Growth and decay of streaks and waves on the forming table

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ABSTRACT

Surface waves and streaks on the forming table can lead to nonuniform basis weight and moisture profile across the sheet. In this study, we analyze the effects of viscosity and drainage on the decay of surface waves and streaks on the forming table. Large scale variations in physical properties that scale with the width of the paper machine are relatively well understood. In contrast, the cause of the small scale nonuniformities in the physical properties are not well understood. The length scales we refer to as "small scale" in this study are relative to the machine width and range from 0.2 m to 0.5 m. Specifically, ultrasonic stiffness measurements of linerboard samples from commercial paper machines show small scale fluctuations in addition to the overall variations in the cross-machine direction. Analysis of the paper machine shows no physical features with the same length scale that could cause these nonuniformities except the wet streaks at the dry line on the forming table. We show the mechanism by which the streaks on the forming table evolve to form the characteristic nonuniformities in basis weight, moisture, and stiffness properties.

Application:

Examines the effects of furnish viscosity and drainage on the formation of wet streaks on the forming table.

THERE ARE SEVERAL LENGTH scales associated with the physical characteristics of commercially manufactured paper and board. The largest of these in the cross-machine direction (CD), obviously, is the width of the paper machine itself. Most paper manufactured today has variations in physical properties with this length scale. The causes include overall imbalance in the pressure manifold, edge effects in the headbox, secondary turbulent structures (1, 2), misalignment of the rolls, nonuniform tension, and edge effects in the press and dryer sections. There are a variety of remedies to correct these larger scale variations.

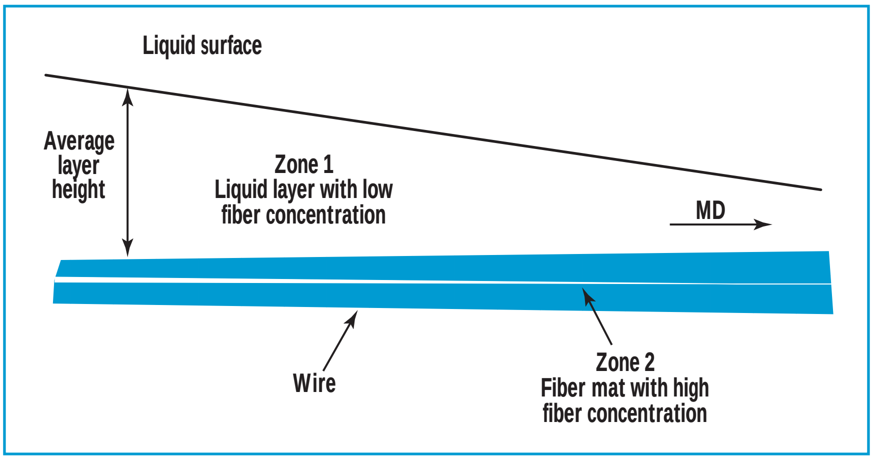
The smaller scale variations in physical properties are not well understood and are, therefore, more difficult to eliminate. In this study, we are concerned with the variations in physical properties in the range of 0.2–0.5 m. Basis weight measurements and ultrasonic stiffness measurements show variations in the order of this length scale. It is not clear at this time if the basis weight variation or the drying rate nonuniformity is the primary cause of the nonuniform elastic stiffness properties. However, the length scale of the fluctuations in physical characteristics is the same as the width of the wet streaks on the forming table.

In particular, extensive measurements of the CD, MD (machine direction), and ZD (direction normal to the paper surface) ultrasonic velocities were collected *(3)* on linerboard samples. The measurements were repeated several times to confirm the existence of small scale fluctuations. The results from similar measurements overlap, showing real variations in the order of 0.2–0.5 m.

