

Fundamentals of flotation deinking

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THE IDEA FOR USING FLOTATION to remove ink particles from secondary fibers was inspired by the flotation separation processes used in the mineral processing industry. Flotation deinking was first used well before the 1950s, but it was not until then that the practice began to gain wider acceptance in the paper processing industry (1).

The mineral processing industry has provided much useful information regarding flotation separation in terms of fundamentals and overviews (e.g., (2, 3)). The literature also contains works that summarize the use of flotation in paper recycling (1, 4–6). However, these latter reviews focus primarily on applications and equipment. The purpose of this paper is to review the current state of flotation fundamentals and show how these fundamentals are used in modeling this complex process.

In flotation deinking, air bubbles rise through agitated liquids in tanks containing suspended cellulose pulp and contaminant particles. The rising bubbles collect hydrophobic contaminant and/or ink particles that typically fall within the size range of approximately 20–300 μm in diameter. The attached particles are then transported to a froth layer, where they are easily removed. Although flotation cell designs vary with respect to geometry, flow configurations, and operating parameters, they all operate on similar principles and incorporate the fundamental processes of pulp aeration, mixing to maximize

bubble-particle interaction, and separation of bubble-particle aggregates from the bulk mixture.

This paper focuses on the bubble-particle interaction and the formation of a stable bubble-particle aggregate that can be transported to the froth layer for removal. To provide context for a discussion of these issues, there is a brief summary of the similarities and differences between mineral flotation and flotation deinking. This is followed by descriptions of the four flotation microprocesses—capture, attachment by sliding, three-phase contact, and stability—that serve as components of overall flotation models. Finally, the current status of flotation modeling is presented.

MINERAL FLOTATION AND FLOTATION DEINKING

Much of the theoretical and experimental work related to the fundamental flotation microprocesses has been developed explicitly for mineral flotation. Nevertheless, many of these fundamental principles are also assumed to be applicable to flotation deinking, as long as one is aware of the main similarities and differences between the two processes. **Table I** lists some of the similarities and differences between flotation deinking and mineral processing (7). Deinking differs in that the particle density is relatively low and can be comparable to water; deinking systems typically have a wider distribution of particle size and characteristics (i.e., surface energy, geometry, etc.); and pulp suspensions consist of

ABSTRACT

Flotation deinking is a complex process. A better understanding of its fundamentals would help in developing models that can predict whether a given process change would help or hinder flotation performance before attempting an expensive mill trial. Knowledge of flotation fundamentals can also be used to improve equipment design and performance. This paper reviews the fundamentals of flotation separation. Because the use of flotation in paper recycling was adopted from the mineral processing industry, the discussion begins with a comparison of the similarities and differences between flotation deinking and mineral flotation. The flotation macroprocess is presented as a function of four sequential microprocesses: particle capture (or interception), attachment by sliding, three-phase contact, and stability. These microprocesses are reviewed in detail, followed by a discussion of selected flotation deinking models.

Application:

Better knowledge of flotation fundamentals can enhance deinking performance and improve the design of future flotation cells.

