PAPER PHYSICS

PEER REVIEWED

ABSTRACT

The authors measured machine direction and cross direction tensile strengths of various papers containing circular holes and straight slits and predicted the results using the point stress criterion. First developed for fiber composites, this predicts that a specimen containing a hole or crack will fail when the stress some characteristic distance away from the edge of the defect reaches the tensile strength of the material. This simple criterion can predict the failure of paper specimens containing defects of various sizes.

Application: *The point stress criterion model can predict the strength of paper webs containing holes of any size with a simple set of experiments.*

Predicting the strength of paper containing holes or cracks with the point stress criterion

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RUNNABILITY DURING CONVERTING is a key quality parameter for many grades of paper and board. The presence of defects ranging from pinholes and shives to large slime holes and crepe wrinkles can affect runnability. Very large defects will usually cause failure during converting, but an on-line hole detector can find them. Small crack-like defects such as shives continue to create problems in the newsprint industry. Efforts since the 1970s (1–4) have attempted to use fracture mechanics to deal with the problem.

Unfortunately, fracture toughness can be a difficult property to measure. Making predictions of failure even knowing this property is also difficult.

A paper machine hole detector can detect defects with a wide range of sizes, but the final product may contain small holes not detected. Fracture mechanics only considers sharp cracks and is unsuitable for sheets containing holes. No satisfactory method of predicting the effect of these holes on the tensile strength of a paper sheet presently exists. The holes nevertheless do create stress concentrations and may serve as fracture initiation points in use.

In this paper, we adapt a simple engineering approach developed by Whitney and Nuismer *(5)* for composite materials. The point stress criterion (PSC) can predict the strength of various paper sheets containing circular holes and sharp cracks. The approach should give paper makers a useful method to predict the effect of holes in a web during converting.

THE POINT STRESS CRITERION

Inglis *(6)* first described the effect of an elliptical hole on the stress field in an infinite isotropic sheet and derived the stress concentration factor of the following equation:

$$K_t = \sigma_{max}/\sigma_\infty = 1 + 2a/b \qquad (1)$$

where

K_t = stress concentration factor
σ_{max} = maximum amplified stress at the edge of the hole
σ_∞ = remote or applied stress
a = transverse direction axis of the ellipse
b = load direction axis of the ellipse.

The maximum stress at the edge of a circular hole where a = b is 3 times the remote, applied stress. The stress concentration factor for a circular hole is therefore 3. The stress field and the stress concentration factor are functions of the hole geometry only, although they do vary for anisotropic sheets such as machine-made sheets where the ratio of machine direction (MD) to cross direction (CD) is typically greater than one. A sharp crack is like a flattened ellipse with b = 0. Equation 1 therefore predicts that the stress at the tip of the crack is infinite.

Equation 1 shows that σ_{max} does not depend on the absolute value of a and b but only on the ratio between them. This means σ_{max} depends only on the shape of the hole not its size. For composite materials and for paper, a small hole will not degrade the strength as much as a large one. We cannot therefore simply set $\sigma_{max} = \sigma_0$, the strength of the material, to predict when the sheet will fracture. Whitney and Nuismer *(5)* suggested that the sheet would fail when the stress at some characteristic distance, d_0, from the edge of the hole reached the tensile strength of the unnotched material. This ensures that a certain volume of the material has a stress greater than the failure stress. The criterion based on this statement is the PSC illustrated in **Fig. 1.**

