

ABSTRACT

A method to measure curvatures (curl) of thin papers is examined. The method is based on measurement of the out-of-plane deflection of a curled sheet with a noncontact laser distance meter. The specimen is arranged so that its plane is vertical to minimize additional deflections caused by gravity. A cylindrical shell was used to evaluate the method. The classical relation between evaluated curvature and inverse radius of the shell was verified, and the utility of transformation equations for the curvatures was demonstrated by rotation of the cylinder. The influence of gravity on the curvature was investigated using finite element analysis. The error in the evaluation of curvature due to gravity is estimated to be less than 4% for sheets with basis weights larger than 40 g/m².

THE DIMENSIONAL STABILITY AND curl of paper and board have received much attention lately because of increased production rates and reduced tolerances in printing, xerography, and converting operations (1-5). Curl is defined as departure from a desired flat shape of a paper and is caused by changes in the ambient humidity and/or temperature. The causes for curl are well documented in the literature (2, 3, 6, 7), but complexities influencing curl, such as history-dependent material properties and geometrical nonlinearities (8, 9), are still under investigation. A curled sheet commonly assumes a cylindrical shape with the cylinder axis along the direction of minimum hygroexpansion and maximum stiffness, i.e., the direction of manufacture (machine direction) (6, 7), but frequently other forms of curl are observed where the shape deviates from a sim-

Measuring curl of thin papers

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ple cylinder (5, 10). Any complex but uniform curl shape may be described by the three curvature components shown in Fig. 1. Figure 1(a) illustrates the two bending curvatures, K_x and K_y , and Fig. 1(b) shows the twisting curvature K_{xy} . Further discussions of the analysis and evaluation of the curvatures are given in the literature (11-13).

The evaluation and analysis of curl are further complicated by the change in curl shape that may be brought about by a change in sheet dimensions or humidity as discussed by Nordström *et al.* (9). As demonstrated by their analysis and illustrated in Fig. 2, an initially flat sheet at dry conditions may, as a consequence of increased moisture content, transition to a saddle shape, which eventually becomes unstable and transitions to one of two stable cylinder shapes at very high humidities. The same scenario applies for sheets that are initially flat at high humidities during decreasing humidity. Uesaka *et al.* (3, 14) suggested that the components K_x , K_y , and K_{xy} can be determined by cutting narrow strips of paper in the x , y and 45° directions, respectively, and measuring the curvature of the strips. Such a procedure, however, would not accommodate the complex nonlinear interactions that exist among the various components of curvature in a normally sized sheet of paper (9). Consequently, a curl test specimen representative for a sheet should be of similar dimensions in the x and y directions.

The research effort to minimize curl of paper is hampered by the lack of suitable and accurate methods for the measurement of curl curvatures.

Some curl test methods have been devised for paperboard (12, 15), but such tests may not be appropriate for thin papers because of the influence of gravity on the deflection. For the purpose of measuring curl of thin papers, a special apparatus was developed and critically examined by experiments and finite element analysis.

PRINCIPLE AND APPARATUS

Each point (x , y) of a curved surface may be characterized by three curvature components: K_x , K_y , and K_{xy} ; these are defined with respect to the sheet coordinates (x , y), as shown in Fig. 1. The curvatures can be shown to be tensor quantities; they will be discussed later and transformed as such. The general expressions for the curvatures at a point are complicated functions of the displacement of that point and the spatial coordinates giving the position of the point (11, 16, 17).

For deformed surfaces with small slopes (much less than 1), the linearization of the general expressions for the curvatures yields the following familiar expressions for the curvature components (11):

