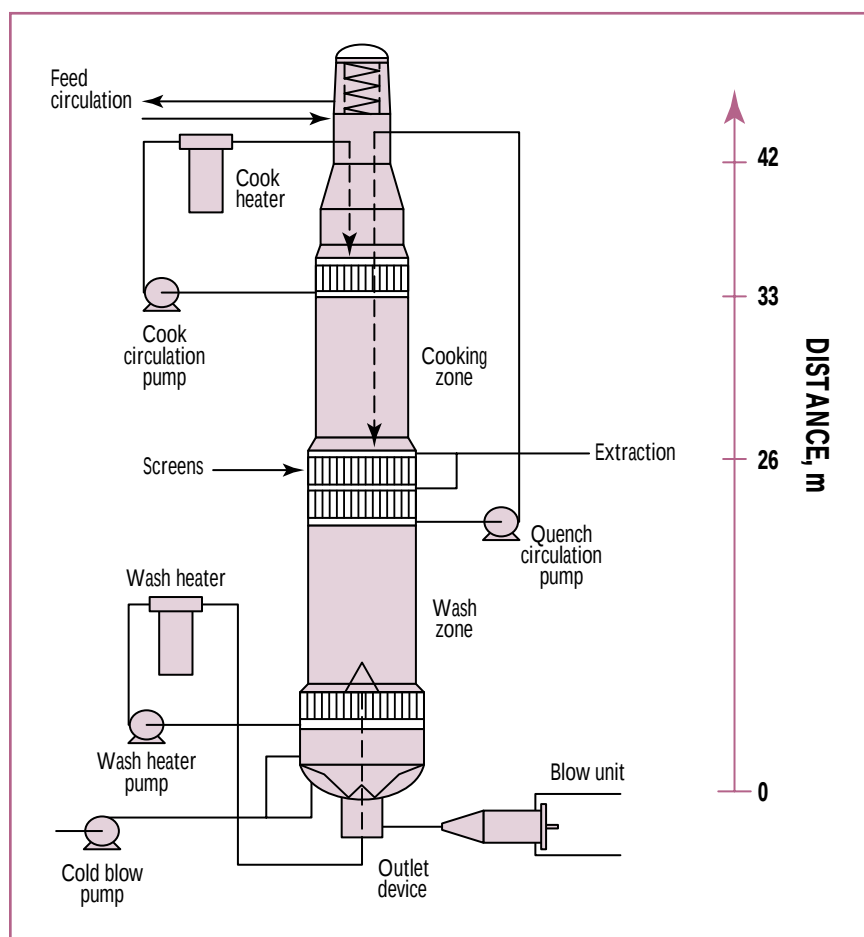
The momentum balance for the chip plug for computing the chip velocity is a dynamic equation. It includes the force of inertia, the force of gravity, the chip and liquor pressure forces, dry friction force between the chip plug and the vessel walls, and the viscous friction force between the two phases. The momentum balance for the free liquor phase for computing the liquor pressure has a steady-state form. It includes the same contributions as the chip plug except the chip pressure force and dry friction against the vessel walls.

Steady-state results show that all the pressure and friction terms are important. The forces of inertia are negligible compared to the other forces.

Energy balances

We have derived energy balances for both phases. The energy balance for the chip plug includes heat of convection (heat connected to vertical convection of mass), heat of chemical reaction, and heat of conduction. This has a vertical component between the two phases and another component through the side walls of the vessel. The energy balance for the free liquor includes the same contributions as the chip plug except the heat of chemical reaction.

