

Modeling paper strain in a calender nip

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CALENDERING IS A WEB-FINISHING process that applies uniform nip pressures to increase sheet smoothness and improve the uniformity of thickness. A better understanding of how paper responds to the stresses applied during calendering has practical implications because it could improve process efficiency. Initial efforts to simulate sheet response to calendering were based on standard linear viscoelastic models, which assume that the treated material is smooth, continuous, and uniform. However, Browne *et al.* (1) noted that such models are not well suited for paper, which is a rough, structured material with substantial nonuniformities in thickness and density.

This article—an excerpt from the book *Treatment of Paper Under Pressure* (2)—corroborates the finding that linear viscoelastic models are inadequate for describing paper deformation in the calender nip. Furthermore, the article proposes a nonlinear modification of the standard viscoelastic model based on experimental measurements of paper compression during calendering. The main difference between the proposed model and the standard linear model is that it quantifies sheet deformation nonlinearly as a function of applied stress and the plastic (irreversible) deformation that occurs immediately upon load application.

BACKGROUND

As the web passes through successive calendering nips, its density in-

creases, thereby increasing the web's compression resistance. The principal strain observed during calendering is compression strain perpendicular to the web plane. Web thickness decreases by 50–60% in the nip and recovers 70–80% of the lost thickness after exiting the nip. The calendering process also produces considerable changes in sheet porosity and the modulus of elasticity, E .

A paper sheet can be considered as a frame formed by the fiber layers and their smallest branches (fibrils), which bind the fibrous frame via hydrogen bonds. The porous volume of air in an uncalendered sheet is usually more than 50% of the total volume. In this case, the initial modulus of elasticity of paper is stipulated by the small initial (or origin) paper strain under slight loads. Calendering consolidates the sheet by moving and deforming the fibers in the original sheet structure while displacing air from the pores. The result is an increase in the sheet's modulus of elasticity, E . Previous investigations (2–5) have shown that when calendering pressure is increased, fibrils and fibers are transferred from a freely packed configuration to a compressed structure. The increase in the modulus of elasticity, E , for calendered sheets is nonlinear and depends on the applied stress, σ . Equation 1 is a linear function that approximates the change in E within the range of typical calendering stresses (5–30 MPa).

$$E_0 = E_i + N\sigma \quad (1)$$

ABSTRACT

A method of modeling paper deformation in a rolling nip is proposed. This nonlinear viscoelastic model was developed to predict the strain response of paper subjected to a compressive pressure pulse in a calender nip. The proposed model was verified using data gathered from tests of 110-g/m² writing paper and 150-g/m² paperboard. The analytical description of paper deformation in a calender nip has practical applications and can be used to predict such web characteristics as thickness, density, compressibility, and thickness uniformity after calendering.

Application:

Describes a nonlinear viscoelastic model that predicts sheet deformation in the calender nip.