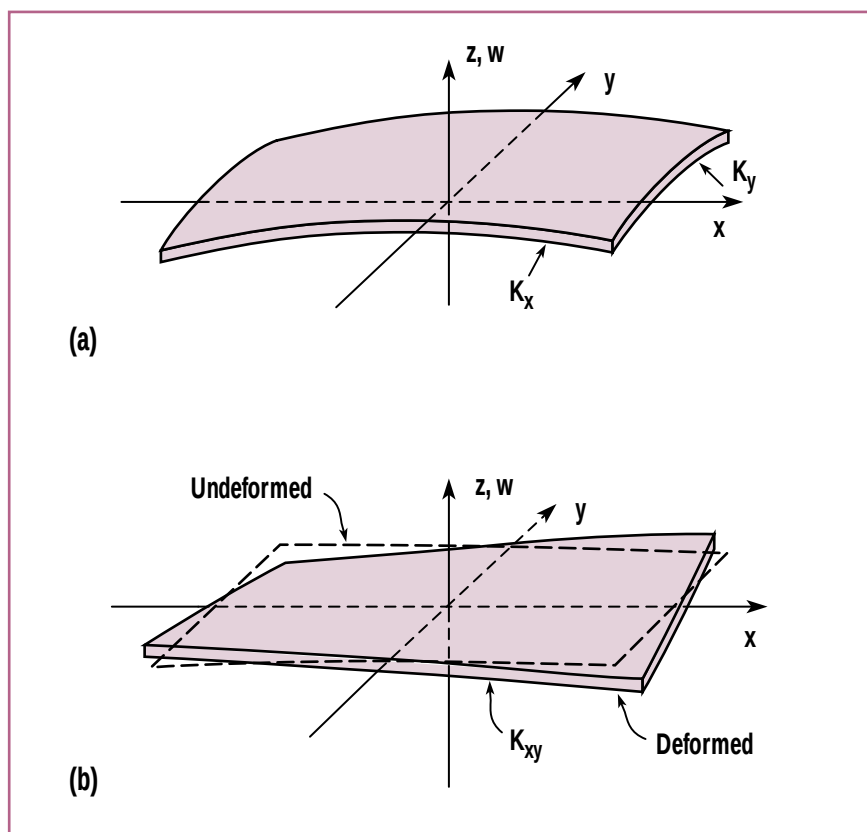
$$K_x = -(\partial^2 w / \partial x^2) \quad (1a)$$

$$K_y = -(\partial^2 w / \partial y^2) \quad (1b)$$

$$K_{xy} = -(2\partial^2 w / \partial x \partial y) \quad (1c)$$

where w is the deflection (z -direction displacement) of the sheet (mid-surface), and x and y are the planar coordinates of the sheet. The curvatures are illustrated in Fig. 1.

Figure 3(a) shows a schematic of the principle for the curl meter. A



1. Definition of curvatures K_x , K_y , and K_{xy} . (a) Bending curvatures, K_x and K_y , (b) twist curvature K_{xy} . All curvatures are shown positive as defined by Eq. 1.

square sheet of paper is attached vertically at its center using a 5-mm-diam. magnetic pin at the region of contact. The magnetic pin attachment of the sheet is detailed in Fig. 3(b). The constraint of the out-of-plane deflections of the specimen due to the attachment pin is expected to be minimal by virtue of its small diameter. Basically, the out-of-plane deflections of the paper sheet, $w(x, y)$, brought about by a change in relative humidity from a reference state, are measured at several positions (x, y) using a noncontact laser displacement sensor attached to an x-y table. The control unit, shown in Fig. 1(a), is connected to a personal computer (PC) that provides the desired positions (x, y) of the sensor as specified by the operator. The PC also reads and stores the deflections $[w(x, y)]$ and subsequently performs data reduction to evaluate the curvatures K_x , K_y ,

and K_{xy} . Details of the equipment, measurements, and evaluations are provided in the following paragraphs. The specimen size is selected from considerations of being representative for a macroscopic sheet while at the same time being small enough to reduce the influence of gravity on deflection. The vertical arrangement of the sheet further minimizes gravity influence on the deflection, as suggested by Viitaharju and Niskanen (5).

The topology data of the deflected surface are obtained by noncontact laser beam measurements of the deflection $[w(x, y)]$ at each of several $(m \times m)$ regularly spaced points (x, y) over the surface of the sheet. (See Fig. 4.) The laser analog displacement sensor (LAS-8010V, Nippon) has a measuring range of 100 ± 40 mm. The laser transmits a visible beam with a spot size less than 1 mm in diameter. To allow

for positioning of the displacement sensor in the x-y plane, the sensor is attached to a stiff vertical aluminum beam fitted with a linear translator for positioning of the sensor in the vertical direction (y axis). The vertical beam, in turn, is attached to a similar but horizontally arranged beam fitted with a linear translator for horizontal (x-axis) positioning. Translation and positioning of the sensor along each of the two coordinate directions is accomplished by computer-controlled stepping motors capable of achieving any targeted position within ± 0.0125 mm. The translators and control unit (C10) were developed by ISEL Electronics, Hisselberg, Germany.

The size of the measuring area ($b \times b$ in Fig. 4) and the number of measuring points are arbitrarily selected by the user. Because the deflection sensor is attached to a vertical beam, vibrations of the beam (and sensor) may influence the measurements. Vibration was only a problem when the displacement sensor was accelerated and stopped in the horizontal direction (x direction). Therefore, to minimize the influence of vibrations on the measurements, the sensor is controlled so that the vertical (y-axis) displacements are measured at a fixed x coordinate. Then the sensor is moved one step along the x axis. Before taking the next set of vertical displacement measurements, a short delay (1 second) was implemented to allow vibrations to damp.

