PEER REVIEWED

# A comparison of two test methods for estimating the fracture energy of paper

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## ABSTRACT

*Two tests for estimating the specific fracture energy of paper are compared. The "essential work of fracture" (EWF) method and STFI's J-integral method—based on one notch length—reflect fiber length and the proportion of chemical reinforcement pulp in the furnish quite similarly. However, the two methods differ in the effect of beating. Both measures are proportional to each other for brittle papers, but this proportionality does not hold for tough, ductile papers, for which the J-integral gives much lower fracture energy values. The EWF decreases more with hard-nip calendering than the J-integral. These differences are partly explained by the fact that the two methods measure different quantities. The J-integral estimates the critical value of the total energy release rate, whereas the EWF measures the energy release rate reaching the inner fracture process zone. Another possible explanation for the differences is that the fracture resistance of the material changes with crack growth.*

***Application:***

*A theoretical and practical review of two methods for characterizing crack propagation in sheets under stress. Differences in the results for both methods are explained, and the authors discuss the relevance of both measures as estimates of web durability.*


THE STRENGTH OF AN OBJECT CAN be greatly reduced by defects. This is because stress concentrations develop in the vicinity of flaws when the object is loaded. A crack propagates if there is enough energy available to overcome the material's resistance in the fracture process zone. This resistance is "specific fracture energy," a material property.

The specific fracture energy of a material is independent of the specimen geometry and dimensions. However, the amount of energy *available* for crack propagation does depend on specimen geometry, crack size, and external load. With a small increment in crack growth, some potential energy is released. The crack cannot propagate unless this released energy is at least equal to the energy needed to overcome the resistance of the material. The condition for crack propagation in a linearly elastic body with a crack of length $a$ is then

$$G = -(d\Pi/dA) = \beta \sigma^2 a/E \geq R \qquad (1)$$

where

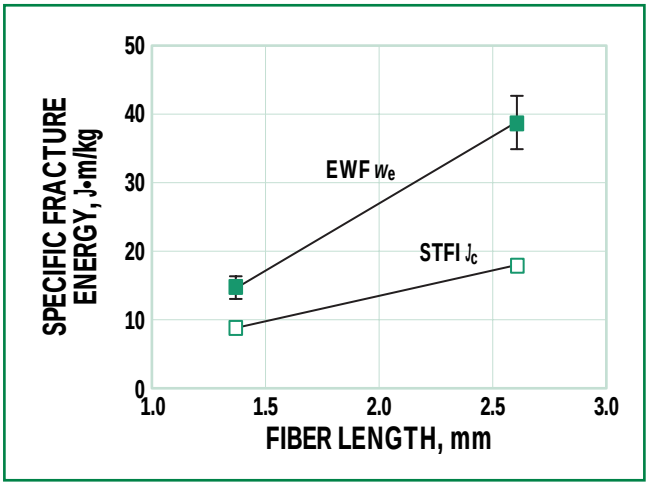
$G$ = energy release rate (or the "crack driving force")
$d\Pi$ = change in potential energy with a small increase of crack area $dA$
$\beta$ = a geometrical factor, which can be calculated given the dimensions of the object *(1, 2)*
$\sigma$ = remote stress far from the crack
$E$ = Young's modulus
$R$ = specific fracture energy of the material.

The specific fracture energy equals the critical value of the energy release rate $G$, i.e., $G_c$. The square root of the product of the specific fracture energy and Young's modulus is the critical stress intensity factor, which is also called fracture toughness.

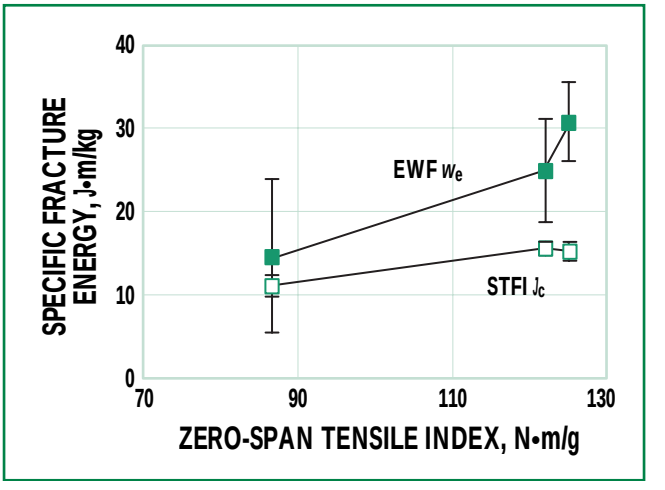
Equation 1 is the basis for what is commonly called linear elastic fracture mechanics (LEFM). The relative simplicity of the LEFM analysis has brought about a few applications for the measurement of the specific fracture energy of paper *(3–6)*. Some of the results have shown a capability of predicting the failure of test specimens with different dimensions as well as commercial paper webs *(5, 6)*. However, to achieve unbiased results—despite unrealistically assuming that the material is linearly elastic up to failure—requires that the specimens be carefully designed and large enough to confine plastic deformations to the vicinity of the crack tip *(4, 5, 7–10)*.