where

E_i = initial modulus of elasticity, MPa

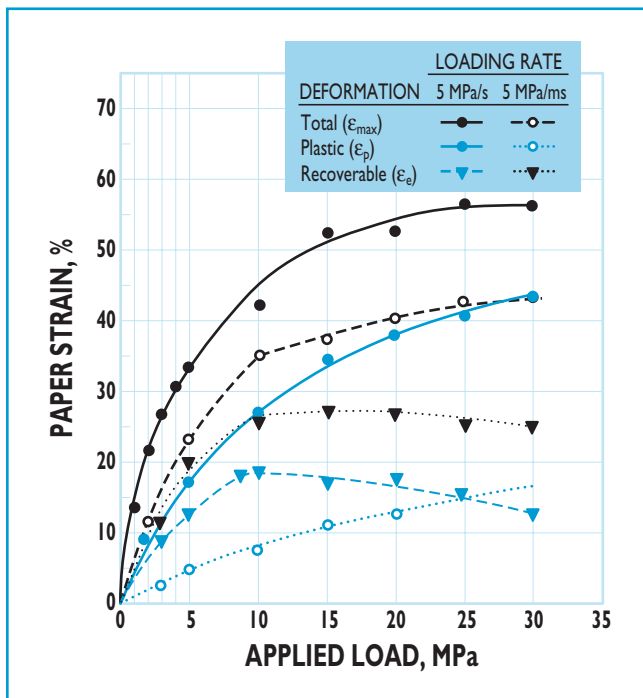
N = angular factor of the modulus

σ = applied stress, MPa

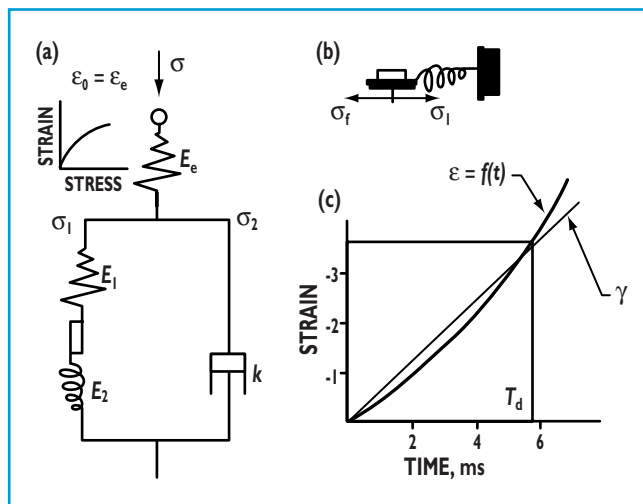
The initial modulus of elasticity and the angular factor of the modulus are determined by the paper type as well as by its moisture and temperature. For instance, for printing paper at 8% moisture and 20°C, Eq. 1 translates to:

$$E_0 = 13 + 2.6\sigma \quad (2)$$

At low pressures where the residual (plastic) strain ϵ_p is less than 10–15%, the modulus of elasticity increases only slightly, and Eq. 2 can still be represented by the linear function but with less angular factor N . Experimental investigations (2) show that even at low pressures that do not exceed a material's limit of elasticity, both plastic (irreversible) and elastic (reversible) strains are developed during paper packing. At



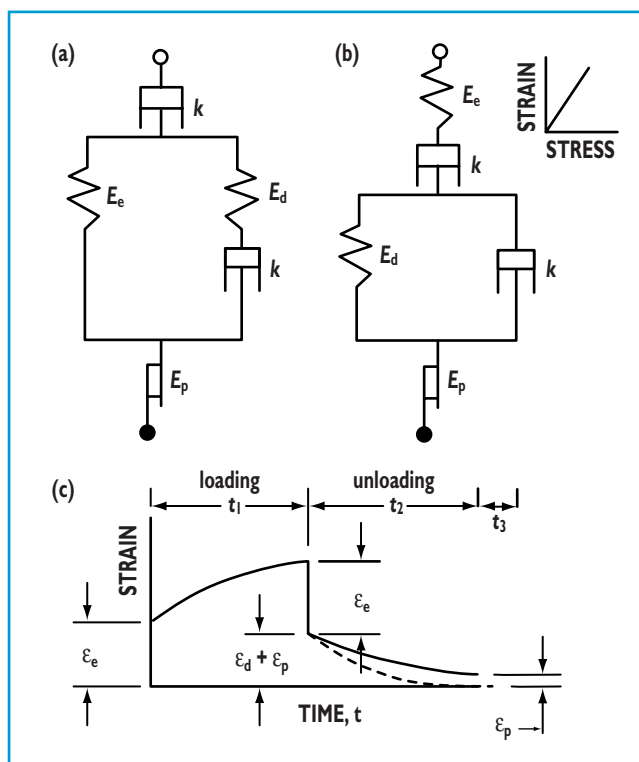
1. Paper strains as a function of applied load for writing paper (110 g/m²) at two loading rates



3. Nonlinear viscoelastic model and method of time-delay definition (top) and a graph showing how to determine the time-delay factor T_d

pressures exceeding the limit of elasticity, the elastic strain tends to decrease while the plastic strain increases. Thus there is a basis to assume that elastic bonds are “re-generated” into plastic ones as a result of microrupture, destabilization of the frame cells, or repacking of the cellulose macromolecules.

