The effect of pulp fiber properties on the tearing work of paper

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ABSTRACT

When modeled as the elastic energy dissipated in paper failure, the maximum tear strength of paper depends on the product of fiber length and the square of fiber strength. The effect of fiber length vanishes with increased bonding. Dependence on fiber strength does not necessarily mean fiber failure is the mechanism of energy dissipation. Elastic energy released when fibers or bonds fail depends on the average load within the fibers and the strength and number of elements failing. All these entities relate to fiber strength.

Runnability of paper in converting or printing depends on basis weight, wetness, treatments like calendering, and flaws in the web. The frequency of web breaks naturally depends on the imposition of loads on the web. An Elmendorf-type, out-of-plane tearing or in-plane tearing test has traditionally evaluated the ability of a web to tolerate flaws *(1–8)*. Tests of in-plane fracture work using other concepts also are available *(4, 9–13)*.

The properties of pulp fibers determine paper strength. Mechanical treatment of fibers, chemical additives, or introduction of suitable fines *(14, 15)* can alter the degree of interfiber bonding. These affect strength properties considerably. During manipulation of interfiber bonding, some technical properties of the paper improve and others worsen. There are also other ways to change the structure of the sheet. These include fiber orientation distribution, concentration distributions of different kinds of constituents, etc. The properties of pulp fibers determine the limits for the combinations of paper properties achievable from changing interfiber bonding and the structure of the sheet.

Previous workers analyzed the effect of forces between and within individual fibers on tear strength *(16)*. They assumed tearing work was due to frictional forces when pulling individual fibers from a matrix. Although the force needed to break a fiber is considerable, the work needed for fiber failure is probably negligible compared with frictional work. This is because the displacement necessary to break a fiber is smaller than the displacement necessary to pull the fiber. This approach resolved most of the deficiencies of the early work of Brecht and Imset *(17)*. Giertz and Helle *(18, 19)* confirmed many phenomena predicted. Kane *(20)* refined the theory by introducing a distribution of embedded lengths of fibers. Shallhorn and Karnis *(21)* followed this, and recent work applied it further *(22)*. This author working with others attempted to consider fiber orientation distribution and fiber properties *(23, 24)*. Page *(25)* recently examined the energy dissipated in fiber failure.

Working with Elmendorf-type, out-of-plane tearing, Brecht and Imset *(17)* found that the bending stiffness of the sheet affects the lever of the tearing force at the crack tip. It therefore contributes to the required force *(26)*. Since other theorists neglected the effect of the tearing mode, their findings apply primarily to in-plane tearing.

Shallhorn and Karnis *(21, 22)* have modeled tensile strength and tearing work. This allows experimental verification of their theory using these two macroscopic properties. The bond shear strength contributing to these strength properties cannot be the same, however, since bonds release gradually in the tensile test. Only those bonds remaining active until tensile failure contributing to the tensile strength, the tensile strength is not as sensitive to fiber strength as fracture work is *(13, 27–30)*. Most theorists have assumed fibers are uniform and the work necessary to break fibers is negligible. As Page noted *(25)*, one mechanism of energy dissipation must be bond rupture rather than friction.

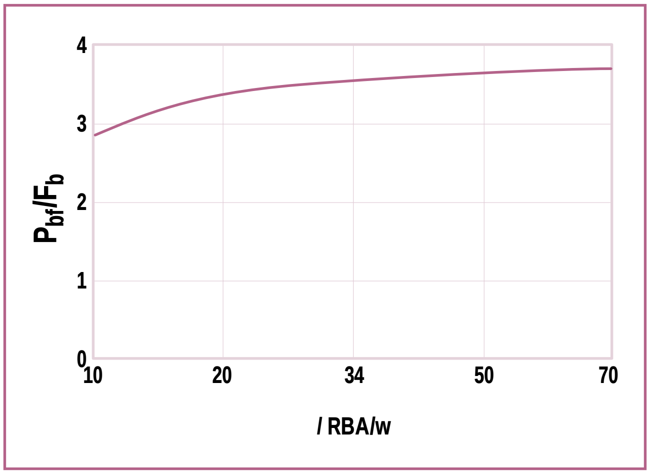
One can question whether the tearing tests or any theories discussed above deal with the toughness of paper. The fundamental work of Griffith *(31)* proposed the condition for continuing crack propagation in tensile loading of a linearly elastic body. He noted that the rate of released elastic energy exceeds the rate of energy consumption needed to create new molecular surfaces. The following equation shows this:

$$G \geq 2g \tag{1}$$

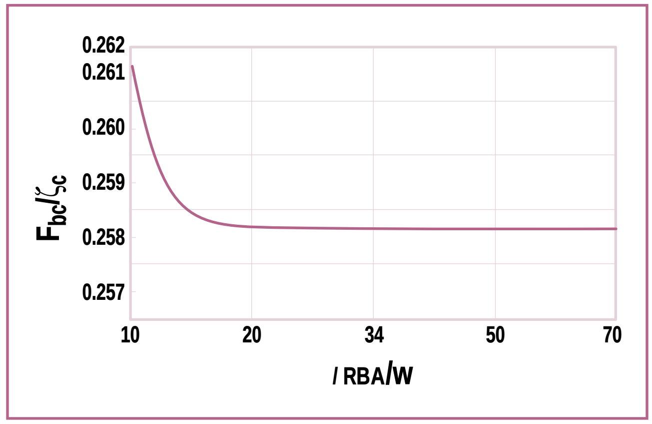
where

G = energy release rate
g = work necessary to overcome molecular attractions for a small increment of crack surface area.

Depending on the tearing conditions for a test sample, the rate of energy dissipated in the failure of the specimen may be much greater than needed for crack propagation. The critical energy release rate



1. The effect of the product of fiber length and relative bonded area divided by fiber width on the average fiber load needed to break a bond according to Eq. 9, E_f/G_f = 30.



2. The ratio of critical bond strength to fiber strength according to Eq. 14, E_f/G_f = 30.

needed to propagate a crack is a material property—fracture toughness. This property determines the material's ability to carry flaws.

The approach of Griffith expanded for other than ideally linear, elastic materials *(32, 33)* has provided experimental methods for determining the critical energy release rate *(4, 9–13)*. The tear test does not meet the requirements of a fracture toughness test. Much of the elastic energy dissipated in the tearing remains in the specimen. Such tests do not measure fracture toughness. To understand the tearing work and the effect of pulp fiber properties on it, we must examine the total energy dissipated rather than the critical energy release rate needed to propagate a crack.

TEARING WORK OF PAPER— A MICROMECHANICAL THEORY

The strain energy of any elastic body is directly proportional to the square of the load within the body. It is inversely proportional to the Young's modulus of the material. The strain energy also depends on the dimensions of the body as the following equation shows:

$$W = k(P^2/E) \qquad (2)$$

where

W = elastic strain energy
k = a parameter depending on the dimensions of the speci men but not on the properties of pulp fibers
P = the load within the body
E = Young's modulus.

Consider straining the specimen sufficiently to break an element, i.e., breaking a fiber or a bond between fibers. Breaking the element releases the following amount of elastic energy:

$$\Delta W_e = k[P^2 - (P - P_e)^2]/E \qquad (3)$$

Solving Eq. 3 provides the following:

$$\Delta W_e = -k(2PP_e - P_e^{\,2})/E \qquad (4)$$

where

P_e = load the element carries before failing.

For fiber failure, P_e equals fiber strength, zC, expressed as the product of specific cell wall strength, z, and coarseness C. For bond failure, P_e equals bond (shear) strength, F_b.

The change in the elastic strain energy is negative, since the failure of the element decreases the potential energy of the specimen. Since P is much larger than P_e, the released energy for element failure is the following:

$$\Delta W_e = -k(2PP_e/E) \qquad (5)$$

One can consider the load within the specimen P as the average load of one fiber within the crack tip, $\overline{P}_f$ multiplied by the number of these fibers. In turn, this is inversely proportional to coarseness. The average load of one fiber within the crack tip depends on the strength of the fibers that are about to fail. It also depends on the average load within the fibers where bonds are about to fail. This leads to the following equation:

$$P \sim \overline{P}_f\,/C \sim [p_f zC + (1 - p_f)P_{bf}]/C \qquad (6)$$

where

p_f = the probability of fiber failure within crack propagation
P_{bf} = average fiber load where bonds break.

	l, mm	C, mg/meter	z, Nm/g
Mean	2.24	0.18	434
Median	2.23	0.17	437
Coefficient of variation	11%	18%	7%
Minimum	1.38	0.14	357
Maximum	2.60	0.29	507

I. Descriptive statistics of fiber properties in the dataset

To determine the average fiber load where bonds are about to break, we must examine the distribution of stress within the length of a Hookean fiber *(34–36)*. Previous work *(37, 38)* shows that the second differential of fiber load, P_f, with respect to the distance from fiber end, x, is the following:

$$P_f{}^2x/dx^2 = 8G_f RBA^2 t/w[(P_f/E_f tw) - e] \quad (7)$$

where

G_f = shear modulus
E_f = Young's modulus
t = thickness
w = width of the fiber
RBA = relative bonded area
e = the macroscopic sheet strain in the fiber direction.