Even with considerable plastic deformation near the crack tip, the energy release rate can be described as a line integral around the crack tip *(11)*. The $J$-integral can be evaluated as a function of displacement on the basis of the load–elongation curves of samples with cracks of differing lengths *(12)*. Alternatively, one may elongate a few specimens for various crack extensions, calculate $J$ for any extension, and then estimate a critical "near-initiation" $J$ at a small crack growth *(2, 13, 14)*. To avoid the work of analyzing specimens with different crack sizes, a few applications for approximating $J$ from specimens with one crack size have also been attempted *(15–17)*.

A $J$-integral application for measuring the specific fracture energy of paper has been recently developed by the Swedish Pulp and Paper Research Institute *(18–20)*. First a nonlinear constitutive material model is created from the load–elongation curves of ordinary tensile specimens assuming anisotropic elasticity and isotropic strain hardening. The $J$-integral for a




*1. Effect of fiber length on the fracture energy estimates*




*2. Effect of fiber strength on the fracture energy estimates*

center-notched tensile specimen as a displacement function is then calculated using numerically determined geometry factors *(20)*. The critical value of the *J*-integral is determined at the displacement where the center-notched tensile specimen shows greatest resistance. A device implementing such a test is now used to study the effect of fiber properties and furnish design on the specific fracture energy of paper.

When plasticity is excessive, single-parameter fracture mechanics breaks down. In other words, the specific fracture energy becomes an ill-defined quantity because much energy is expended on plastic deformations even far from the crack tip. The total work needed to extend a crack depends on the geometry and size of the object, and the specific fracture energy cannot be described by a critical energy release rate as in Eq. 1 *(21–23)*. One single material parameter cannot describe the conditions of failure even if plasticity effects near the crack tip would be included. However, excluding plasticity effects remote from the fracture process region, the local circumstances necessary for crack propagation are solely determined by local material properties if an autonomous end region develops at the crack tip. Then one can define the specific essential work of fracture ($w_e$), a material property for given thickness, as

$$w_e = (W_f - W_p)/Lt \qquad (2)$$

where

$W_f$ = total work consumed by fracture
$W_p$ = work consumed by plastic deformations outside the autonomous end region (nonessential work)
$L$ = crack propagation length
$t$ = thickness of the specimen.

The condition for failure now becomes

$$G_e \geq W_e \qquad (3)$$

where

$G_e$ = rate of energy release reaching the end region.

For a few materials and a given specimen thickness, the specific essential work of fracture has been shown to be independent of specimen geometry *(24–26)*. However, only some geometries enable experimental separation of the essential and nonessential work of crack propagation. One such geometry is the double-edge notched tension specimen (DENT), whose area of plastic deformation is confined to a semicircular region of diameter $L$ (unnotched ligament length). The work consumed by plastic deformations is thus proportional to the square of the ligament *(23)*. The specific essential work of fracture for the DENT geometry can then be written as

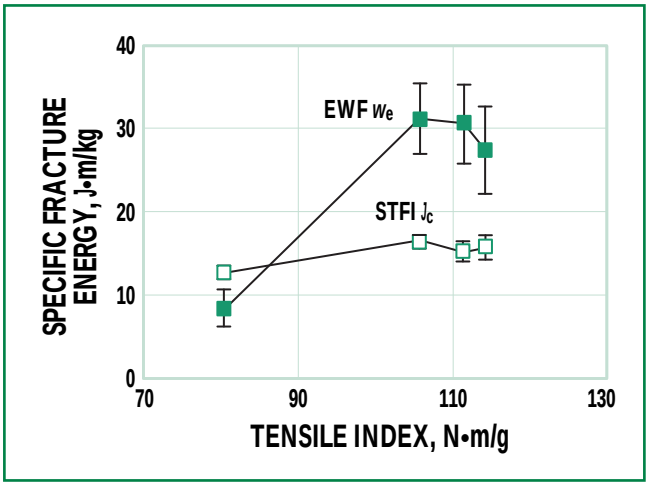
$$W_e = (W_f - W_p)/Lt = w_f - (bLw_p) \qquad (4)$$