The duration of the applied load affects the amount of residual strain in the sheet. **Figure 1** shows the change in total deformation and its constituents (plastic and elastic deformation) with applied load over time intervals of



2. Two linear viscoelastic models (top) and a graph (bottom) depicting strain development over time during load application and unloading

6 s and 6 ms for writing paper (110 g/m²). The relationship between deformation and applied load is nonlinear, with the growth of deformation slowing down as the applied stress increases. It is characteristic that—in contrast to the paper extension in the machine direction at the deformation of the cross compression—plastic and elastic deformation are developed under slight loads applied over short periods of time. As seen in Fig. 1, the total deformation decreased slightly when the duration of the applied load was reduced from 6 s to 6 ms. At the same time, the residual (plastic) deformation decreased substantially at the shorter loading time of 6 ms. Note that despite these differences, the curves for total, plastic, and elastic strains maintained their characteristic shapes under the two regimes. These results, as well as similar results observed for many other types of paper (3, 5), show that the laws governing deformation under long-term or instantaneous loading are similar. Moreover, this similarity can be quantified by the ratio of the long-term modulus of elasticity H and the instantaneous modulus of elasticity E_0 .

With repeated loading cycles, the paper undergoes an increase in rigidity and modulus of elasticity in the direction of loading. When modeling paper deformation during compression under high stress over a wide range of time, one must consider the phenomena of time-delayed recovery of strain and nonlinear deformation from

the applied load. The time-delayed recovery is included in the linear models used by Brecht (3), Meinelcke (4), and others, as seen in Fig. 2. These models explain, to a certain extent, the kinetics of sheet deformation as the applied load on the sheet increases.

Performance of the two models in Figs. 2a and 2b can be explained as follows. During sheet compression, the elastic strain ϵ_e (Fig. 2c) corresponds to the elastic compression E_e . This strain appears instantaneously upon application of the load, as seen in Fig. 2c. Then, during the period of the applied load, t_1 , two other deformation constituents are increased nonlinearly:

1. Delayed elastic deformation ϵ_d , which corresponds to the compression element E_d in the model of Kelvin-Focht, consisting of parallel inserted elastic compression E_e and viscous element k
2. Plastic strain ϵ_p , which corresponds to plastic compression E_p and viscous element k .

As seen in Fig. 2c, from the moment of unloading, t_2 , until the moment t_3 (representing the total relaxation of stresses in the material), the reversible components of the deformation (ϵ_e and ϵ_d) decrease to zero, but the residual strain ϵ_p remains to reflect the irreversible changes in the paper structure after compression. The models for short-time loading include the phenomenon of plasticity only when the external load exceeds a given value. In reality, however, even compression of small magnitude can cause plastic deformation in paper sheets. Another weakness of these models is that they do not reflect the fact that cellulosic materials increase in density with increasing deformation, thus raising the possibility of converting elastic deformation into irreversible plastic deformation when the load or the duration of loading are increased.

The phenomenon of irreversible sheet deformation under small loads can be modeled, as seen in Fig. 3. Deformation is modeled in Fig. 3b by the elastic and plastic elements (that follow the law of dry friction) that are connected in parallel or in sequence, provided that the elastic element is preliminarily stretched with the stress σ_1 , which is equal to the stress of friction σ_f of the plastic component. Then it is obvious that any minimum external action $\Delta\sigma$ will lead to irreversible deformation, as in the following case:

$$\Delta\sigma + \sigma_1 > \sigma_f$$

In the case of unloading, because the stress of tension decreases proportionally with the displacement (extent of deformation), the stress of friction turns out to be greater than the stress of elasticity ($\sigma_f > \sigma_1$) and hinders further displacement. Upon further loading, the residual strain does not increase simultaneously with the elastic strain but, rather, increases only when the applied stress is greater than the difference between the forces of friction and spring tension. With this component included in the linear model of sheet relaxation, the mathematical description corresponds much more closely with the paper reaction at compression. The effect of packing within the sheet, which leads to the growth of the modulus of elasticity as the sheet is compressed, can be taken into account by the nonlinear response of elastic components to the applied stress.