In the actual system, shown in Fig. 5, up to 15 test specimens can be examined. The system is placed in a dedicated environmentally controlled room where the relative humidity (RH) can be varied over a large range (10–80% RH). The temperature was held constant at 23°C. Once the coordinates and distances between the laser head and each point on the surface are collected and stored by the computer, the deflections, $w(x, y)$, defined with respect to an arbitrary flat vertical reference surface, are calculated.

Equations 1(a)-1(c) may be integrated assuming that the curvature components are constants over the sheet surface. This leads to the following equation for the deflected surface:

$$w(x, y) = a_1x^2 + a_2y^2 + a_3xy + a_4x + a_5y + a_6 \quad (2)$$

A least-squares regression analysis was performed using Matlab (18) to determine the coefficients a_1 through a_6 from the measured coordinates (x, y) and the corresponding deflection, w . Once the coefficients are determined, Eqs. 1 and 2 are combined to yield the curvatures:

$$K_x = -2a_1 \quad (3a)$$

$$K_y = -2a_2 \quad (3b)$$

$$K_{xy} = -2a_3 \quad (3c)$$

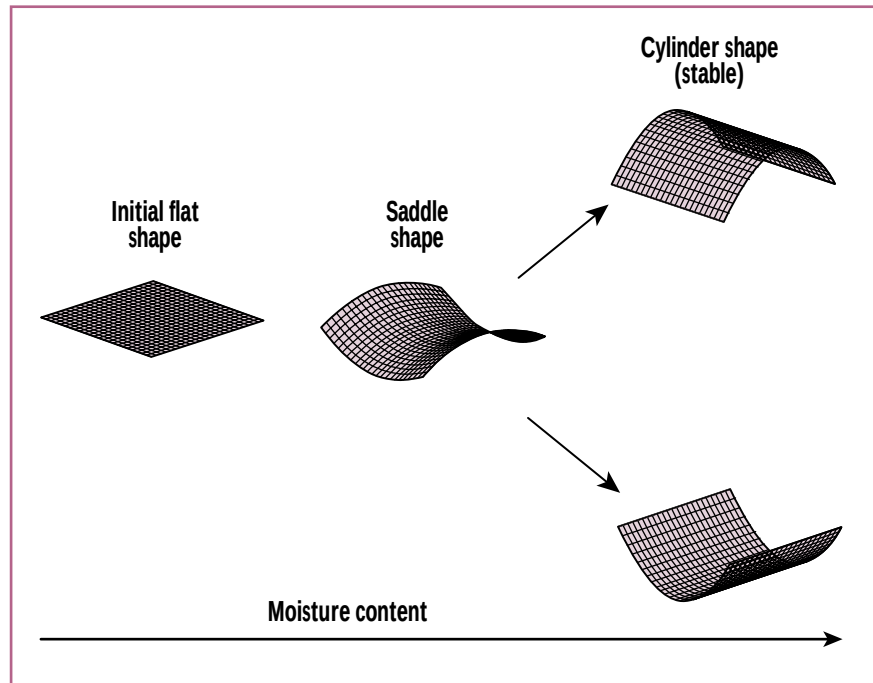
The curvatures refer to the coordinate system (x, y) shown in Fig. 1. If the plate is rotated counterclockwise by an angle Θ around the z axis, as shown in Fig. 6, the curvatures in the new (primed) coordinate system become as follows (11):

$$\begin{bmatrix} K'_x \\ K'_y \\ 1/2 K'_{xy} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} K_x \\ K_y \\ 1/2 K_{xy} \end{bmatrix} \quad (4)$$

where $m = \cos \Theta$ and $n = \sin \Theta$. This equation is useful for finding the direction of the cylinder with respect to the x axis that is free of twist curvature. Solving Eq. 4 for $K'_{xy} = 0$ yields:

$$\Theta_0 = 1/2 \tan^{-1} (K_{xy}/K_y - K_x) \pm 90^\circ \quad (5)$$

The above equation may be used to calculate the angle Θ_0 , through which the cylinder must be rotated for the twist curvature to vanish. The curvatures K'_x and K'_y correspond-



2. Changes in curl shape brought about by increased humidity (9)

ing to this angle (calculated from Eq. 4) may be shown to be the overall maximum and minimum curvatures and are an analogy to principal stresses and strains (19), denoted *principal curvatures*. The angle Θ_0 gives the direction of the “axis of curl.” If the curl shape is cylindrical, it is experimentally difficult to align the cylinder axis with the x axis of the measuring system. Evaluation of Θ_0 reveals the degree of alignment of the specimen, and Eq. 4 allows the calculation of the principal curvature of the cylinder.