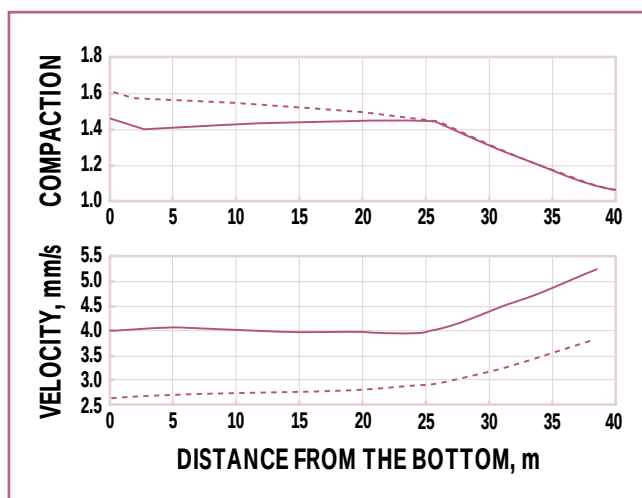
Steady-state calculations for the chips show that the heat of reaction and the heat of conduction from the liquor are the most significant terms. For the free liquor phase, the convection and the heat of conduction from the chips are the most significant. The vertical heat of conduction and the heat of conduction through the walls are negligible for both phases.



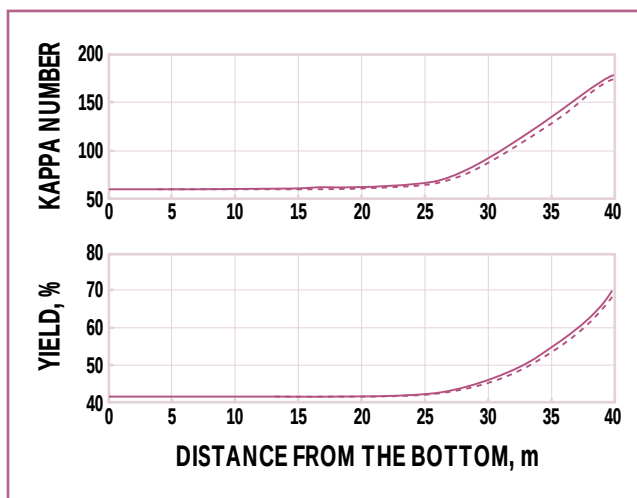
1. A steam/liquor phase digester for continuous kraft cooking

Operating point	LPR	HPR
Blow flow, L/min	2500	3800
Chip meter speed, rpm	15.6	21.3
Liquor to wood ratio, m ³ /ton	3.0	3.0
Steam pressure, kPa	685	685
Effective alkali feed, g/L	23.0	25.0
Extraction flow, L/min	2927	4015
Side nozzle flow, L/min	1850	2800
Bottom nozzle flow, L/min	850	1300
Nominal chip level, m	41.5	41.5
Nominal liquor level, m	39.5	39.5
Transport circ. temp., °C	137	137
Steam temp., °C	165	175
Cooking circ. temp., °C	160	170
Low. wash circ. temp., °C	90	90
Wash water temp., °C	90	90
Ambient temp., °C	20	20

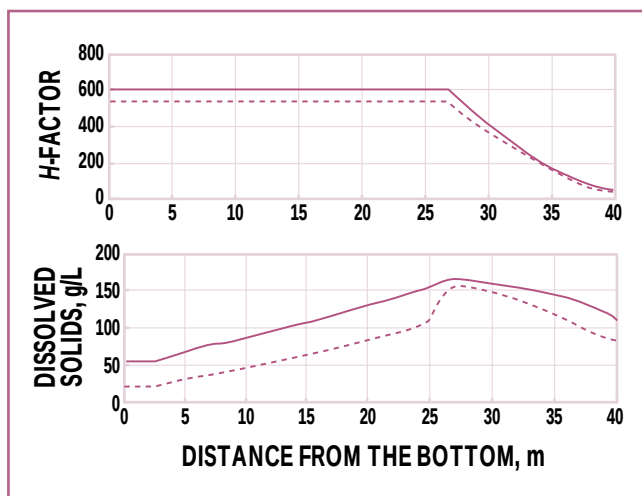
1. Two operating points for production of kraft softwood pulp