Equation 7 is also applicable for fibers torn sharply at the crack tip when the value of the macroscopic strain in other parts of the sheet is nearly zero.

In such circumstances, the value of the second differential is positive. One can consider the condition for bond failure as the differential of fiber load over the bond exceeding the strength of the bond. Integrating Eq. 7, the critical fiber load for bond failure then becomes

$$P_{bf}(x) = F_b(E_f/2G_f)^{1/2} \tanh[(2G_f/E_f)(2RBAx/w)] \quad (8)$$

Integrating Eq. 8 from zero to half fiber length gives the following average critical fiber load for bond failure:

$$P_{bf} = F_b(E_f/2G_f)(w/RBAl)\ln\{\cosh[(2G_f/E_f)^{1/2}(lRBA/w)]\} \quad (9)$$

where

l = fiber length.

Although the shear modulus of a wood pulp fiber is much less than the longitudinal Young's modulus *(39–44)*, the effect of relative bonded area and fiber length on the probability of the fiber failing is small, unless the fibers are quite short or the bonded area is very small. Since fiber length is greater than fiber width, the value of the tanh-expression of Eq. 8 is often near 1 *(38, 45)*. Figure 1 is an illustration of this where fiber length changes from 1.5 mm to 3 mm with *RBA* changing from 0.2 to 0.68. The width of the fiber remains at 30 µm. We find that the average fiber load needed to break a bond is 3–4 times the strength of the bond. Fiber length and relative bonded area only slightly affect this load. These results agree well with experimental observations for extracting single fibers from a handsheet *(46)*.

On the other hand fibers failing or the bonds breaking give the average elastic energy released when removing a fiber from resisting the crack as follows:

$$\Delta W_{fiber} \sim -k\{P[p_f zC + (1 - p_f)nF_b]\}/E \quad (10)$$

where

n = number of bonds between the crack tip and the closer end of the fiber.

One can substitute Eq. 6 into Eq. 10 and consider that the number of fibers is inversely proportional to coarseness. Combining this with $P_{bf} = 3\,F_b$ from Fig. 1 gives the following for the tear index of paper:

$$W_{paper} \sim [(p_f zC)^2 + p_f(1 - p_f)zC(n + 3)F_b + (1 - p_f)^2\, 3nF_b{}^2]/EC^2 \quad (11)$$

Since the average length of the fiber portion where bonds are breaking is a quarter of the length of the fiber, the number of bonds breaking is a quarter of the fiber length divided by the average length of a fiber segment between bond centroids. The average length of the fiber segment in turn is $w/(2RBA)$ *(38)*. This results in the number of bonds being fiber length and *RBA* divided by twice the fiber width. The bond (shear) strength is the product of the area of the bond as the square of fiber width and the specific bond shear strength (*SBSS*). The continuum form of Eq. 11 is the following:

$$W_{paper} = (p_f z)^2/E + \{p_f[1 - p_f]\, z\, SBSSw[(RBAl/2) + 3w]\}/EC + [(1 - p_f)^2 (3/2)RBAl\, SBSS^2 w^3]/EC^2 \quad (12)$$

This equation readily implies that sheets with tightly bonded and weak fibers have the tear index proportional to the square of fiber strength and inversely proportional to the stiffness of the paper. The proposed dependence on fiber strength agrees with experimental observations with sheets of fibers weakened with acid vapor *(25, 47, 48)* and with the theoretical consideration of Page *(25)*.