EVALUATION OF APPARATUS

The equipment was evaluated using a 10-cm-long and 10-cm-wide cylindrical shell, with an internal radius of 47 mm, made from aluminum. The internal surface of the shell was covered with a sheet of copy paper to reduce the reflectance. The influence of the number of coordinate pairs (x, y) was examined first. For an area of $8 \times 8 \text{ cm}^2$, 4×4 and 6×6 coordinate points gave $K_x = 26.5$ and 26.4 m^{-1} , respectively, indicating that 4×4 points are adequate for this type of surface. Next, the influence

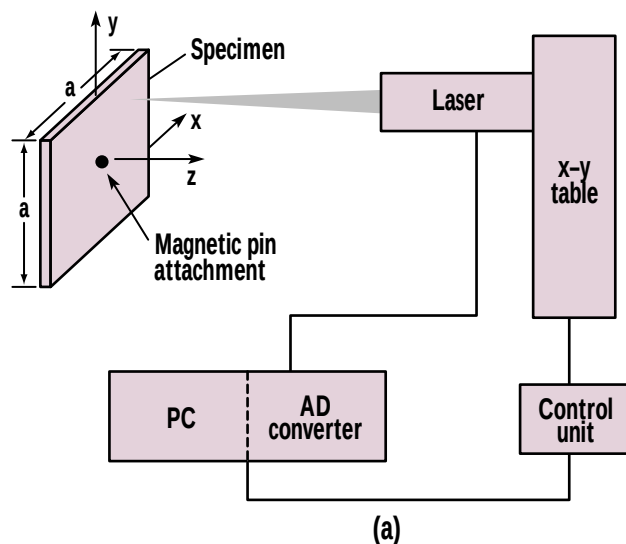
of the size of the measuring area was examined using 4×4 points. Table I summarizes the results.

Inspection of the data in Table I reveals that the curvature increases with increased size of the measuring area. For areas less than $6 \times 6 \text{ cm}^2$, there seems to be an approximately constant curvature that approximately coincides with the inverted radius of the curvature of the shell, $1/R = 1/0.047 = 21.3 \text{ m}^{-1}$. This result shows that the relation $\partial^2 w / \partial x^2 = 1/R_x$ is valid only locally when the overall deflections are large. Consideration of a deflected surface with the shape of a circular arc with its center on the z axis (see Fig. 7) will further illustrate the importance of selecting measuring area. The deflection of the surface is given by:

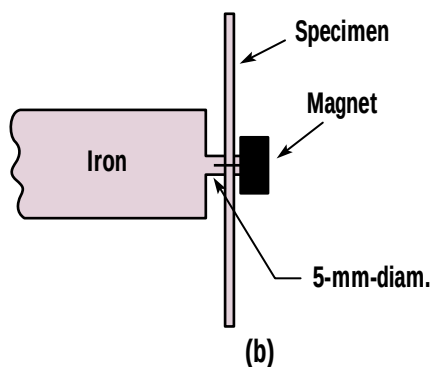
$$w = R - \sqrt{R^2 - x^2} \quad (6)$$

For small values of x (x much less than R), a Taylor series expansion (20) of Eq. 6 yields:

$$w \approx x^2 / 2R \quad (7)$$



(a)



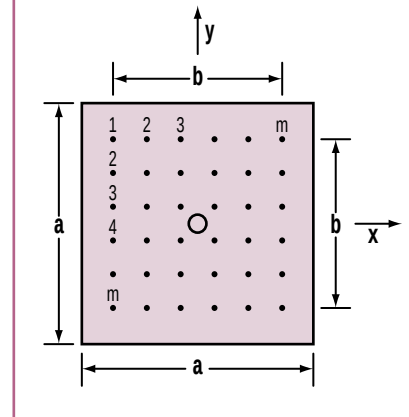
(b)

3. Principle of curl meter

Equations 1(a) and 7 yield $K_x = -1/R$, which shows that the magnitude of the curvature is the inverse of the radius of curvature only when the displacement w is small, i.e., when x is much less than R . For larger measuring areas, such a relationship is not valid, as also reflected by the data in Table I. This effect has the consequence that the size of the measuring area needs to be reduced to be consistent with the linearized equations (Eqs. 1) if the deflections and slopes are excessive, such as for the case considered.