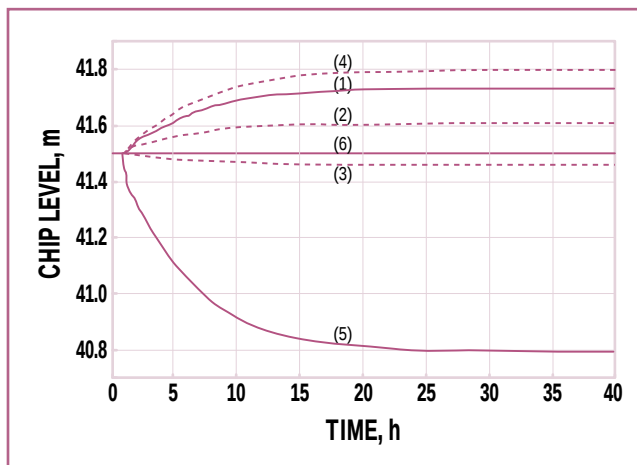
2. Steady-state profiles of the compaction of chips and the chip velocity at high (solid line) and low (dashed line) production rates



3. Steady-state profiles of the kappa number and yield at high (solid line) and low (dashed line) production rates



4. Steady-state profiles of the H-factor at high (solid line) and low (dashed line) production rates and the concentrations of entrapped (solid line) and free (dashed line) dissolved solids at high production rate



5. Chip level after a 5% reduction in the steam temperature (1), cooking circulation temperature (2), lower wash circulation temperature (3), alkali charge (4), and wash water flow (5) with (6) showing the setpoint

SIMULATION RESULTS

Investigation of the properties of the model used two typical operating points for a steam and liquor phase digester. Table I has a definition of these for a low production rate (LPR) and a high production rate (HPR). The table lists two typical operational conditions for the steam

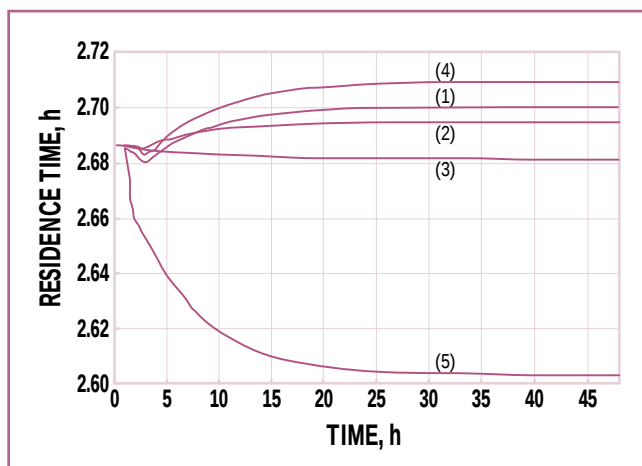
and liquor phase digester at Peterson A/S, Moss, Norway.

We chose the exogenous temperatures in the cooking zone—the steam temperature and the cooking circulation temperature—and the effective alkali feed to obtain approximately the same kappa numbers in the discharged pulp at both

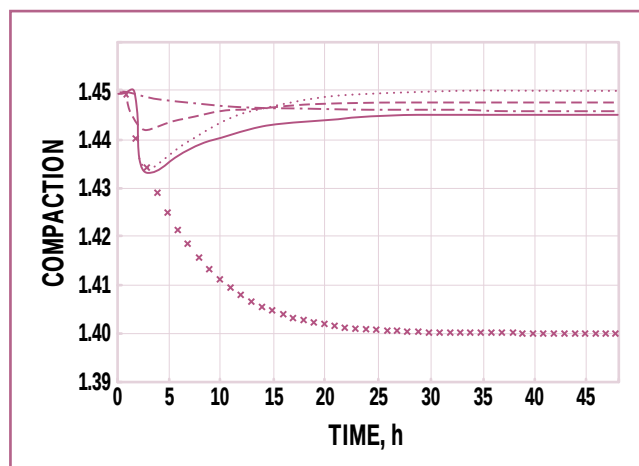
operating points. There is no air in the chips in this investigation.