To apply the PSC, one must know the stress distribution in front of the hole. For an infinite plate containing a circular center hole, Eq. 2 shows the stress *(7)*:

$$\sigma^*_y = \sigma_\infty[1 + (1/2)(R/x)^2 + (3/2)(R/x)^4] \qquad (2)$$

where $x \geq R$ and

R = radius of the hole
x = the distance from the center of the hole

Paper type	Length, mm	Width, mm	Grips, mm	Max. hole, (R, mm)	Other holes (R, mm)							Comment
TMP-1, MD	640	128	20	16	12	8.5	5	3	1.5	1	0	
LP-1, MD	240	48	20	6	5	4	3	1	1.5	1	0	Paper size limit
TMP-2, MD	640	128	20	16	12	8.5	5	3	1.5	1	0	
B-1, MD	640	128	20	16	12	8.5	5	3	1.5	1	0	
TMP-3, MD	640	128	20	16	12	8.5	5	3	1.5	1	0	
TMP-1, CD	260	52	20	6.5	5	4	3	2	1.5	1	0	Roll width limit
B-1, CD	320	64	20	8	6.5	5	4	3	2	1	0	Roll width limit
TMP-3, CD	260	52	20	6.5	5	4	3	2	1.5	1	0	Roll width limit
TMP-1, slit	6401	128	20	16	12	8.5	5	3	1.5	1	0	
TMP-3, slit	640	128	20	16	12	8.5	5	3	1.5	1	0	

I. Sample dimensions

σ^*_y =stress in the direction of the applied load in an infinitely wide sheet.

Figure 1 depicts the stress distribution in a finite width sheet. When x = R, the ratio of σ^*_y to σ_∞ in Eq. 2 is 3—the stress concentration factor for a circular hole. The equation for the strength of a sheet with a defect, σ_N, is obtained by letting $\sigma^*_y = \sigma_O$—the strength of a normal tensile specimen—at a distance x = R + d_O from the center of the hole:

$$\sigma_N/\sigma_O = 2/(2 + \xi_1^{\,2} + 3\xi_1^{\,4}) \qquad (3)$$

where

$$\xi_1 = R/(R + d_O) \qquad (4)$$

Whitney and Nuismer found that Eq. 3 accurately predicted the strength of notched glass fiber reinforced epoxy. Since Eq. 2 is for an infinitely wide plate, one must modify the equation for stress in front of a hole in a finite width sheet. Tan *(8)* has accomplished this using the finite-width correction factor:

$$\sigma_y(x,0) = Y\sigma^*_Y(x,0) \qquad (5)$$

where

σ_y = the stress in a finite width sheet
Y = the finite width correction factor.

In Eq. 5, Y has the following definition:

$$Y = 2/[2 - (2R/W)^2 - (2R/W)^4] \qquad (6)$$

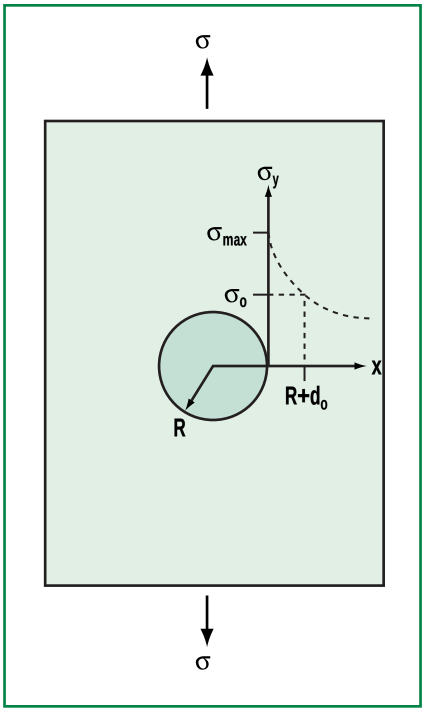
where

W= width of the specimen.

Y is near one if the specimen width is much larger than the diameter of the hole. Combining Eqs. 3 and 6 gives the following expression for notched strength as a function of unnotched strength:

$$\sigma_N = 2\sigma_O/(2 + \xi_1^{\,2} + 3\xi_1^{\,4})Y \qquad (7)$$

Equation 7 can be tested by plotting it against experimental data for σ_N vs. hole size. The parameter d_O results from optimizing the fit of Eq. 7. The effect of MD/CD ratios greater than one on Eq. 7 will be very small for most papers. We therefore ignore it.