The accuracy of the test procedure is mostly governed by the accuracy of the deflection measurement, which in turn is governed by the res-

olution of the optical distance meter (50 μm). Because the curvatures are determined from a regression analysis, it is difficult to calculate the error in the curvatures corresponding to an error in the deflection measurements. If the error in the deflection measurements is random, however, it is expected that the statistical regression analysis procedure used will produce accurate and reliable averages. The closeness of the directly measured and evaluated values of the radius of the cylindrical shell (47 and 46 mm, respectively) indicates that the procedure is accurate. To further examine the method, the previously mentioned cylindrical shell was aligned with the axis of the



4. Depiction of square measuring area by $m \times m$ points

cylinder along the vertical (y) axis. Evaluation of the curvature using a $4 \times 4\text{-cm}^2$ measuring area gave $K_x = 21.1 \text{ m}^{-1}$ and $K_y = K_{xy} = 0$.

The cylinder was then rotated 45° clockwise, and the curvatures were determined using the same ($4 \times 4\text{-cm}^2$) measuring area. This evaluation gave $K'_x = 11.4 \text{ m}^{-1}$, $K'_y = 11.3 \text{ m}^{-1}$, and $K'_{xy} = -21.4 \text{ m}^{-1}$. Substitution of $K_x = 21.1 \text{ m}^{-1}$ and $K_y = K_{xy} = 0$ and $\Theta = -45^\circ$ in Eq. 4 yields $K'_x = 10.6 \text{ m}^{-1}$, $K'_y = 10.6 \text{ m}^{-1}$, and $K'_{xy} = -21.1 \text{ m}^{-1}$, which are in good agreement with measured curvatures. The direction of the axis of curl, Θ_0 , (with respect to the x axis) is obtained by substituting the measured K'_x , K'_y , and K'_{xy} values in Eq. 5; this yields $\Theta_0 = 44.9^\circ \pm 90^\circ$, i.e., $\Theta_0 = 134.9^\circ$ or -45.1° . The cylinder would thus need to be rotated clockwise 45.1° for the twist curvature to vanish; this agrees well with the physical configuration of the cylinder. **Figure 8** shows the fitted function $w(x, y)$ for the cylinder rotated 45° . The cylinder axis (axis of curl) is easily identified as a line $x = y$.

INFLUENCE OF GRAVITY FORCE

An analysis of the effect of gravity on the deflection of curled papers was performed using the finite element method. Two-ply papers with plies of identical orientation, mechanical properties, and density were consid-

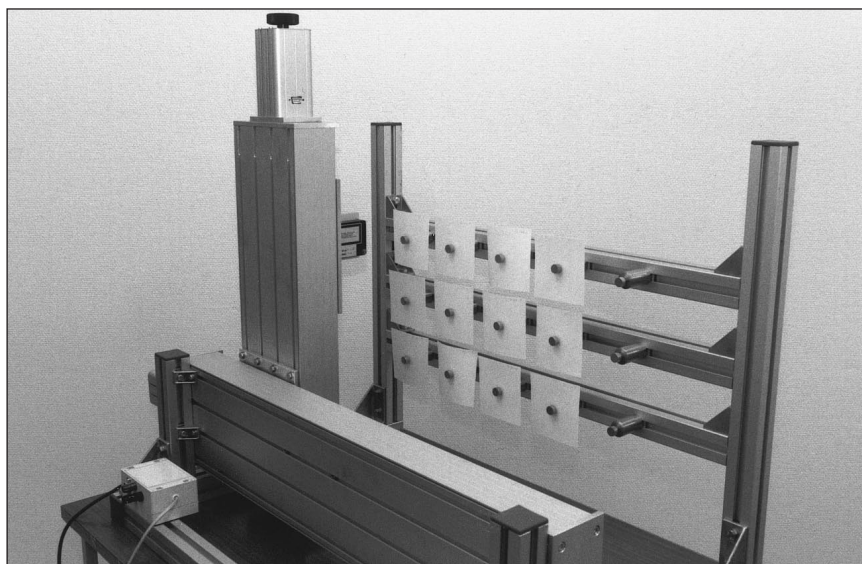
Measuring area, cm ²	Curvature, m ⁻¹
2 × 2	21.8
4 × 4	21.1
6 × 6	23.0
8 × 8	26.9

1. Curvature of cylindrical shell as a function of measuring area

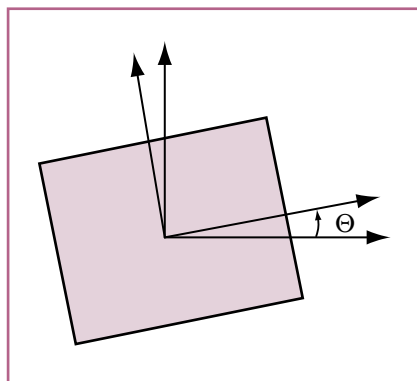
ered (see Table II). The properties listed in Table II are representative for copy paper (9). The moisture expansion coefficient β_{MD} was set to zero to generate a cylindrical curl shape.