where

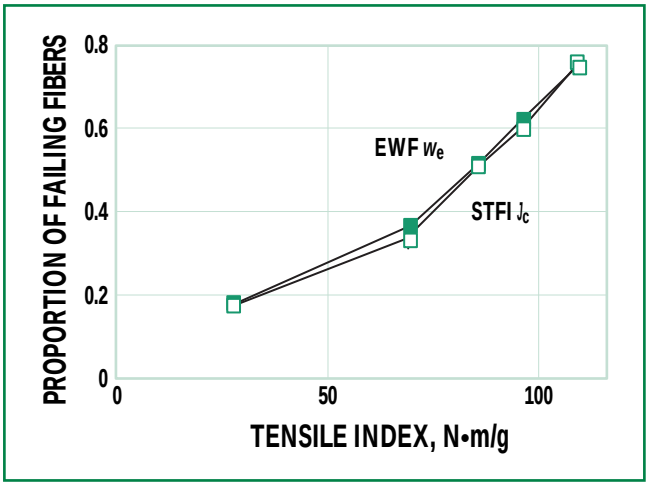
$w_f$ = specific total work of fracture
$b$ = a shape parameter of the plastic region
$w_p$ = specific nonessential work of fracture.

Extrapolating $w_f$ to zero ligament length will obviously yield $w_e$. For Eq. 4 to hold, the ligament must yield prior to crack propagation, ensuring the quadratic dependence of $W_p$ on $L$. The fracture process must be stable to ensure that plastic deformation consumes the external work done.

Even for high-toughness and low-yield-stress materials, the total work of fracture would be independent of specimen dimensions if they were large compared with the size of the plastic deformation zone. The plastic work can then be described as a small deviation from linear elasticity, and the specific fracture energy of the material can be viewed as the critical value of the total energy release rate. This equals the critical value or the *J*-integral, provided that the integration path is taken far from the crack, yielding the critical value of the total energy release rate.




*3. Effect of beating on the fracture energy estimates*




*4. Proportion of breaking fibers in the two fracture energy tests*

$$J_c = -(d\Pi/dA)_c = dW_f/dA = d[(Ltw_e) + W_p]/dA \qquad (5)$$

We denote the elastic–plastic total energy release rate with $-(d\Pi'/dA)$ instead of $-(d\Pi/dA)$, since some energy is now expended for irrecoverable deformations taking place prior to crack propagation, and this energy is not released at the moment of crack extension. The critical value of this rate and the specific essential work of fracture are clearly two different quantities, with $J_c \geq w_e$. However, these two properties are related, since the essential work of fracture equals the critical value of $J$ where the integration path runs just around the autonomous end region, or where the material is linearly elastic *(1, 25–28)*. Where the integration path runs around the plastic deformation zone of a nonlinear material, $J_c > w_e$. In the case of low-yield-stress and high-toughness materials, the experimental determination of $J_c$ may require very large specimens or complex calculations, the latter applying to the presently employed test. The specific essential work of fracture has recently been used with ductile paper grades (29–31). However, satisfying the prerequisites of Eq. 4 may be difficult for brittle grades. Further, the latter test is somewhat cumbersome in practice.

In this paper, we will compare the *J*-integral application developed by the Swedish Pulp and Paper Research Institute (STFI $J_c$) and the specific essential work of fracture (EWF, or $w_e$). We will study the effects of fiber length, fiber strength, pulp beating, proportion of reinforcement pulp, as well as paper calendering on the fracture energy estimates produced by these two methods. Finally, the relevance of these measures in estimating paper web durability is discussed.

## RESULTS

### *Fiber length*

Theoretical analysis shows that the contribution to fracture resistance from a fiber should depend linearly on fiber length in lightly bonded sheets (where few fibers fail) and linearly on the work of fiber failure with heavily bonded fibers *(32, 33)*. Multiplying the average contribution to fracture resistance from a fiber by the number of fibers per unit crack length gives the fracture resistance of paper.

There is some evidence that the fracture energy of paper is linearly dependent on fiber length whether evaluated by the $w_e$ *(31, 34, 35)* or by the STFI *J*-integral *(32, 36)*. Let us now study if the magnitude of the fiber-length dependency differs between methods. A simple experiment was conducted by guillotining beaten, fractionated, unbleached softwood kraft fibers to reduce fiber length. Handsheets made of shortened and non-shortened fibers were tested by each method. **Figure 1** shows the result, the specific fracture energy being indexed by paper density.