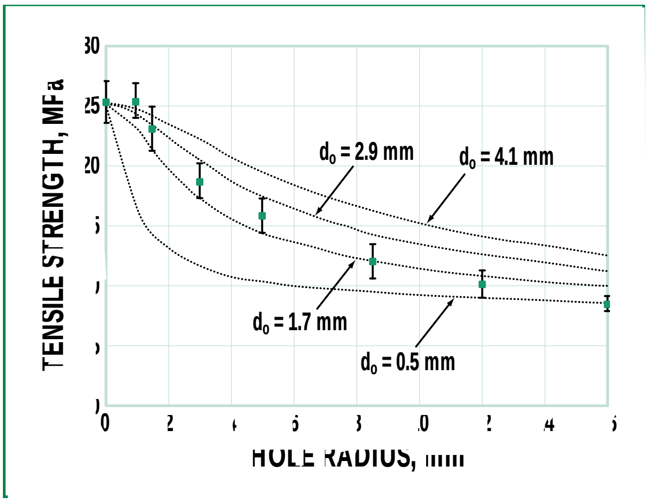


1. Schematic of the PSC

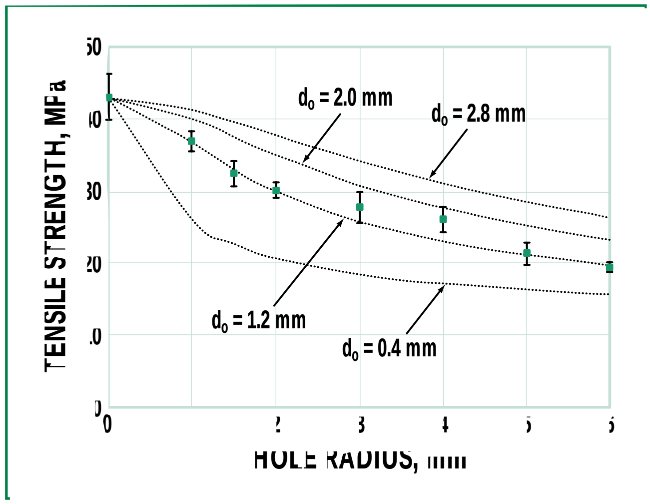
EXPERIMENTAL

To test the application of the PSC for paper, we tested five different papers containing circular holes. TMP-1 and TMP-2 were newsprints containing shives, and TMP-3 was newsprint with minimum shives. LP-1 was a high quality laser/photocopy paper, and B-1 was a white bond paper. All five paper types underwent testing in the MD. TMP-1, TMP-2, and B-1 also underwent testing in the CD. We also tested TMP-1 and TMP-3 with a MD orientation and various center slit sizes to determine the suitability of the Whitney and Nuismer criteria for modeling strength reduction due to the presence of hair cuts and shives.

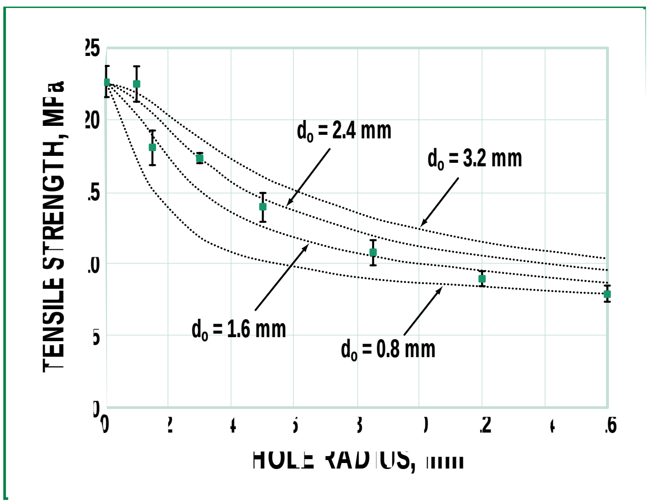
For each paper type, seven hole sizes were used, with control specimens not containing a hole. To reduce the effects of finite-width and length, all specimens had a width to maximum hole diameter radio of 4:1 or greater and a length to width ratio of 5:1. We also used very large—32-mm diameter where possible—and very small—2-mm diameter—hole sizes to determine the ability of the PSC to predict the strength of a wide range of holes. Because of constraints imposed by roll width and sheet dimensions, not all papers were tested using the maximum 32-mm hole diameter. For