According to this model, the processes of deformation and recovery occur as the plastic deformation and the delayed elastic deformation both begin to grow. After unloading, the process of deformation ceases and the process of recovery begins. The elastic deformation disappears immediately, the delayed elastic deformation recovers gradually, and the

plastic deformation remains. Thus the element of dry friction plays the role of a "lock" when the deformation sign is changed.

DEVELOPMENT OF MODEL

The model under consideration is distinguished by its simplicity, so the mathematical descriptions at loading and unloading are not difficult. The main equation describing deformation for this model is derived from the laws of deformation because of their consistent elements and conditions of equilibrium and continuity.

$$\epsilon = \epsilon_0 + \epsilon_1 + \epsilon_2 \quad (3a)$$

$$\epsilon_d = \epsilon_1 + \epsilon_2 \quad (3b)$$

$$\epsilon_0 = \frac{\sigma}{E_0} \quad (3c)$$

$$\epsilon_1 = \frac{\sigma_1}{E_1} \quad (3d)$$

$$\epsilon_2 = \frac{\sigma_1}{E_2} \quad (3e)$$

$$\sigma = \sigma_1 + \sigma_2 \quad (3f)$$

$$\dot{\epsilon}_d = \frac{\sigma_2}{k} \quad (3g)$$

where

E_0 = instantaneous elastic modulus of elasticity

E_1 = delayed elastic modulus of elasticity

E_2 = modulus of plasticity (characterizes the relationship between the initial internal microstresses in the material and the plastic deformation)

k = viscosity factor

Having eliminated the stresses and deformations relating to the separate elements in the expressions grouped in Eq. 3, we obtain Eq. 4.

$$\begin{aligned} \epsilon_d &= \epsilon_1 + \epsilon_2 \\ &= \frac{\sigma_1}{E_1} + \frac{\sigma_1}{E_2} \end{aligned} \quad (4)$$

when

$$\sigma_1 = \left(\frac{E_1 E_2}{E_1 + E_2} \right) \varepsilon_d \quad (5)$$

Manipulation of the definitions in Eq. 3 leads to

$$k \dot{\varepsilon}_d = \sigma - \sigma_1$$

Having expressed σ_1 in Eq. 5, we can substitute this expression to obtain Eq. 6.

$$k \dot{\varepsilon}_d = \sigma - \left(\frac{E_1 E_2}{E_1 + E_2} \right) \varepsilon_d \quad (6)$$

Bearing in mind that

$$\varepsilon_d = \varepsilon - \left(\frac{\sigma}{E_0} \right) \text{ and } \dot{\varepsilon}_d = \dot{\varepsilon} - \left(\frac{\dot{\sigma}}{E_0} \right)$$

we obtain Eq. 7.

$$k \left[\dot{\varepsilon} - \left(\frac{\dot{\sigma}}{E_0} \right) \right] = \sigma - \left(\frac{E_1 E_2}{E_1 + E_2} \right) \left(\varepsilon - \frac{\sigma}{E_0} \right) \quad (7)$$

Final transformation of Eq. 7 leads to Eq. 8.

$$\varepsilon + k \dot{\varepsilon} \left(\frac{1 + \frac{1}{E_1}}{E_1 E_2} \right) = \sigma \left(\frac{1 + \frac{1}{E_1} + \frac{1}{E_2}}{E_0} \right) + k \left(\frac{1 + \frac{1}{E_1}}{E_1 E_2} \right) \left(\frac{\dot{\sigma}}{E_0} \right) \quad (8)$$

The product

$$k \left(\frac{1}{E_1} + \frac{1}{E_2} \right) = T_d$$

has the units of time and characterizes the deformation time delay with the change in stress.

The sum

$$\frac{1}{E_0} + \frac{1}{E_1} + \frac{1}{E_2} = \frac{1}{H}$$

characterizes the system response at long-term loading ($t \rightarrow \infty$), and the value H is called the long-term modulus of elasticity.