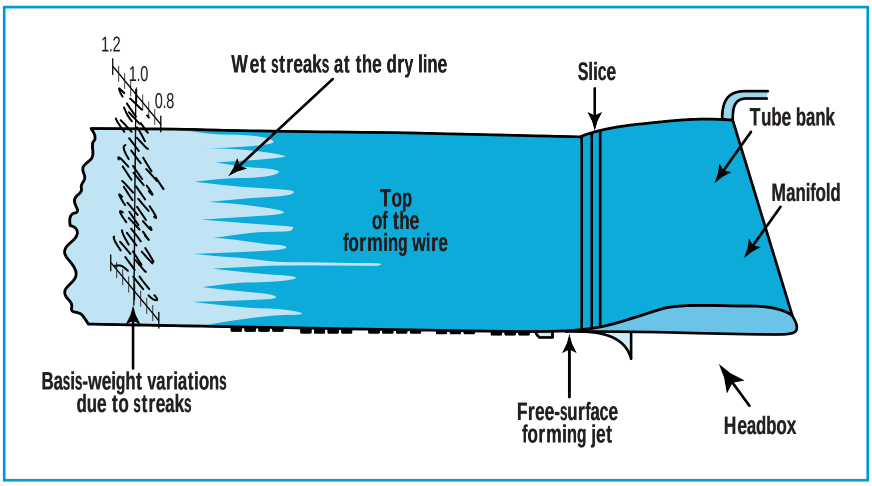
Typical measurements of the ZD ultrasonic velocity of full CD strips are presented by Hall *(3)*. In some cases, the magnitudes of the small scale fluctuation in stiffness and polar angle are as large as the magnitude of the full CD variations. Since the performance and strength of the product usually depend on the magnitude of the variation in physical properties, the small scale fluctuations could be more important than the overall averaged nonuniformities. The small scale fluctuations in elastic stiffness can be caused either by variations in the local sheet density or by the nonuniform drying rate resulting from moisture variation in the sheet. Both phenomena can be caused by the wet streaks on the forming table.

Consider the drainage model on the forming table outlined in **Fig. 1**. Zone 1 has the same fiber concentration as the liquid inside the headbox, while Zone 2 has a much higher fiber concentration (≈5–7%). The dry line forms at the point along the MD where Zone 2 is completely drained. The shape of the dry line, visible on most forming tables, depends on the CD variation in the thickness of Zone 1. If Zone 1 has nonuniform thickness in the CD, then the dry line becomes streaky, as demonstrated schematically in **Fig 2**.

Wet streaks are regions at the dry line that contain more liquid—and with a lower concentration (≈headbox level) of dispersed fiber—than the rest of the layer, as demonstrated in Fig. 2. Assuming that the fibers are uniformly dispersed, this results in variation in mass density of the fibers in the sheet. Also, since the wet streaks contain more liquid with dispersed fiber, the moisture content and drying rate vary according to the width of the streaks. In most cases, the wet streaks result in detectable



1. Cross-sectional view of the liquid layer model on the forming wire



2. Schematic presentation of wet streaks on the forming table

basis weight variations in the sheet. The length and the width of the streaks are important factors in the overall effect on nonuniform physical properties. A very short or narrow streak has no detectable effect on the physical characteristics. In contrast, long and relatively wide streaks result in detectable and often troublesome nonuniformities.

KEYWORDS

Defects, drainage, forming sections, models, moisture content, physical properties, rheological properties, ultrasonic frequencies, viscosity, wave lengths, wet streaks.

Streaks on the forming table are caused by nonuniform or secondary flows in the headbox, the jet interaction with the wire, and the fluid dynamics of the free surface flow on the forming table. With most forming tables, one can immediately see the narrow streaks that form near the slice. These streaks are 1–5 cm wide and correspond to the jets emanating from the tubes inside hydraulic headboxes or the wake of the holes in the slice rectifier roll. The streaks at the dry line are 0.2–0.5 m wide, depending on the headbox, the forming section design, and flow parameters.

In order to eliminate the nonuniformities in physical properties caused by wet streaks in the forming section, we need to understand the origin of streak/wave formation and the physical mechanism of their evolution on the forming table from narrow streaks near the slice to the relatively wide structures at the dry line. In the following section, we explain the physics of streak/wave formation and evolution on the forming table.

STREAK/WAVE ANALYSIS ON THE FORMING TABLE

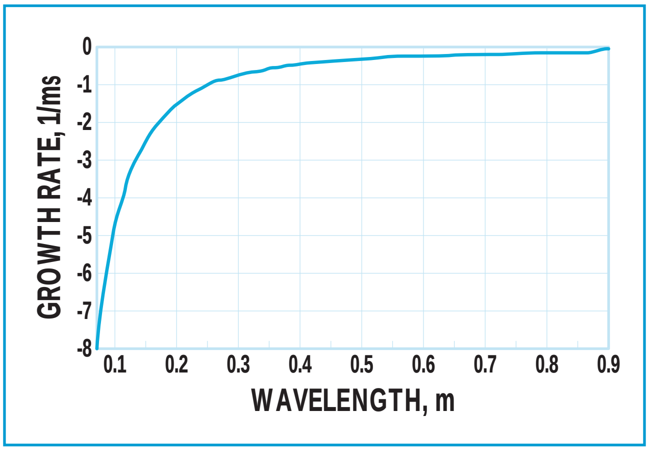
The wave analysis is based on the spectral decomposition of the liquid surface height near the slice and computation of the wave evolution on the wire from the slice to the dry line. The purpose of this analysis is to explain the physical mechanism of the evolution of the apparently narrow streaks near the slice to the relatively wide streaks (0.2–0.5 m) at the dry line.