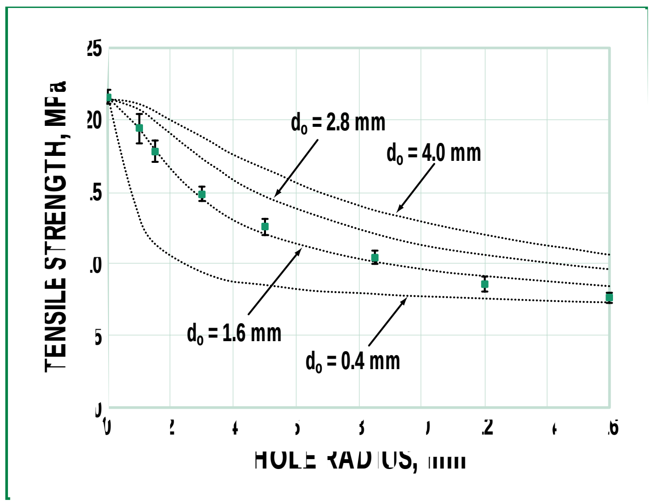
2. Tensile strength vs. hole radius for 128-mm wide sheets of TMP-1 newsprint in MD



3. Tensile strength vs. hole radius for 48-mm wide sheets of LP-1 photocopy paper in MD



4. Tensile strength vs. hole radius for 128-mm wide sheets of TMP-2 newsprint in MD



5. Tensile strength vs. hole radius for 128-mm wide sheets of B-1 bond paper in MD

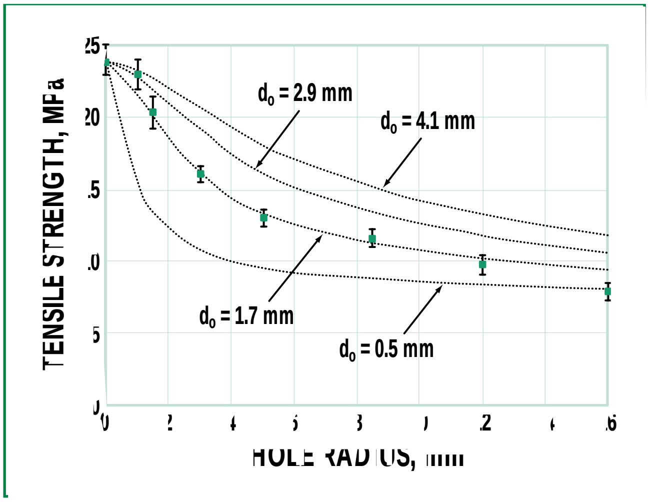
example, the LP-1 paper was precut to 8.5 x 11 in. dimensions allowing for a maximum specimen length of 240 mm with an extra 20 mm in each grip. This in turn allowed a maximum width of 48 mm and a maximum hole diameter of 12 mm. Table I summarizes the dimensions and hole sizes for each sample.

We cut holes greater than 6 mm in diameter using a fresh scalpel and a standard hole template. Smaller holes were cut using a specially manufactured set of punches. Testing used a commercially available tensile testing machine under standard test conditions of 50% RH and 22°C. All specimens used a crosshead speed of 2 mm/min. Each hole size tested involved 7 specimens.