Taking these relationships into consideration, as well as the fact that the modulus of elasticity of the paper at compression depends on the applied stress, Eq. 8 can be rewritten as Eq. 9.

$$\varepsilon + T_d \dot{\varepsilon} = \frac{\sigma}{H} + T_d \left(\frac{\dot{\sigma}}{E_0} \right) \quad (9)$$

The solutions of this equation for the case of instantaneous loading ($t = 0$) and long-term loading ($t \rightarrow \infty$) are as follows:

$$\varepsilon_{t=0} = \frac{\sigma}{E_0}$$

$$\varepsilon_{t \rightarrow \infty} = \frac{\sigma}{H}$$

Because of the similarity of the instantaneous and long-term moduli of elasticity (as noted previously), it is possible to state that

$$E_0 = \gamma H \quad (10)$$

where

γ = similarity factor

Substituting this equality into Eq. 9, and assuming that T_d is constant, leads to Eq. 11.

$$\varepsilon + T_d \dot{\varepsilon} = \sigma \left(\frac{\gamma}{E_0} \right) + T_d \left(\frac{\dot{\sigma}}{E_0} \right) \quad (11)$$

With experimental data describing the relationship between changes in the modulus of elasticity and the applied stress in the process of paper packing, the solution to Eq. 11 is found in the elementary functions. This allows us to determine stress as well as residual and delayed elastic deformation in time as the paper total deformation changes.

Modeled results were verified by comparing them with experimental data for writing paper (110 g/m²) and paperboard (150 g/m²) with loads applied over a time range of 10⁻⁴-10⁻² s, which corresponds to the paper loading regime in the calender. It has been established that the equations reflect the character of paper compression and recovery with a deviation $\pm 10\%$ (depending on the accuracy of the obtained moduli E_0 , H , and γ).

The parameters E_0 , γ , and T_d are determined experimentally. The modulus E_0 is established at the instantaneous loading ($t = 10^{-4}$ - 10^{-3} s). When the paper is loaded over a long period of time ($t \geq 10$ s) and the resistance of the delayed elastic deformation constituent is practically absent, the stress is proportional to the deformation by a factor that is equal to the long-term modulus of elasticity, i.e., $\sigma = H\varepsilon$ at $t \rightarrow \infty$. The parameter γ is determined as a ratio of the instantaneous and long-term moduli of elasticity, i.e., $\gamma = E/H$. As soon as the parameters E and γ have been obtained, the parameter T_d can be determined experimentally with due account of the following considerations. The linear law of the stress growth $\sigma = At$ is specified. Then the relation between the stress and deformation by virtue of the condition $d\sigma/dt = A = \text{constant}$ can be calculated according to Eq. 12.

$$\varepsilon + T_d \dot{\varepsilon} = t \left(\frac{\gamma A}{E_0} \right) + T_d \left(\frac{A}{E} \right) \quad (12)$$

where $\varepsilon = 0$ at $t = 0$

The expression that determines the deformation growth is given as Eq. 13.

$$\varepsilon = \frac{A}{E_0} \left\{ \gamma t - [(\gamma - 1) T_d (1 - e^{-t/T_d})] \right\} \quad (13)$$

It is obvious that at $t = T_d$, the deformation is described by Eq. 14.

$$\varepsilon = \frac{A}{E_0} \left[1 + \left(\frac{\gamma - 1}{e} \right) T_d \right] \quad (14)$$

Thus, if we plot the line using the known parameters A , γ , and E_0 ,

$$Y = \frac{A}{E_0} \left[1 + \left(\frac{\gamma - 1}{e} \right) \right] t \quad (15)$$

then at $t = T_d$ it will meet the deformation curve ε .

Figure 3c shows an example of how to determine the parameter T_d . The experiment was carried out on samples of sulfite cellulose with a mass of 100 g/m² and volume density of 0.65 g/cm³, brought up to dry-

ness 91% and 94% by pressing and subsequent drying on a cylinder. The rate of stress growth upon loading of the specimens was $A = 10^4$ MPa/s. The instantaneous modulus of elasticity for the 91% dry specimens was 20 MPa, and γ was 1.7. Then, according to Fig. 3c, the delay time will be 6×10^{-3} s.