Steady state profiles

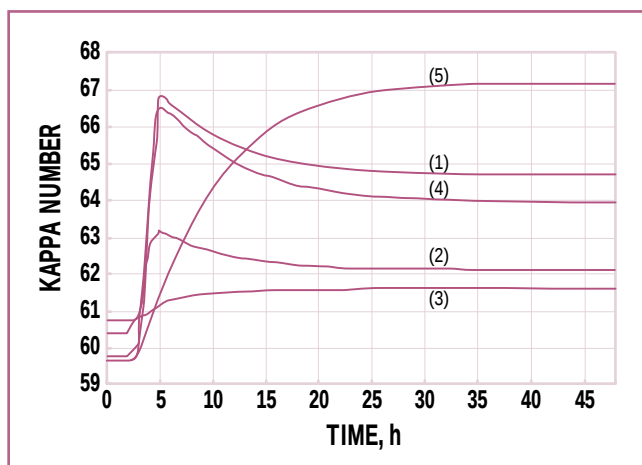
Figures 2–4 show some steady-state profiles. In these figures and those of the following discussion, the horizontal axis denotes the vertical distance from the bottom of the vessel of Fig. 1. Figure 2 shows the



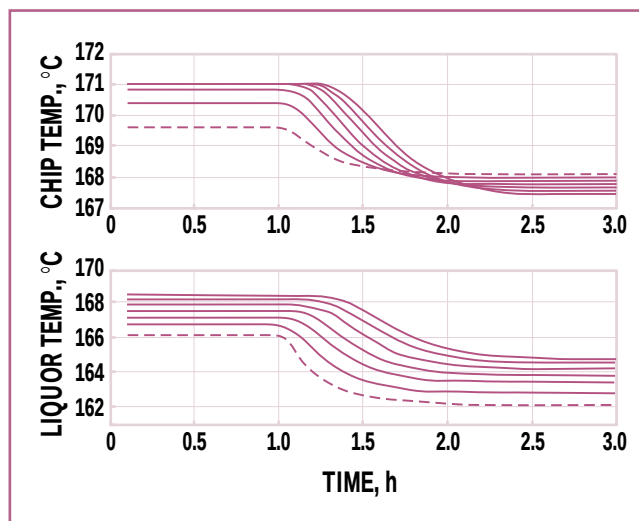
6. Total residence time for the chips after a 5% reduction in the steam temperature (1), cooking circulation temperature (2), lower wash circulation temperature (3), alkali feed (4), and wash water flow (5)



7. Chip packing at the extraction screens after a 5% reduction in the steam temperature (solid line), cooking circulation temperature (dashed line), lower wash circulation temperature (dashed-dotted line), alkali feed (dotted line), and wash water flow (x line)



8. Kappa number in the blow flow after a 5% reduction in the steam temperature (1), cooking circulation temperature (2), lower wash circulation temperature (3), alkali feed (4), and wash water flow (5)



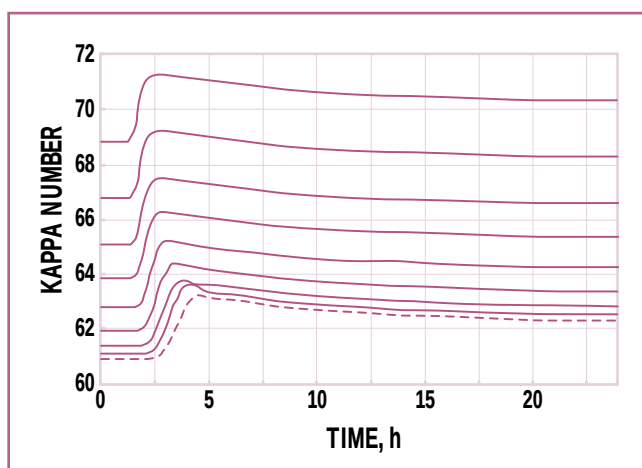
9. Cooking zone temperature at six grid points downstream from the cooking circulation screens when the cooking circulation temperature is reduced with the temperature at the screen plotted as a dashed line