For loosely bonded sheets failing entirely through bond failure, Eq. 12

	l	C	z	z^2	lz^2	$T(r_c)$	$T(1.04r_c)$	$T(1.08r_c)$	$T(1/12r_c)$	$T(1.16r_c)$	$T(1.20r_c)$
l	1.00										
C	0.23	1.00									
z	0.19	-0.62	1.00								
z^2	0.21	-0.61	1.00	1.00							
lz^2	0.65	-0.36	0.86	0.87	1.00						
T(rc)	0.58	-0.26	0.63	0.63	0.75	1.00					
T(1.04rc)	0.64	-0.07	0.59	0.59	0.76	0.86	1.00				
T(1.08rc)	0.59	0.12	0.39	0.39	0.59	0.54	0.87	1.00			
T(1.12rc)	0.45	0.17	0.26	0.25	0.42	0.33	0.70	0.93	1.00		
T(1.16rc)	0.40	0.26	0.15	0.15	0.31	0.26	0.63	0.87	0.94	1.00	

II. Linear correlation coefficients between the fiber properties and tear indices at different apparent paper densities

KEYWORDS

Bibliographies, bonding strength, cell dimensions, fiber bonding, fiber dimensions, fiber length, mechanical properties, paper properties, tear strength, test methods.

shows the tear strength to be proportional to the following:

$$W_{paper} \sim RBAl\ SBSS^2\ w^3/EC^2 \quad (13)$$

One can interpret the ratio of coarseness and fiber width as cell wall thickness. Then the tear strength of loosely bonded sheets should be inversely proportional to the square of cell wall thickness. Equation 13 disagrees with previous theories where the tear strength of loosely bonded sheets was proportional to the square of fiber length *(20–24)*.

CONSEQUENCES OF THE THEORY

A fiber fails when the fiber load needed to break a bond exceeds fiber strength. Equation 8 shows that the load needed to break a bond is greatest in the middle of the fiber.

Fibers start to fail when the fiber load necessary to break a bond in the middle of a fiber equals or exceeds fiber strength. The condition for the probability of fiber failure differing from zero is therefore the following:

$$zC \approx F_{bc}(E_f/2G_f)^{1/2} \tanh[(2G_f/E_f)^{1/2}(lRBA/w)] \quad (14)$$

Figure 2 shows the critical bond strength for the initiation of fiber failure, F_{bc}, solved from Eq. 14 as a function of *lRBA/w*. We find from Fig. 2 that fiber length and *RBA* only contribute slightly to the bond strength where fibers start to fail. The reason is that the value of the tanh-expression of Eq. 14 is near 1. The bond strength where fibers start to fail must therefore be near $(2G_f/E_f)^{1/2}zC$. This varies between 20–40% of fiber strength. [For the moduli of wood pulp fibers, see *(39-44)*]. This result agrees with Fig. 2 and with experimental observations showing that the fiber load needed to break a bond is approximately 3–5 times the strength of the bond *(46)*.

When almost all the fibers fail in Eq. 11, the tear index depends on z^2/E. In such a degree of interfiber bonding with the probability of fiber failure differing only slightly from zero, the tear index depends on the following:

$$3nF_{bc}^2/EC^2 = 3[(lRBA/w)(G_f/E_f)z^2]/E = 0.1(lRBA/w)(z^2/E) \quad (15)$$

Both these formulae appear as comparable terms in Eq. 11. Consider the value of *lRBA/w* varying from 20–50. The tear strength for bonding where the first fibers start to break should be 2–5 times greater than the tear strength of the same paper where all the fibers break in the tear failure. In addition, increased bonding may further increase the Young's modulus of the paper and make the difference greater.

EXPERIMENTAL OBSERVATIONS

The above theoretical analysis leads to three hypotheses. For loosely bonded sheets failing through bond failure entirely, Eqs. 12 and 13 suggest that tear strength depends on

$$W_{paper} \sim RBAl\ SBSS^2\ w^3/EC^2 \quad (16)$$

The tear strength found in the range of bonding where few fibers break depends on

$$W_{paper} \sim [(lRBA/w)(G_f/E_f)z^2]/E \quad (17)$$

With increased bonding, the tear strength asymptotically approaches a value proportional to

$$W_{paper} \sim z^2/E \quad (18)$$

It is easy to verify the last hypothesis experimentally by changing the strength of fibers by hydrolyzing cel-

	I	C	T(rc)/z^2	T(1.04rc)/z^2	T(1.08rc)/z^2	T(1.12rc)/z^2	T(1.16rc)/z^2	T(1.20rc)/z^2
T(rc)	0.31	0.21	1.00					
T(1.04rc)/z^2	0.41	0.44	0.83	1.00				
T(1.08rc)/z^2	0.39	0.51	0.57	0.89	1.00			
T(1.12rc)/z^2	0.26	0.51	0.36	0.71	0.91	1.00		

III. Linear correlation coefficients between the fiber properties and tear indices normalized according to the square of fiber length

lulose with acid vapor. The results confirm that the tear strength is proportional to the square of fiber strength *(25, 47, 48)*.