The $w_e$ seems somewhat more sensitive to fiber length. As fiber length increased by 114%, the $w_e$ fracture energy estimate increased from 112% to 228%, while the STFI $J_c$ estimate increased from 85% to 125%, considering the 95% confidence limits of fracture energy increment. Although, the $w_e$ seems more sensitive, the difference is not great, since the relative changes given by the confidence limits overlap. The absolute values of $w_e$ are also greater than the STFI $J_c$-values, disagreeing with Eq. 5 but agreeing with previous observations *(19, 29, 37)*. This discrepancy appears systematically in the results presented here.

It is quite possible that the fracture work consumed within the autonomous end region depends heavily on fiber length. Where the fibers do not break extensively, fiber length may give a measure of the width of the inner fracture process zone. The work consumed by plastic deformations in the outer plastic zone, however, may not depend on fiber length, and in such circumstances the *J*-integral should be less sensitive to it.

### *Fiber strength*

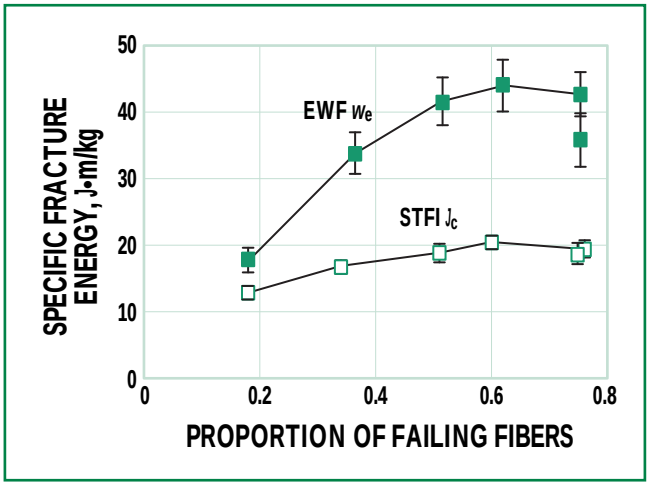
From Eq. 1, we see that the work required to break a brittle object is directly proportional to the square of the critical tensile stress and inversely proportional to the tensile stiffness. This also applies to fiber fracture

work, fiber strength, and fiber stiffness. The work required to break a fiber may thus depend on the square of fiber strength or linearly on fiber strength if this and fiber modulus are correlated. Some experimental observations seem to interpolate between these mechanisms, since the fracture energy of paper has been shown to depend on fiber strength to a power of 1.6 *(31)*. However, a recent study *(32)* could not find any clear effect of fiber strength on the specific fracture energy, although the in-plane tear strength was affected *(38)*.

To see if these measures of paper fracture energy differ in this respect, a vaporizing experiment with HCl was conducted *(31, 39–41)*. Whole handsheets were exposed to vapor in a closed glass container, aerated, and conditioned for 24 hours before testing. The results in **Fig. 2** confirm the previous findings that the effect of the fiber strength on the STFI $J_c$ fracture energy estimate is not great *(32)*. The zero-span tensile index increases by 44%, and the fracture energy estimate increases by 14–65%, with the mean values showing a 37% increase. The very wide confidence limits of the $w_e$ estimate make it difficult to compare the results between tests. However, the mean values represent an increase in fracture energy of 109% with the 44% fiber-strength increment. Thus the EWF $w_e$ result seems to follow the theoretical argument given by Eq. 1: The acid vapor does not affect the tensile stiffness of the fibers, and the change in specific fracture energy of paper equals the square of the change in the zero-span tensile index. However, one must keep in mind that the decrement of fiber strength is likely to affect the relative tendencies to fiber failure and debonding. We do not know the proportion of failing fibers in the vaporization experiment.

With well-bonded fibers, weakened with acid vapor, fiber failure probably dominates the work consumed in the essential fracture process zone. On the other hand, the work consumed by plastic deformations in the outer plastic zone does not necessarily depend on the square of fiber strength.

Equation 5 thus might explain the differing effect of fiber strength on the fracture energy estimates, the *J*-integral characterizing the total fracture work instead of the work done in the autonomous end region, which is the only part considered by $w_e$.




*5. Effect of beating on the fracture energy estimates as a function of the proportion of failing fibers*

### *Beating*

Most experimental observations *(3, 31, 42, 43)* confirm the recent theoretical prediction *(32, 33)* that the specific fracture energy of paper should first increase with increased fiber bonding, reach a maximum, and then decrease. This theory examines phenomena in the essential fracture process zone, but we can reasonably assume that the fracture work criteria should correlate. However, some recent observations with the STFI *J*-integral indicate that beating only slightly increases fracture energy *(32)*.