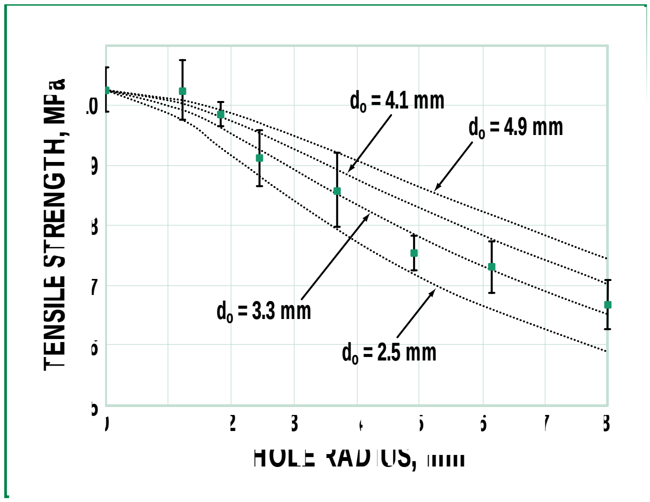
RESULTS AND DISCUSSION

Figures 2–11 show the results for tensile strength as a function of hole size. The data points represent the mean experimental results with standard deviation bars. Plotting Eq. 7 using a variety of values for d_o gives the theoretical lines. If the theoretical curve for a particular value of d_o passes through all the data points for a given material, then d_o is a material property, and Eq. 7 is a valid failure criterion. In this case, narrow specimens with a single, small hole size could be used to estimate d_o for a given material, and Eq. 7 could then predict the strength for sheets of any size containing holes of any size.

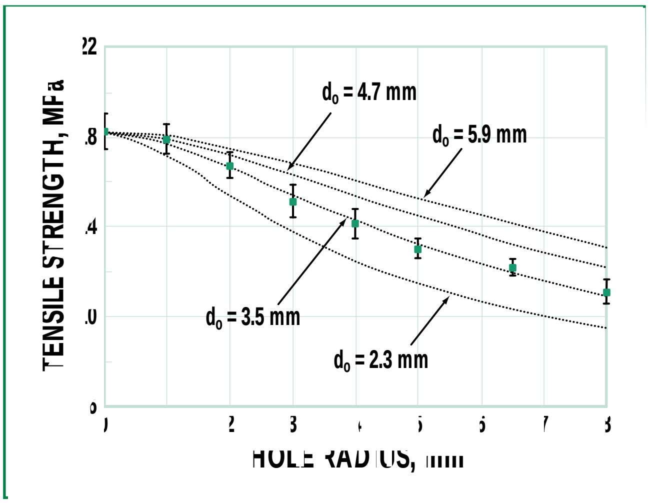
The parameter d_o is a measure of the damage tolerance of the material. A large value corresponds to a damage tolerant material, since a large area of material in front of the hole has a stress greater than the tensile strength before the specimen fails. This criterion has some attributes of a fracture toughness criterion, although its derivation uses strictly empirical grounds and has no basis in physics other than the qualitative argument



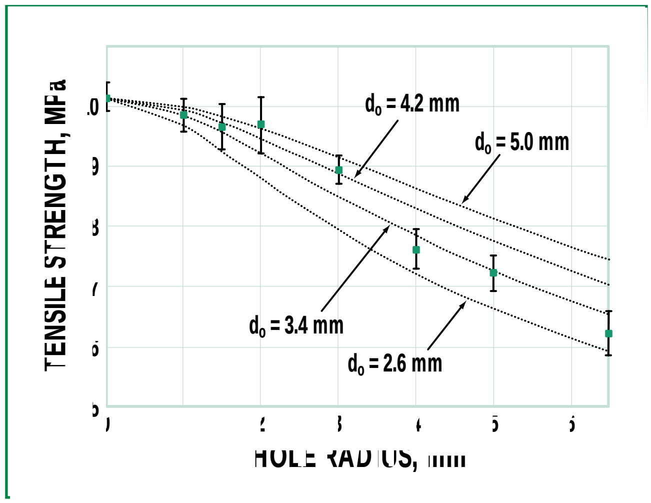
6. Tensile strength vs. hole radius for 128-mm wide sheets for TMP-3 newsprint in MD