resulting profiles of the compaction of chips—defined as the volume fraction of chips in the vessel compared with that in the feed—and the chip velocity. Figure 3 shows that the kappa number in the discharged pulp is approximately 60 for liner grade and the yield is approximately 42%. The compaction of chips

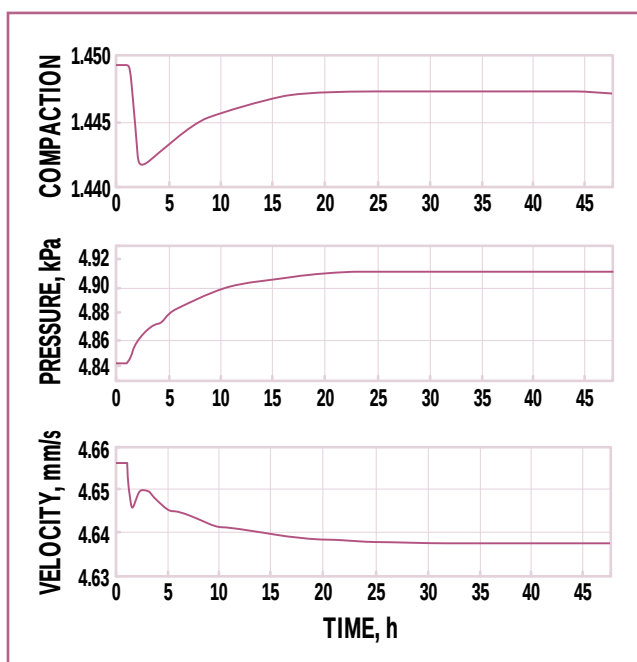
increases down toward the extraction screens in the cooking zone (42–26 m). This is primarily due to gravity and the delignification. The physical interpretation of the latter is that the chips become softer during the cook due to increased porosity inside them. Then they change their shape but not their volume.

They are therefore packed tighter (4). This reduces the chip velocity.

In the wash zone (26–0 m), the gradients are considerably smaller than in the cooking zone. This is due to the reduced (stopped) delignification and the lift from the wash water. The figures indicate that the profiles depend on the operational



10. Kappa numbers in the wash zone when the cooking circulation temperature is reduced with the dashed line denoting the discharged pulp



11. Compaction of chips and the chip pressure at the extraction screens and the chip velocity at the cooking circulation screens when the cooking circulation temperature is reduced

KEYWORDS

Continuous digesters, dynamic tests, flow, mathematical models, phases, pulping, reaction kinetics, simulation, time, vertical design.

conditions. Total residence time in the digester is approximately 2.6 h at the high production rate and 3.8 h at the low production rate. The H -factor is higher at high production rate than at low production rate. This is because the reaction rates are higher at high production rate due to higher cooking temperature despite the shorter residence time.

The lower part of Fig. 4 shows how the concentrations of dissolved solids increase smoothly in the cooking zone. Wash liquor extracts the residual solids in the chips up toward the extraction screens by the countercurrent "diffusion wash" in the wash zone. The concentration of free dissolved solids is below the

concentration of entrapped dissolved solids in the whole vessel due to the dynamics caused by the interphase mass diffusion. It appears that the steady-state difference between the concentration in the chips and in the free liquor is considerable. Note that the chips in the discharged pulp entrap approximately a third of the amount of dissolved solids at the extraction screens. The same result appears at both production rates. This means that the diffusion of dissolved solids is a steady-state limiting factor with the possibility of better wash efficiency. For the concentrations of alkali, however, the results are nearly the same in the two phases. This paper does not show these values. The results depend on operational conditions such as the dilution factor and the purity of wash water at the entrance.