The effect of Valley beating on the specific fracture energy estimates as a function of tensile index for bleached softwood kraft handsheets is shown in **Fig. 3**. We find that the effect of beating is more considerable on the EWF $w_e$ than the STFI $J_c$. The $w_e$ estimate increases more than threefold, but the STFI *J* estimate increases only slightly.

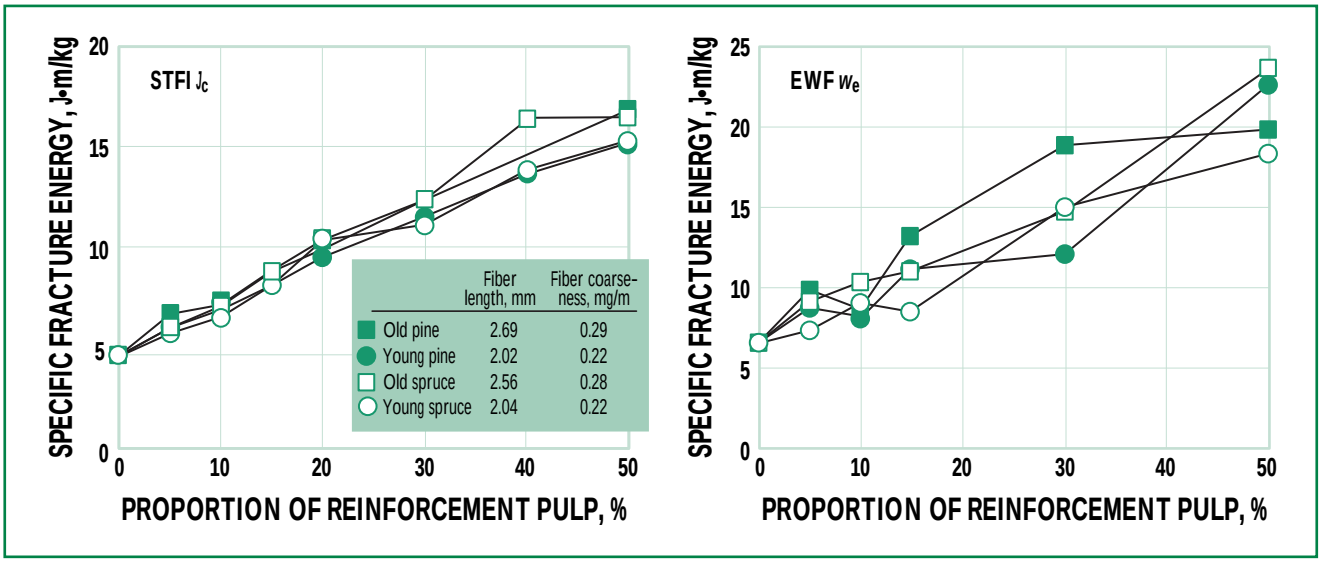
The large difference in the effect of beating on the fracture energy estimates might be explained by differences in test configuration. The shape of the stress field does differ. If the actual crack tip conditions differed enough to change the microscopic mechanisms of failure, this should be reflected in the frequency of fiber failure.

Another beating series was conducted, 2% of the fibers being dyed with Congo Red. **Figure 4** shows that the proportion of failing fibers between the two tests coincides. Thus the discrepancy shown in Fig. 3 probably results from something other than a major difference in the actual crack tip conditions. The specific fracture energy of this beating series is shown as a function of the proportion of failing fibers in **Fig. 5**. We find that there is indeed a maximum in fracture energy as a function of beating, but not necessarily where 50% of the fibers fail *(42)*. However, it is difficult to find the maximum using the STFI *J*-integral for measuring fracture energy.

Previous observations on the effect of pulp beating, produced by other estimates of specific fracture energy, agree more with the $w_e$ than with the STFI $J_c$ *(19, 42–44)*.

### *Reinforcement pulp and calendering*

Recent results *(36)* indicate that the critical value of the STFI *J*-integral of paper made from a mixture of pressure groundwood and softwood chemical pulp is a linear function of the chemical pulp content *(45–49)*. In that study, the fracture energy was evaluated by the STFI $J_c$ method, and the treatment has now been verified with the EWF. We find in **Fig. 6** that the differences found between different




*6. Effect of the proportion of four reinforcement pulps on the fracture energy evaluated with the STFI $J_c$ and the EWF $w_e$ for lightly calendered papers (30 kN/m)*

reinforcement pulps with the STFI $J_c$ cannot be distinguished with the EWF, obviously because of the greater scattering of the $w_e$ estimate. Despite 36 ligaments torn in each test—the coefficient of variation of the total work for each ligament length being 5–10%—the 95% confidence limits for the $w_e$ estimate were typically ±10–20%.