To analyze the evolution of the waves on the forming table, we use a filtration model of paper forming on the table, as demonstrated in Fig. 1. We distinguish two zones on the wire: one where the fiber mat is forming on the wire, and a second zone where the liquid has a low concentration of fiber (equal to that of the headbox). The analysis takes the amount of fluid transfer through the boundary of Zone 1 to Zone 2 (from drainage rate measurements) to compute the average height of Zone 1. The waves or streaks are variations in the height of Zone 1 in the cross-machine direction.

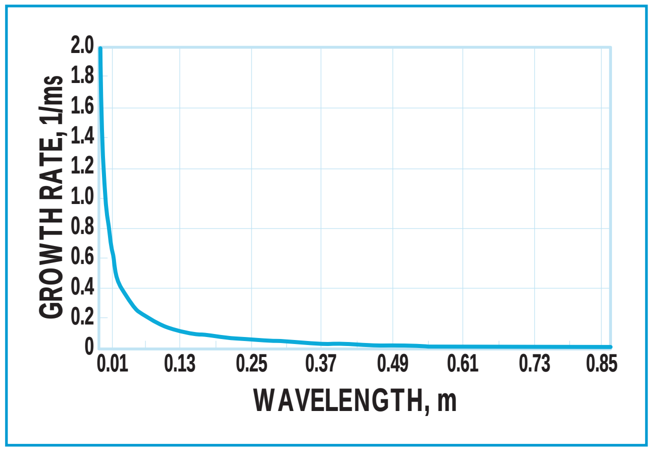
Since the wave amplitude is smaller than the wavelength of the range of streaks with which we are concerned, one can show that in this case, the convective terms in the momentum equation are much smaller than the other terms. Therefore, for this application, the momentum equation can be reduced to

$$\rho \frac{\partial v}{\partial t} = \nabla p + \nabla \cdot [2\mu^* E_{ij}] \qquad (1)$$

(Note: All variables are defined in the Nomenclature section at the end of this paper.)



3. Rate of decay of streaks on the forming table



4. Rate of growth of streaks on the forming table

We use the Prandt's mixing length theory, given by

$$\mu_t = \rho \ell_m [2E_{ij} : \nabla v] \qquad (2)$$

where

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Because of conservation of mass and the incompressibility condition, the velocity vector is a divergence-free function.

The height of the fluid surface, ζ, is expanded in terms of Fourier modes as

$$\zeta(x,y) = \sum_{n=1}^{N} \frac{k_n}{\alpha_n}\left[A_n(e^{k_n\bar{\zeta}} - e^{-k_n\bar{\zeta}}) + iC_n(e^{l_n\bar{\zeta}} - e^{-l_n\bar{\zeta}})\right] \cdot \exp\left(ik_n x + \alpha_n \frac{y}{U}\right) \qquad (3)$$

where

$$k_n = \frac{2\pi}{\lambda_n}$$

$$l_n^2 = k_n^2 + \frac{\alpha_n}{\bar{\nu}_t}$$

The free surface conditions are the balance of tangential and normal stresses along with the dynamic condition at the free surface, which constrains the free surface to a material surface, as it physically is.

The expanded series is substituted in the governing equations, and using the free surface conditions, the relation for the coefficients of the modes is derived. This relation is in the form of a set of nonlinear algebraic equations that are solved using the nonlinear equation solver in IMSL (international mathematical and statistical language) software.

The relation between the exponential growth rate, α, and the wavelength, λ_n, or the wave number, k_n, is given by a nonlinear algebraic equation:

$$f(\alpha, k, \bar{\nu}_t, \bar{\omega}_0, \bar{\zeta}) = 0 \qquad (4)$$

where:

$$\bar{\nu}_t = \nu_{eff} + \nu_t$$

$$\nu_{eff} = \nu\left[1 + \frac{6}{5} r_p^2 \, \phi(\eta_T + 2\bar{\bar{\eta}} + 2\eta_\perp)\right]$$