In the process of press finishing the paper sheet (at calendering and sometimes at printing), it is useful to know the functional and quantitative relationship between the inlet parameters of the loading process (stress deformation) and the residual strain in the paper sheet.

Let's represent Eq. 11 in a form that is convenient for calculating the residual strain ε_p . For this purpose, we express ε_p in terms of the total deformation ε and the stress σ . According to the model,

$$\varepsilon = \varepsilon_p + \frac{\sigma_1}{E_1} + \frac{\sigma}{E_0} \quad (16)$$

$$\varepsilon = \varepsilon_p + \left(\frac{E_2}{E_1} \right) \varepsilon_p + \frac{\sigma}{E_0} \quad (17)$$

or

$$\frac{\sigma}{E_0} = \varepsilon - [\varepsilon_p(1+n)] \quad (18)$$

where $n = E_2/E_1$

After substituting this expression into Eq. 11, we obtain Eq. 19.

$$\varepsilon + T_d \dot{\varepsilon} = \gamma [\varepsilon - \varepsilon_p(1+n)] + T_d [\dot{\varepsilon} - (n+1) \dot{\varepsilon}_p] \quad (19)$$

or, after reduction of similar items, Eq. 20.

$$\gamma \varepsilon_p + T_d \dot{\varepsilon}_p = \left(\frac{\gamma-1}{n+1} \right) \varepsilon \quad (20)$$

By writing the equation in this way, it is possible to find the residual strain in cases where the law of the material deformation is known, as it is for the calendering process. When the web enters the calender nip, the law of its loading with respect to deformation is determined by the max-

imum deformation value $\varepsilon_{\max}$ and loading time τ_L . **Figure 4** shows the paper deformation in the calender nip. The law of the paper deformation alteration in the nips along the contact arc of the rolls with the paper can be represented by Eq. 21.

$$\varepsilon = \varepsilon_{\max} \frac{\cos \phi - \cos \phi_0}{1 - \cos \phi_0} \quad (21)$$

where

ε = current value for paper deformation

$\varepsilon_{\max}$ = maximum value for paper deformation

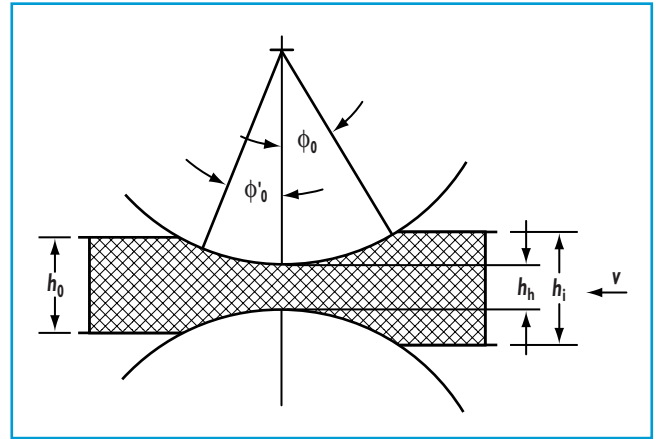
$$= (h_i - h_n)/h_i$$

h_i = initial paper thickness

h_n = paper thickness in the calender nip

ϕ = arc of the roll contact with the paper web within the range of changes ($0 < \phi < \phi_0$)