Blending softwood kraft pulp with mechanical pulp increases the results of both fracture energy measurements over threefold, with the proportion of the reinforcement pulp increasing from zero to 50% (Fig. 6). The tested sheets had been lightly calendered in a laboratory calender. As a function of the proportion of chemical softwood pulp, the results are highly correlated and proportional to each other through a constant. However, **Fig. 7** shows that this proportionality changes with calendering load, and it also changes with increasing specific fracture energy.

Heavy hard-nip laboratory calendering appears to decrease the $w_e$ estimate more than the STFI $J_c$ estimate, as seen in Fig. 7. Thus the essential work of fracture in the autonomous end region is more affected by such treatment than the total work of fracture. In this particular case, the calendering did not affect tensile stiffness greatly, but it did decrease tensile rupture strain and tensile strength considerably. The effect of calendering on the fracture energy estimates might change with changed calendering conditions *(50–52)*.

Figure 7 also shows that observations follow the theoretical prediction of $J_c > w_e$ given by Eq. 5 only for the most brittle papers (mostly heavily calendered). Still more remarkable is that tough papers, like the beaten softwood kraft handsheets and sack papers, show the most considerable difference in the fracture energy estimates, the estimate of $J_c$ being much smaller than $w_e$. This discrepancy is in agreement with the development of the fracture energy estimates as a function of beating of softwood kraft pulp fibers (Figs. 3 and 5). In principle, either of the two methods may yield biased results, but in the case of tough, ductile papers, the specific essential work of fracture behaves like a material property, since it equals the specific fracture energy determined from displacement of the crack tip opening *(29, 30)*. However, it may be very difficult to achieve stable cracking and semicircular plastic zone shape in EWF testing of brittle papers.

KEYWORDS
*Fracture, cracks, break resistance, test methods, energy, release, paper webs, durability, fiber length, toughness, bibliographies, ductility, geometry, deformation, STFI, R and D, beating, plastic deformation.*

## DISCUSSION

The critical value of the *J*-integral as measured by the STFI method and the specific essential work of fracture, $w_e$, are two different properties. $J_c$ describes the critical value of the total energy release rate, while $w_e$ depicts the critical release rate of energy reaching the inner fracture process zone, thus excluding work consumed for plastic deformations elsewhere.

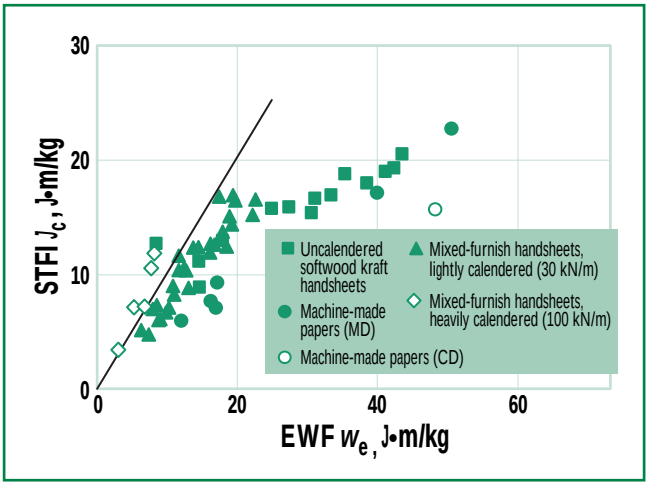
Which definition of the specific fracture energy is most relevant from the point of view of the runnability of paper? Consider a single loading situation: The plastic deformations outside the inner fracture process zone absorb energy, thus increasing the resistance of the paper. However, if the paper is subjected to cyclic loading so that the vicinity of a flaw yields plastically well before the load impulse that finally leads to failure, the resistance provided

by the autonomous end region dominates the actual resistance of the paper. In other words, if no loading, deloading, and reloading precedes the critical process stage, $J_c$ may be more relevant. If the critical process stage occurs after intensive prestraining causing yielding, $w_e$ describes the resistance of paper better.

The fracture energy estimates differ with changes in fiber properties and papermaking treatments. This seems to reflect the principal difference between the methods. Fiber length contributes to the width of the inner fracture process zone, and fiber strength contributes to the critical load within this zone. The essential work of fracture, $w_e$, is more sensitive to these fiber properties. However, there is a considerable discrepancy between the theoretical arguments and the experimental material. According to Eq. 5, $J_c > w_e$, but experimental observations disagree with this relationship (cf. Figs. 1–3 and 5–7 and the literature *(19, 29, 37)*).