7. Tensile strength vs. hole radius for 52-mm wide sheets of TMP-1 newsprint in CD



8. Tensile strength vs. hole radius for 64-mm wide sheets for B-1 bond paper in CD



9. Tensile strength vs. hole radius for 52-mm wide sheets for TMP-3 newsprint in CD

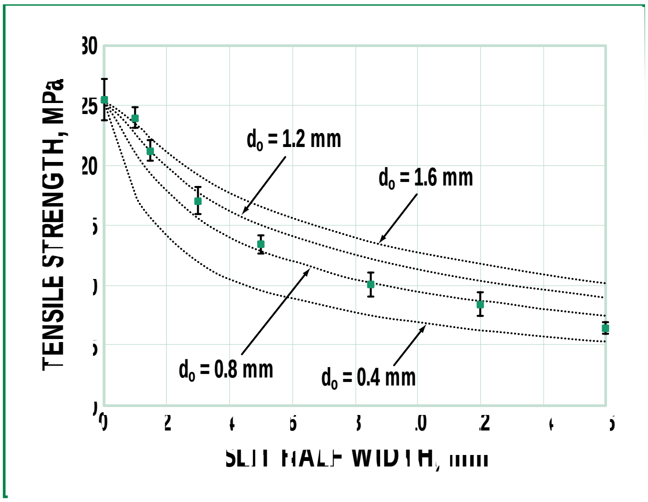
provided above. Because of its simplicity, the PSC could be an extremely useful tool for monitoring product quality.

MD results

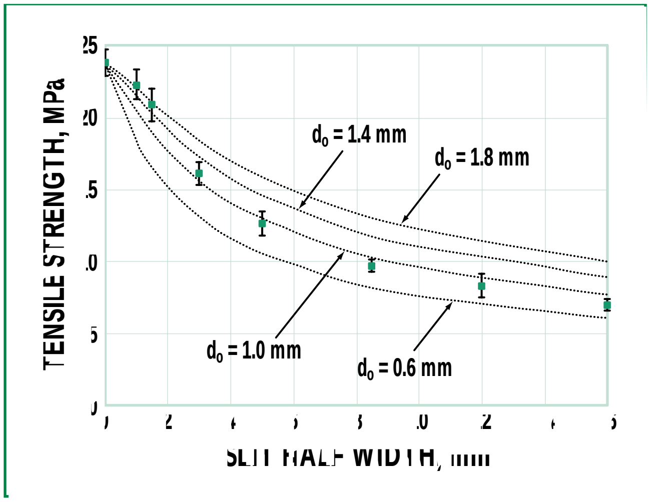
The strength always drops quicker than predicted by the simple reduction in cross-sectional area due to the presence of the holes. All five paper types showed good correlation between the experimental results and those predicted by the PSC. The LP-1 paper in Fig. 3, the B-1 paper in Fig. 5, and the TMP-3 paper in Fig. 6 showed the best correlation with characteristic d_O values of 1.3 mm, 1.6 mm, and 1.7-1.8 mm, respectively. In each case, all specimens fractured at the hole, confirming the presence of a stress concentration in those areas of the paper. The TMP-1 and TMP-2 samples showed a lower degree of correlation with the PSC particularly for small holes as Figs. 2 and 4 show. For those samples, characteristic d_O values were approximately 1.7 mm—similar to that for the TMP-3 specimens.

The TMP-1 and TMP-2 papers had a relatively large fraction of visible shives and tended to fracture in areas away from the smaller holes. A poorly bonded and relatively sharp shive should cause failure before a hole of similar size, since Eq. 1 predicts higher stresses at the tip of the shive—b would be very small for a shive. This was evident for several TMP-1 and TMP-2 samples upon closer examination of the fracture line. In one instance, a 6-mm diameter hole was not sufficiently large to initiate fracture. The fracture occurred at a shive that was actually longer.

Even without excluding specimens that did not fracture at the hole from the data set, the correlation was still quite good. Note that for the papers that experienced shive dominated behavior the unnotched tensile strength specimens used to determine σ_0 would also fail at shives to produce



10. Tensile strength vs. slit width for 128-mm wide sheets for TMP-1 newsprint in MD



11. Tensile strength vs. slit width for 128-mm wide sheets for TMP-3 newsprint in MD

an artificially low value of σ_0. This would then distort the theoretical curves to some degree. When using the PSC to predict the effect of shives in papers such as these, using the true, unnotched strength of a shive free region as a basis for the theory would probably be better.