ϕ_0 = ingoing nip arc



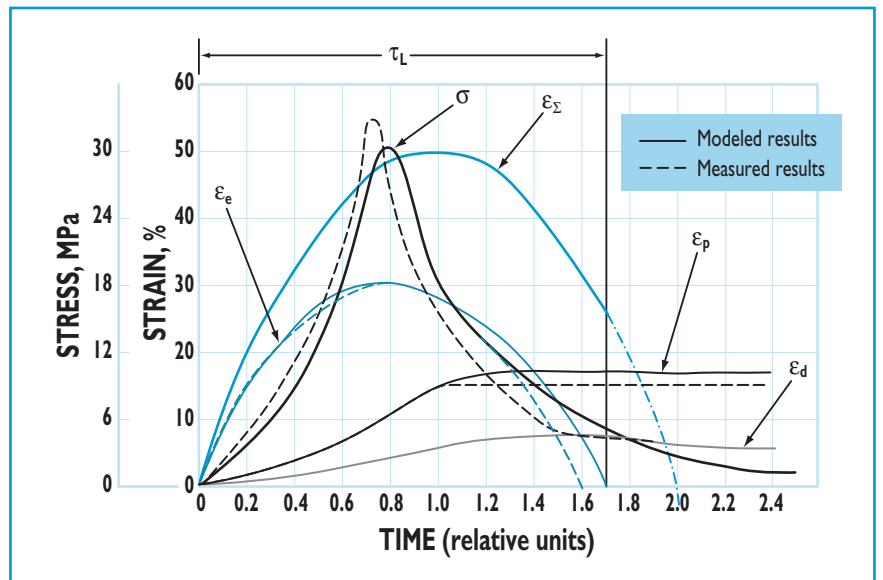
4. Sheet deformation in the calender nip

ϕ'_0 = outgoing nip arc (see Fig. 4)

After expansion of this function into the series of McLawren and passing from the dimensional coordinate on displacement to the dimensionless coordinate on time (τ), we come up with Eq. 22.

$$\varepsilon = \varepsilon_{\max} \tau(2 - \tau) \quad (22)$$

By substituting Eq. 22 into Eqs. 11 and 20, we obtain the equations that can be used, respectively, in the case of loading and unloading paper in the calender nip. **Figure 5** shows the change in applied stress (σ) and



5. Comparison of analytical and experimental results showing applied stress and resultant strains over time for paperboard (150 g/m²)

strains ($\epsilon_s, \epsilon_c, \epsilon_d, \epsilon_p$) over time along the nip arc length and beyond for paperboard (150 g/m²).

These equations can also be used to solve practical problems. For example, with knowledge of the calender dimensions and the sheet moduli, one can determine the linear pressure in the nip required to achieve a target packing value, i.e., the residual (plastic) strain, ϵ_p . The theory can also be used to predict sheet caliper after calendering, taking into account the relaxation in strain that occurs after the sheet exits the calender nip. Solutions to both problems are provided in the book *Treatment of Paper Under Pressure* (2).

CONCLUSION

Web deformation in response to an applied load can be modeled mathematically. Models of media with complex rheologies are widely used in

the plastics industry. In the paper industry, this approach is commonly used to model sheet deformation in the machine direction (linear model). This article presents a method of modeling sheet deformation in the cross-machine direction (nonlinear model). The proposed model of sheet response to calendering accounts for the nonlinear, stress-dependent phenomenon of plastic deformation at the moment of load application. When the model is applied correctly, the accuracy of the analytical solutions depends on the reliability of measured paper elastic rheological constants.

Recent investigations of paper deformation performed by Peel (6), Rodal (7), and Browne *et al.* (1) suggest that further research can improve the correlation between analytical and practical results. **TJ**

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LITERATURE CITED

1. Browne, T. C., Crotagino, R. H., and Douglas, W. J. M., *J. Pulp Paper Sci.* 22(5): J170(1996).
2. Feygin, V. B., *Treatment of Paper Under Pressure at Finishing Process*, Forest Industry, Moscow, Russia, 1989.
3. Brecht, W., *Papier* 10: 563(1971).
4. Meinecke, A., *Wochbl. Papierfabr.* 113(22): 858(1985).
5. Fljate, D. M., *Paper Properties*, Forest Industry, Moscow, Russia, 1986, p. 680.
6. Peel, J. D., *Mechanical Engineering Publications*, London, Vol. 2, 1989, pp. 979-1025.
7. Rodal, J. J. A., *TAPPI 1993 Finishing and Converting Conference Proceedings*, TAPPI PRESS, Atlanta, pp. 321-331.

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