Dynamic behavior

The steady-state point at the high production rate was the starting

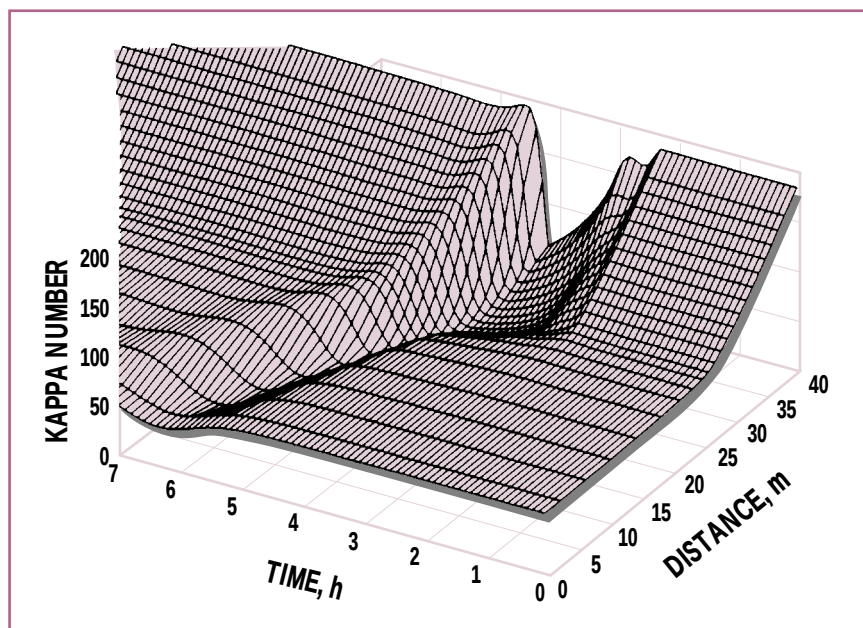
point for investigating the dynamic qualities. There is a control loop for the liquor level where the extraction flow is the manipulated variable. At simulation time 1 h, there is a first order step of 5% in the respective manipulated variables that primarily controls the temperature, concentration of alkali, and the wash effect. The filter time constant is 12.5 min. The manipulated variables are the steam temperature of 175–166.25°C, cooking circulation temperature of 170–161.5°C, lower wash circulation temperature of 90–85.5°C, the alkali concentration in the feed of 25.0–23.75 g/L, and the wash water flow through the side nozzles of 2800.0–2660.0 L/min. From the results, we may deduce something about the relative importance of these variables and their influence on the pulp quality and the other key modeled variables. As opposed to normal operation, the chip level is not controlled to include the effect of level changes.

Figures 5–8 show the effects on the chip level, the residence time, the chip packing, and the kappa number. The figures show that the wash water flow (the manipulated variable that controls the wash effect) has the strongest gain to these variables. Because the wash water flow lifts the chips and has a strong impact on the chip level, we may conclude that the manipulated variables that have the strongest influence on the residence time for the chips also have the strongest impact on the pulp quality. This shows the importance of modeling the residence time for the chips in the digester.

In addition, the influences from the steam temperature and the alkali charge are comparable. As one might expect, the influence from the lower wash circulation temperature is weak compared to the others. The effect of the reduction in the cooking circulation temperature is weaker than from the steam temperature.

The long-term chip level dynamics have the most influence on the settling times. Figure 5 shows that they are about the same.