Because bending stiffness of paper increases more rapidly with its basis weight than its mass, it is expected that low-basis-weight papers would be more influenced by gravity than heavier papers. Furthermore, it is expected that sheets with large curl curvature and large deflections would be influenced more by gravity than nearly flat papers. To cover a practical range of curvatures and deflections, 10 × 10-cm sheets with “small” and “large” curl magnitudes were considered [maximum deflections in the absence of gravity were 5.2 mm (small) and 17.4 mm (large)]. Basis weights of 30, 40, 60, 80, and 100 g/m² were considered. Finite element analysis was performed first in the absence of gravity so that the above deflections were obtained irrespective of the basis weight by adjusting the moisture content.

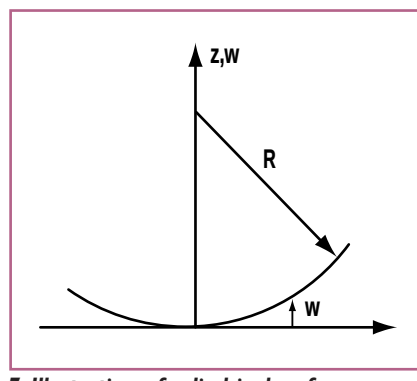
An initial estimate of the moisture content corresponding to a certain deflection (and curvature) was obtained according to the two-ply linear analysis presented in the literature (21). To represent a worst-case scenario, the cylinder axis was configured to coincide with the horizontal x axis, as shown in Fig. 3. In actual curl measurements, the axis of minimum curl would be aligned vertically to minimize the influence of gravity on curl. With the gravity force acting, the deflections of the sheet are not



5. Photograph of curl-measuring system



6. Positive (counterclockwise) rotation of the sheet by an angle Θ around the z axis



7. Illustration of cylindrical surface

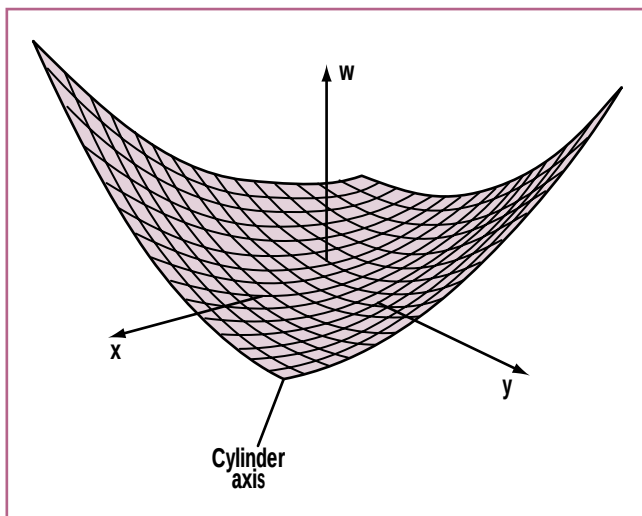
expected to remain symmetrical with respect to the x - z plane. Symmetry with respect to the y - z plane, however, is preserved, and only half the sheet was modeled. The gravity force was applied on the deflected paper, and the deflections (including the effect of gravity) were calculated using finite elements. All finite element calculations featured the geometrically nonlinear option of the ABAQUS code (22). Details of the mesh and element types are provided in the literature (9).

Figure 9 shows the deflection shapes with and without gravity acting. Because of gravity, the deflected shape of the sheet will not remain

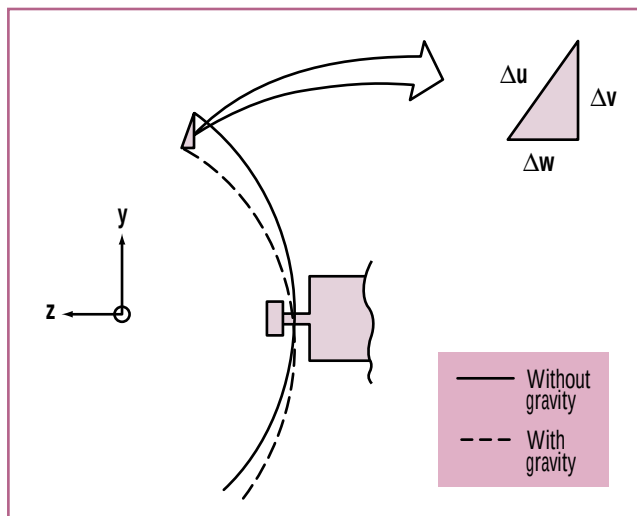
symmetric with respect to the plane $y = 0$. The influence of gravity on the shape of the curled sheet may be quantified by the displacement due to gravity of the upper edge of the sheet, Δu , as shown in Fig. 9. The characteristic Δu is given by the Pythagorean relation:

$$(\Delta u)^2 = (\Delta w)^2 + (\Delta v)^2 \quad (8)$$

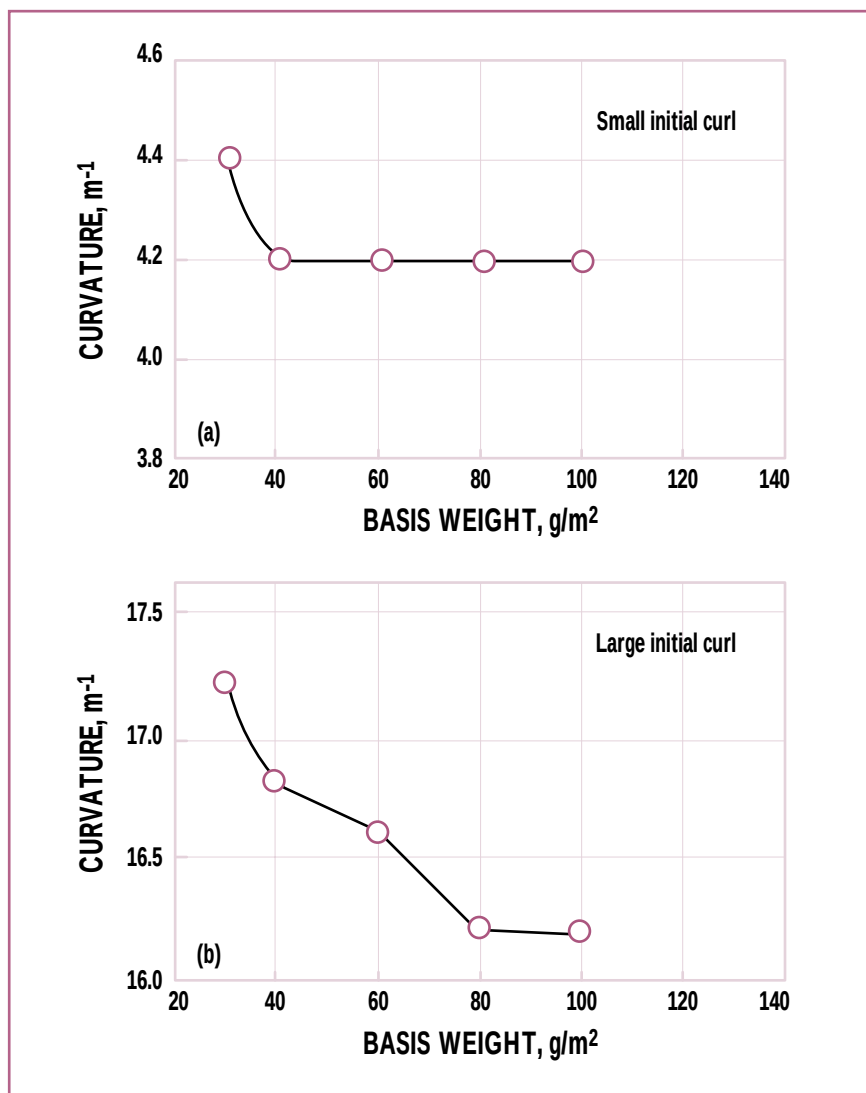
where Δw and Δv are the horizontal (z -axis) and vertical (y -axis) displacements of the edge brought about by application of gravity. Table III lists the results for different basis weights corresponding to the two initial deflections (5.2 and 17.4 mm). The



8. Out-of-plane deflections for a cylinder rotated 45°



9. Influence of gravity on the curl shape



10. Effect of gravity for sheets with small and large initial curvatures. (a) Small initial curvature, (b) large initial curvature.

additional deflection due to gravity, Δu , decreases with increasing basis weight as a result of the increased bending stiffness of the sheet. Δu is also smaller for the sheets with small initial curl than for those with large initial curl. The difference in Δu between sheets with small and large curl is approximately a factor of three (which is also approximately the ratio of the initial deflections).

To examine the influence of gravity on the evaluation of the curvatures, the calculated deflections, $w(y)$, were fitted to Eq. 2, and the curvature K_y was evaluated. Note that Eq. 2 is valid only for a sheet that is free from body forces, with a uniform curvature. When gravity is a factor, this assumption is not realized, as indicated in Fig. 9. Consequently, when gravity force is present, the evaluated curvature K_y represents an overall (apparent) curvature of the sheet.