The *J*-integral method discussed in this study approximates the displacement where crack growth "initiates" from the greatest load resistance of the notched specimen. In brittle materials, to which LEFM is applicable, crack propagation generally does initiate at maximum load, but this is not so in ductile materials in which subcritical crack growth may precede the maximum load *(22, 53)*. The *J*-integral method studied here seems to work best with brittle papers; with tough, ductile papers it obviously underestimates the fracture energy (Fig. 7).

In ductile fibrous materials, the entire concept of crack growth is somewhat ill-defined. Localized damage occurs in the fracture process zone, but fibers behind the tip of the process zone still bridge the opening crack *(54)*. This is often seen in EWF specimens in which fibers hanging over the crack may induce considerable cohesion, even if the external strain is large and there is a clearly visible displacement between the two parts of the specimen. Macroscopically, this leads to a "rising *R*-curve," i.e., the fracture resistance of the material increases as a function of crack growth.




*7. Correlation between the EWF $w_e$ and the STFI $J_c$ estimates for different kinds of papers. [$J_c > w_e$ only in the case of heavily calendered (and thus brittle) papers.]*

Fracture resistance determined at the moment of instability is not strictly a material property, since it depends not only on the shape of the *R*-curve, but also on the "driving force curve," which in turn depends strongly on the test configuration (*(2)*, p. 46). Thus results determined from the point of instability are not generally transferable to other kinds of structures of the same material.

At large crack extensions, even rising *R*-curves typically turn flat, and the resistance of the material no longer changes with crack growth (*(2)*, p. 48). The EWF test measures this asymptotic steady-state fracture resistance *(25, 27, 55)*, which can more easily be interpreted as a material property. This difference between the methods appears to be a logical explanation for the common experimental observation $w_e > J_c$.

The results of the two methods of measurement differ most clearly in the case of tough, ductile papers (Fig. 7). The variation in the effects of fiber strength and beating (Figs. 2, 3, and 5) also seem to be associated with ductility. The more ductile the paper, the greater the difference between the *J*-integral and the EWF-method-based estimates for the specific fracture energy. The stronger and the more beaten the fibers are, the more ductile the paper, and the greater the difference between the fracture energy estimates. TJ