$$\bar{\omega}_0 = \sqrt{Gk + \frac{\sigma k^3}{\rho}}$$

$$G = g + \bar{\omega}_0'$$

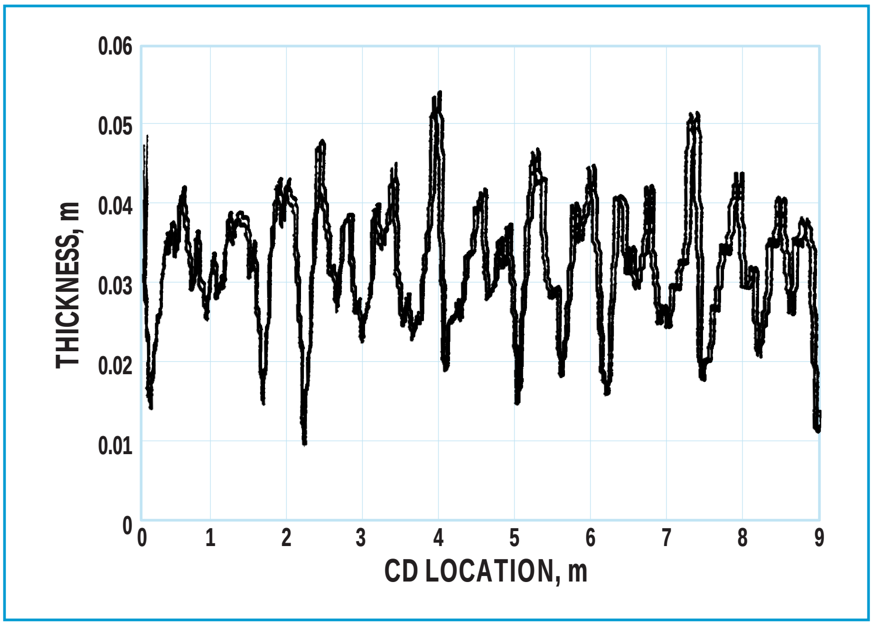
[The equation for ν_{eff} is from the work of Claeys *(4)*.]

RESULTS

By calculating the growth and decay rates of the waves in the machine direction, i.e., the *y* axis, along the forming wire, we compute the surface height of the liquid along the full forming table. Note that the results are functions of several parameters on the forming wire, including the draining rate, the kinematic and turbulent viscosities, the average layer height, the machine speed, and the location of the drainage element. Therefore, the growth and decay rates for each mode vary along the forming table.

The streaks at the slice have a broad spectrum of wavelengths ranging from a few centimeters to more than a meter. The characteristic length scale of the streaks at the dry line is determined by the critical mode, which becomes dominant as the streaks are evolving on the forming table. The reason for this is that the modes with smaller wavelength grow and decay very rapidly before they reach the dry line. The streaks with larger wavelengths take too long to grow, and therefore they are of no consequence. **Figures 3 and 4** show the respective decay and growth rates of the waves at a given MD location on the wire. Only the critical mode grows to its maximum amplitude just in time to reach the dry line and form the dry-line streaks.

For the headbox and the forming table considered in this study, the wave amplitudes on the wire at about 2 m from the slice are plotted in **Fig. 5**. Note that the streaks quickly evolve to the dominance mode, which in this case is about 0.45 m wide. The measurements of the layer thickness on the wire at the same thickness from the slice show good qualitative agreement with the calculations.



5. Computation of surface height across the S8 machine at about 2 m from the slice

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The streaks that are computed at the dry line have the same 0.4–0.5-m width as observed on the machine. Moreover, the results show the same qualitative length scale of fluctuations as were found in the ZD ultrasound measurements obtained from the literature *(3)*. TJ

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NOMENCLATURE

A_n = constant that depends on the *n*th-mode initial disturbance amplitudes
C_n = constant that depends on the *n*th-mode initial disturbance amplitudes
$E_{i,j}$ = rate-of-strain tensor
g = force of gravity
G = $g + \omega_0'$
i = index
k_n = *n*th-mode wave number
l_n = *n*th-mode wave number
ℓ_m = mixing length
p = pressure
r_p = ratio of fiber length to diameter
t = time
u_i = velocity vector
u_j = velocity vector
U = machine speed
v = velocity vector
x_i = position vector
x_j = position vector
x = denotes cross-machine direction
y = denotes machine direction
α_n = *n*th-mode wave amplitude
ζ = height of fluid surface
$\bar{\zeta}$ = average layer thickness (varies in MD because of drainage along the table)
$\bar{\bar{\eta}}$ = function of r_p *(4)*
η_T = function of r_p *(4)*
$\eta_\perp$ = function of r_p *(4)*
λ_n = *n*th-mode wavelength
μ = kinematic viscosity
μ_t = turbulent viscosity
μ^* = sum of kinematic and turbulent viscosities
ν = kinematic viscosity of fiber suspension
ν_t = turbulent eddy viscosity of fiber suspension
$\bar{\nu}_t$ = $\nu + \nu_t$
ν_{eff} = effective kinematic viscosity
ρ = density
σ = surface tension of fluid
ϕ = fiber volume fraction
$\bar{\omega}_0$ = see Eq. 4
$\bar{\omega}_0'$ = drainage rate on wire