Figures 9–11 show more of the dynamic properties of the model using reduced cooking circulation temperature. These figures highlight an important property of the model—the coupling between the residence time for the chips and the chemical reactions. Figure 9 shows how the chip and liquor temperatures decrease at the cooking screens and at six discrete grid points downstream from the screens. Because of this, the chemical reaction rates decrease. Figure 10 shows that the kappa number therefore increases. Due to this increase, the chip pressure also increases as Fig. 11 shows. As a result, the chip plug is retarded in the cooking zone upstream from the cooking circulation screens. Figure 11 also indicates



12. Three-dimensional plot of the kappa number lapse during a short stop and a startup

this. This effect limits the kappa number increase through increased residence time for the chips. The compaction of chips consequently decreases as Fig. 11 shows.

After approximately 2–3 h for the discharged pulp, the kappa number decreases. The fact that the retardation of the chip plug increases the chip level causes this—see curve (2) in Fig. 5—thereby increasing the residence time further. The feed flow of chips is constant. Because of the chip level increase, the compaction increases also.

We observed a large spread in the time between important dynamic effects in the digester. Viewing the kappa number of the discharged pulp (Figs. 8 and 10) shows that the initial increase lasts for 2–3 h while steady-state occurs after approximately 24 h.

DISCUSSION AND CONCLUSIONS
This paper presents a novel, dynamic model of the continuous digester. Modeling the residence time for the chips and the depen-

dency on different pulping conditions provides a valuable tool for describing the delignification process in continuous digesters. The simulation results indicate that the residence time for the wood chips in the vessel has a strong impact on the kappa number. By this, we mean it is important to include the interaction between the vertical motion and the chemical reactions in a continuous digester model for kappa number prediction.

From an operational perspective, the model is promising. First, recognized persons within the pulp and paper community have accepted the results (5). Audits have provided this acceptance. These audits have included two types of discussion:

- The model and the assumptions on which it rests
- The results from the simulation scenarios.

Second, the results show that the many interacting effects in a digester may paint a confusing picture to

operating personnel. Using a model as a predictive simulation tool may therefore provide support to making proper operational decisions. An example is the effect of temperature change that Fig. 10 shows. Instead of overreacting after an initial sharp increase in the kappa number, knowledge about the long-term effects can improve operational decisions.

Continuing this thought, a model might have use in a predictive control scheme. Christensen (6) reports one example. In the example above, note that many digesters especially in one-vessel systems run with a loosely controlled chip level or no automatic chip level control. One could therefore argue that chip level

control is not the key problem to control the residence time for the chips. In any event, this study supports the idea that chip level control is not sufficient by itself. One must also consider the chip velocity and the compaction of chips in the residence time control problem.

Besides the scenarios shown in this paper, the model has been tested under operational disturbances (changes in the feed chip reactivity, lignin, and air content), production rate changes, and short stopping of the digester (3). As an illustration of the latter, Fig. 12 shows how the kappa number profile changes as a function of time. During the stopping period of 100 min, the total kappa number reduction in the dis-

charged pulp is about 40 units. The ability of the model to explain the dynamic behavior of the digester mass during such extreme operational conditions like this may be helpful in training simulators and in developing improved control strategies. TJ

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LITERATURE CITED

1. Härkönen, E. J., *Mechanical Engineering Series No. 88*, Acta Polytechnica Scandinavia, Helsinki, 1984.
2. Christensen, T., Albright, L. F., and Williams, T. J., *A mathematical model of the kraft pulping process*, Ph.D. thesis, Purdue University, Lafayette, IN, 1982.
3. Michelsen, F. A., *A dynamic mechanistic model and model-based analysis of a continuous Kamyr digester*, Ph.D. thesis, The University of Trondheim, Trondheim, Norway, 1995.
4. Härkönen, E. J., *Tappi J.* 70(12): 112(1987).
5. Christensen, T., Kleppe, P. J., and Kirkebak, P., *Internal project meetings*, Peterson A/S, Moss, Norway, 1994.
6. Christensen, T., Ivan, M. P., Michaelsen, R., *et al.*, *Pulp Paper Can.* 95(12): 146(1994).