Figure 10 shows the results for the sheets with small and large initial curl. For the sheet with small initial curvature, Fig. 10(a) shows that the apparent curvature is diminishing with increasing basis weight at small basis weights and reaches an asymptotic value (4.2 m^{-1}) at about 40 g/m^2 . This value coincides with the curvature calculated without considering gravity. For the sheet with large

Ply	EMD, GPa	ECD, GPa	GI ₂ , GPa	bMD* x 10 ⁻²	bCD x 10 ⁻²
1	7.05	2.80	1.72	0	13.4
2	7.05	2.80	1.72	0	0

*b_{MD} is set equal to zero to generate a cylindrical shape of curled papers.

II. Mechanical properties and moisture expansion coefficients for sheets considered in finite element analysis. E = Young's modulus, G_{12} = shear modulus, and β = moisture expansion coefficient. Poisson's ratio, ν_{12} , was set to 0.3 and density = 638 kg/m³.

w, g/m ²	Δu , mm Small curl	Δu , mm Large curl
30	1.48	4.54
40	0.86	2.66
60	0.42	1.29
80	0.26	0.79
100	0.18	0.53

III. Change in deformation due to gravity for sheets with small and large initial curl

KEYWORDS

Curl, cylinders, deflection, deformation, density, dimensional stability, finite element analysis, light weight papers, mathematical analysis, measurement, measuring instruments, mechanical properties, test methods.

initial curvature, Fig. 10(b) shows that a basis weight of about 80 g/m² is needed to measure a curvature that is not influenced by gravity. Inspection of the data in Fig. 10(b), however, reveals that the error in curvature is less than 4% for sheets with basis weight of 40 g/m² or larger. This small error indicates that the vertical arrangement of the sheet

should allow curl evaluation for most common paper grades.

CONCLUSIONS

A new apparatus for determining curl on thin papers has been introduced and examined. The closeness of the directly measured and evaluated values of the radius of a cylindrical shell indicates that the procedure is accurate. It was demonstrated that the curvature depends on the size of the measuring area. The classical relationship $K_x = -1/R$ is therefore valid only locally if the overall deflections are large. By using a transformation matrix, the axis of curl was calculated, and by rotating the cylindrical shell a known angle, it was demonstrated that the mathematical evaluation provides accurate results. Examination of the influence of gravity on the deflection of curled

papers showed that the influence of gravity on curvature evaluation is negligible for common paper grades.

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LITERATURE CITED

- Koran, Z., Bussieres, D., and Prud'homme, R. E., *Tappi* 65(8): 88(1982).
- Rutland, D. F. in *Design Criteria for Paper Performance* (P. Kolseth, C. Fellers, L. Salmen, et al., Eds.), STFI, Meddelande A 969, 1987, p. 29.
- Uesaka, T., *J. Pulp and Paper Sci.* 17(2): J39(1991).
- Niskanen, K. J., *Paper and Timber* 75(5): 321(1993).
- Viitaharju, P. H. and Niskanen, K. J., *J. Pulp Paper Sci.* 20(5): J148(1994).
- Glynn, P. and Gallay, W., *Pulp Paper Mag. Can.* 63(8): T418(1962).
- Gallay, W., *Tappi* 56(12): 90(1973).
- Uesaka, T., Kodaka, I., Okushima S., et al., *Rheologica Acta* 28(3): 238(1989).
- Nordström, A., Gudmundson, P., and Carlsson, L. A., under preparation.
- Gray, D. G., *J. Pulp Paper Sci.* 15(3): J105(1989).
- Hyer, M. W., *J. Compos. Mater.* 15(3): 175(1981).
- Eriksson, L. E., Cavlin, S., Fellers C., et al., *Nordic Pulp Paper Res. J.* 2(2): 66(1987).
- Hyer, M. W., *J. Compos. Mater.* 15(7): 296(1981).
- Uesaka, T., Ishizawa, T., Kodaka, I., et al., *Japan Tappi J.* 39(10): 953(1985).
- Wennerblom, A. B., *Tappi J.* 75(4): 97(1992).
- Chia, C. Y., *Nonlinear Analysis of Plates*, McGraw-Hill, New York, 1980,
- Mollman, H., *Introduction to the Theory of Thin Shells*, Wiley & Sons, New York, 1981,
- Anon., *Matlab User's Manual*, The Math Works, Inc., Natick, MA, 1993,
- Timoshenko, S. P. and Goodier, J. N., *Theory of Elasticity*, 3rd. edn., McGraw-Hill, New York, 1970, p. 319.
- Rade, L. and Westeraren, B., *Beta-Mathematics Handbook*, Studentlitteratur, Lund, 1988, p. 171.
- Carlsson, L. A., Fellers, C., and Htun, M., *Svensk Papperstid.* 83(7): 194 (1980).
- Anon., *ABAQUS User's Manual*, Version 5.3, HKS Inc., Pawtucket, RI, 1993.