The other hypotheses are more difficult to confirm because the equations contain various entities that are difficult to measure. In the vaporizing experiment, however, Page *(25)* found that, after maximum tear, the exponent giving the best fit for the equation, Tear = kz^n, was 2. This agrees with Eq. 17. Contrary to Page's tentative proposal *(25)*, this does not mean that the energy released upon fiber failure accounts for much of the work of tear. The elastic energy dissipated when elements fail depends on the product of the average load within the fibers and the average strength of an element multiplied by the number of failing elements. All these factors depend on fiber strength. This is true at least after the maximum tear. Experimental observations reporting that fiber length dominates tear at low degrees of bonding and fiber strength in highly bonded sheets *(21, 25, 47–51)* agree with the suggested hypotheses.

We attempted to verify the hypotheses using an experimental material of 43 commercial bleached softwood kraft pulps produced in 1988 by the Finnish Pulp and Paper Research Institute. All pulps underwent a series of beating treatments. Table I gives descriptive statistics for the fiber properties within the material. We found a considerable variation in all the properties. The specific cell wall strength, z, in the table resulted from multiplying the mean of the two highest zero-span tensile index observations by the factor 8/3 *(27)*.

We can define the apparent sheet density having a maximum tear index within a beating series for any pulp as r_c. Then Table II shows a matrix of linear correlation coefficients between the fiber properties and tear index at apparent densities ranging from 1–1.2. Table II shows that the fiber properties introduced in Table I do not correlate for this material except the negative correlation between coarseness, C, and specific fiber strength, z. We also found that the tear index of papers close to the critical density, r_c, correlates only slightly with tear indices at higher densities. This indicates that there obviously are different mechanisms of energy dissipation, although we are now discussing papers after the maximum tear index only,

Table II shows that the correlation between fiber length and tear index decreases with increased density as Eq. 12 suggests. Close to the critical density r_c, the entity showing the greatest correlation with the tear index is the product of fiber length and the square of fiber strength as Eq. 17 proposes. There is a confusing observation, however. At high densities, the tear index does not correlate well with the square of fiber strength as Eq. 18 would suggest.

There is strong evidence from the vaporizing experiments by Page *(25)*, however, that the tear index follows the equation, Tear = kz^2, after the maximum tear. We can continue this analysis by dividing any observed tear index after the maximum tear by the square of zero-span index of that particular paper. Table III shows a correlation matrix for this normalized tear index.

After normalizing the tear index with the square of the zero-span tensile index, there is still less correlation between the strength of papers at the critical density and at a high density. According to Eq. 12, the effect of fiber length vanishes with increased density. An amazing observation is the positive correlation between coarseness and fiber strength-normalized tearing work. This observation agrees with the previous result of Seth and Page *(47)*. One could easily interpret this as coarser fibers having a lower probability of failure at a specified sheet density. In the present treatment, however, we determined the critical density, r_c, for any pulp as the density of maximum tear. It is therefore difficult to apply this explanation.

DISCUSSION AND CONCLUSIONS

We have modeled the tearing work of paper as elastic energy dissipated when breaking bonds and fibers. The results differ from previous theories where frictional work when pulling out fibers was calculated *(16, 21–24)*. The main refinement to the recent note by Page *(25)* is that fiber failure does not have to be the dominating mechanism of energy dissipation, although tearing strength is proportional to the square of fiber strength through a constant. This is because the elastic energy released depends on the average load within a fiber and on the strength and number of breaking elements. All these entities depend on fiber strength.

The micromechanical theory contains many parameters that are difficult to measure. It is therefore difficult to verify it experimentally. This paper did not examine the effects of delamination and viscoelasticity. Earlier work considered these *(7, 8, 52)*. The experimental data used did not include information on the Young's moduli of the papers. The main predictions agree with experimental findings, however.

Although a relationship between very different kinds of fracture work tests exists for brittle papers *(52)*, the Elmendorf tear correlates poorly with the toughness of paper *(4, 5)*. This discrepancy comes from the tearing mode, delamination, and viscoelasticity. An additional factor is that the tearing test measures dissipated energy instead of the work *consumed* for crack propagation *(31)*. TJ

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