The correlation stabilizes as the hole size increases. Figures 2–6 show that specimens with a hole radius of 5.0 mm could provide reliable estimates of d_O.

CD results

All paper samples tested in the CD showed good correlation between the experimental and the results using PSC as Figs. 7–9 show. Characteristic d_O values were 3.5 mm for the B-1 paper in Fig. 8, 3.3–3.4 mm for the TMP-1 newsprint in Fig. 7, and 3.4–3.5 mm for the TMP-3 paper in Fig. 9. All these values were much higher than the MD results. We would expect this since the more ductile CD sheets are more damage tolerant. The MD and CD sheets are effectively different materials. The d_O values should therefore not be the same. The d_O values were virtually identical for the various CD samples.

The TMP-1 results were less consistent than the B-1 and TMP-3 results and had a greater degree of scatter. Fractures induced by shives were again predominant for the lower radius holes in the TMP-1 sample. Some fracture occurred away from the hole in the B-1 specimens at the lower radius holes. This was surprising since the B-1 paper contained no visible shives. The data set included the specimens that did not break at the hole, and the correlation was still good.

Slit results

The strength analysis of the paper specimens containing center slits follows directly from the analysis of the samples with holes. When used for slits, the PSC is an empirical alternative to the more fundamental fracture-mechanics based failure criteria *(2–4)*. All that is necessary is an expression for the stress distribution around a slit. For these samples, we obtained theoretical results using the following expression for stress distribution where $x \geq a$:

$$\sigma_y^*(x,0) = \sigma_\infty/[1 - (a/x)^2]^{1/2} \tag{8}$$

We can correct for finite-width *(9)* using the following:

$$Y = 1/[1 - (2a/W)^2]^{1/2} \tag{9}$$

Substituting $a + d_o$ for x, the equation for notched strength is the following:

$$\sigma_N = \sigma_0(1 - \xi_2^{\,2})^{1/2}/Y \tag{10}$$

In eq. 10, the definition of ξ_2 is the following:

$$\xi_2 = a\big(a + d_O\big) \tag{11}$$

Results for the TMP-1 and TMP-3 newsprints show good correlation between the PSC and the experimental values of strength in Figs. 10 and 11, respectively. This suggests that the PSC can also apply to papers containing sharp defects such as shives. In this case, all specimens broke at the slit for all slit sizes. The characteristic value of d_O was 0.9 mm for the TMP-1 paper and 1.0 for the TMP-3 paper. These values are different from the average value of 1.7 mm obtained for the identical MD samples with holes. The difference in the value of d_O between the specimens containing holes and slits is due to the difference in the extent of the elevated stresses perpendicular to the slit direction even when the tensile stress d_O from the tip of the flaw is identical. The PSC as presented in Eqs. 7 and 10 is useful because of its simplicity. We have not tried to obtain an improved criterion by considering the full two-dimensional stress field. Based on the results presented here, each cat-

KEYWORDS
Cracks, criteria, defects, dimensions, failure, forecasts, fracture, holes, runnability, tensile strength.

egory of flaw must have its own value of d_O. A more conventional fracture-mechanic approach *(2–4)* can also apply for materials containing sharp cracks.

CONCLUSIONS

The PSC is a useful method of predicting the strength of specimens containing holes and slits of various sizes. Because it is simple and involves routine testing methods and equipment, it may be a useful quality control measure for mills such as newsprint mills concerned about paper breaking from holes during converting. The value of d_O can be measured for specimens containing holes of a single size. The PSC allows prediction of the effect of holes of various sizes on the remote tensile stress at failure. The evidence suggests that one must evaluate separately the value of d_O for both holes and slits and for MD and CD samples. TJ

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