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LITERATURE CITED


1. Kanninen, M. F. and Popelar, C. H., *Advanced Fracture Mechanics*, Oxford University Press, New York, 1985.
2. Anderson, T. L., *Fracture Mechanics: Fundamentals and Applications* (2nd edn.), CRC Press, Boca Raton, FL, 1995.
3. Seth, R. S. and Page, D. H., *Tappi* 58(9): 112(1975).
4. Seth, R. S., *Tappi* 62(7): 92(1979).
5. Swinehart, D. and Broek, D., *J. Pulp Paper Sci.* 21(11): J389(1995).
6. Swinehart, D. E. and Broek, D., *Tappi J.* 79(2): 233(1996).
7. Gurney, C. and Hunt, J., *Proc. Royal. Soc.* A299: 508(1967).
8. Seth, R. S. and Page, D. H., *J. Matl. Sci.* 9: 1745(1974).
9. Mai, Y.-W., *J. Matl. Sci.* 10: 1271(1975).
10. Seth, R. S. and Page, D. H., *J. Matl. Sci.* 10: 1273(1975).
11. Rice, J. R., *J. Appl. Mech.* 35(6): 379(1968).
12. Begley, J. A. and Landes, J. D., "The J-integral as a fracture criterion," in *Fracture Toughness*, ASTM STP 514, American Society of Testing and Materials, Philadelphia, 1972, pp. 1–20.
13. "Standard test method for Jic, a measure of fracture toughness," ASTM Standard E 813-88, American Society of Testing and Materials, Philadelphia.
14. Mai Y.-W., He, H., Leung, R., *et al.*, "In-plane fracture toughness measurement of paper," in *Fracture Mechanics*, ASTM STP 26(1256), American Society of Testing and Materials, Philadelphia, 1995, pp. 587–99.
15. Rice, J. R., Paris, P. C., and Merkle, J. G., "Some further results of J-integral analysis and estimates," ASTM STP 536, American Society of Testing and Materials, Philadelphia, 1972, pp. 231–45.
16. Liebowitz, H. and Eftis, J., *Eng. Fract. Mech.* 3: 267(1971).
17. Yuhara, T. and Kortschot, M. T., *J. Matl. Sci.* 28: 3571(1993).
18. Fellers, C., Fredlund, M., and Wågberg, P., *Tappi J.* 75(4): 103(1992).
19. Fellers. C., *New Available Techniques and Current Trends*, SPCI and ATICELCA Conference Proceedings, SPCI, Stockholm, 1992, part 2, pp. 356–72.
20. Wellmar, P., Fellers, C., Nilsson, F., *et al.*, *J. Pulp Paper Sci.*, 23(6): J269(1997).
21. Broberg, K. B., *Intern. J. Fract.* 4(1): 11(1968).
22. Broberg, K. B., *J. Mech. Phys. Solids* 19: 407(1971).
23. Cotterell, B. and Reddel, J. K., *Intern. J. Fract.* 13(3): 267(1977).
24. Mai, Y.-W. and Cotterell, B., *Eng. Fract. Mech.* 21(1): 123(1985).
25. Mai, Y.-W. and Cotterell, B., *Intern. J. Fract.* 32: 105(1986).
26. Mai, Y.-W., and Powell, P., *J. Polymer Sci. B* 29: 785(1991).
27. Cotterell, B. and Mai, Y.-W., "Plane stress ductile fracture," in *Advances in Fracture Research* (D. Francois, Ed.), Vol. 4, Pergamon Press, Oxford, U.K., 1981, pp. 1683–95.
28. Mai, Y.-W., *Intern. J. Mech. Sci.* 35(12): 995(1993).
29. Seth, R. S., Robertson, A. G., Mai, Y.-W., *et al.*, *Tappi J.* 76(2): 109(1993).
30. Seth, R. S., *Tappi J.* 78(10): 177(1995).
31. Seth, R. S., *Tappi J.* 79(1): 170(1996).
32. Kärenlampi, P., Niskanen, K., and Alava, M., *1995 International Paper Physics Conference Proceedings*, CPPA-TS, Montreal, pp. P39–46.
33. Niskanen, K., Kärenlampi, P., and Alava, M., *J. Pulp Paper Sci.* 22(10): J392(1996).
34. Peng, Y.-X. and Seth, R. S., *China Pulp Paper* 8(4): 10(1994) (in Chinese).
35. Peng, Y.-X., *Appita* 48(5): 343(1995).
36. Kärenlampi, P., Rantanen, R., Hämäläinen, T., *et al.*, *TAPPI J.* 80(4): 119(1997).
37. Åström, A., Nordström, A., Rost, T., *et al.*, *Transactions of the Tenth Fundamental Research Symposium*, Mechanical Engineering Publications, London, 1993, pp. 807–23.
38. Kärenlampi, P., Suur-Hamari, H., Alava, M., *et al.*, *TAPPI J.* 79(5): 203(1996).
39. Seth, R. S. and Page, D. H., *Tappi J.* 71(2): 103(1988).
40. Seth, R. S., *MRS Symposium Proceedings*, Vol. 197, Materials Research Society, Pittsburgh, 1990, pp. 125–41.
41. Page, D. H., *Tappi J.* 77(3): 201(1994).
42. Kortschot, M. T., Trakas, G., and Kazi, S., *1994 International Paper Physics Seminar Proceedings*, TAPPI PRESS, Atlanta.
43. Kazi, S. M. and Kortschot, M. T., *TAPPI J.* 79(5): 197(1996).
44. Uesaka, T., Matoba, M., Murakami, K., *et al.*, *Design Criteria for Paper Performance: Progress in Paper Physics Seminar Proceedings*, STFI, Stockholm, 1984, pp. 110–13.
45. Ritala, R., *Nordic Pulp Paper Res. J.* 2[special issue (Aug.)]: 15(1987).
46. Ritala, R. and Huiku, M., *Fundamentals of Papermaking: BPBIF Transactions of the 9th Fundamental Research Symposium*, Mechanical Engineering Publications, London, 1989, pp. 195–218.
47. Kärenlampi, P., *J. Pulp Paper Sci.* 22(1): J31(1996).
48. Alava, M. J. and Niskanen, K. J., *J. Pulp Paper Sci.* 22(11): J409(1996).
49. Kärenlampi, P., *J. Pulp Paper Sci.* 22(11): J410(1996).
50. Lyne, M. B., *BPBIF Transactions of the 6th Fundamental Research Symposium*, Mechanical Engineering Publications, London, 1978, Vol. 2, pp. 641–69.
51. Soszynski, R. M. and Seth, R. S., *PICONEX '85 International Packaging Conference Proceedings*, 1985, pp. 590–604.
52. Charles, L. A. and Waterhouse, J. F., *J. Pulp Paper Sci.* 14(3): 59(1988).
53. Broberg, K. B., *J. Mech. Phys. Solids* 23: 215(1975).
54. Li, Y. S. and Duxbury, P. M., *Phys. Rev. B* 38(13): 9257(1988).
55. Mai, Y.-W. and Cotterell, B., *J. Matl. Sci.* 15: 2296(1980).