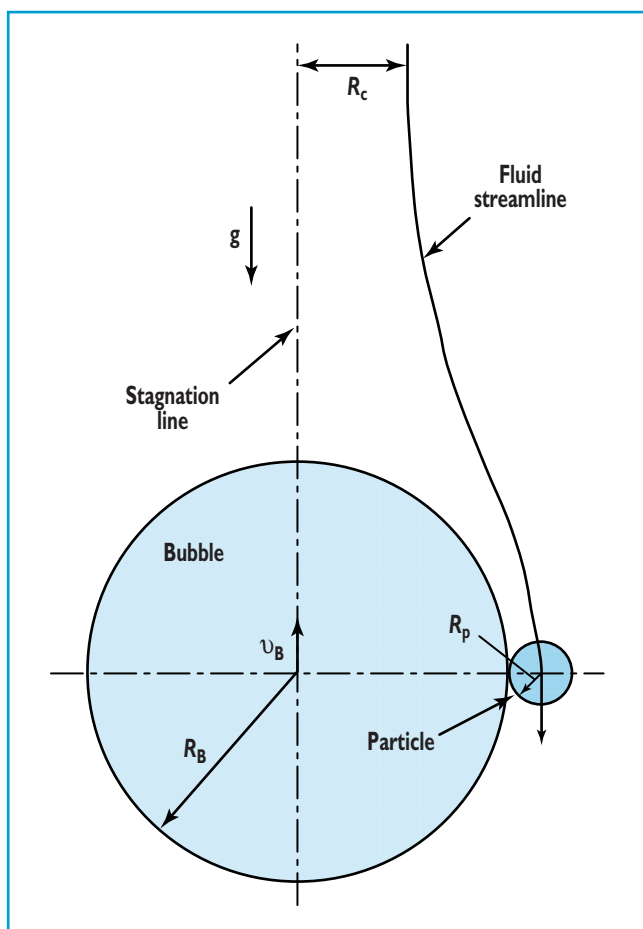
cellulose fibers that can form a network (7, 8). In addition, both the contaminants to be removed and the pulp slurry can be highly heterogeneous. These differences result in much lower separation efficiencies in flotation deinking than in mineral flotation (9).

Despite these differences, it is commonly assumed that the fundamentals of flotation separation are similar for both processes, with the exception of inertial effects, which are typically neglected in flotation deinking. Therefore, the four microprocesses that are applicable to min-

Parameter	Flotation deinking	Mineral flotation
Particle surface energy	Very complex • Low-energy, hydrophobic adhesives • Medium-energy, hydrophobic inks • High-energy, hydrophilic filler and fibers	Fairly uniform • High-energy, hydrophilic materials
Particle size	Broad distribution	Broad distribution
Particle density	Very low, in some cases lower than water	Broad distribution, but higher than water
Particle liberation from stock	By repulping in the presence of chemicals and surfactants	By grinding without added chemicals
Pulp properties	• Very heterogeneous • High temperature (40–60°C) • Possible redeposition of particles • Fiber networks	• Homogeneous • Low temperature • Redeposition is not important
Final product characterization	By means of • Paper sheet brightness • Dirt speck analysis • Stickies content	By means of • Chemical analysis
Impact of efficiency on additional processing steps	Contaminants that are not removed can • Damage downstream equipment • Reduce product quality	Final product purity is affected

* Adapted from Schulze (7).

I. Comparison between flotation deinking and mineral flotation.



I. All particles within the streaming tube of radius R_c will be captured by (or collide with) the air bubble.

eral processing are also used to describe the events occurring in a flotation deinking cell.

FLOTATION MICROPROCESSES

The overall flotation separation process is modeled as a multistage probability process consisting of a sequence of microprocesses with associated probability values. This sequence includes (a) the approach of a particle to an air bubble and the subsequent interception of that particle by the bubble; (b) the sliding of the particle along the surface of the thin liquid film that separates the particle from the bubble and film rupture; (c) the subsequent formation of a three-phase contact between the bubble, particle, and fluid; and (d) stabilization of the bubble-particle aggregate and its subsequent transport to the froth layer for removal from the flotation cell. These microprocesses take place in all flotation equipment.

Probability values for some of these elementary microprocesses are available in the literature (e.g., (2, 9–12)), as are mathematical descriptions (13–25). Most of these models assume that the bubble and particle are spherical. In addition, most of the microprocess analyses consider the interaction of one particle with one bubble. Efforts to include the effects of multiple particles interacting with a single bubble (26, 27) or bubble swarms (26, 28) on the flotation microprocesses complicate the analysis considerably.

In general, the process of collecting contaminant particles by bubbles in a pulp slurry takes place in a complex and highly turbulent environment. However, as the distance separating the particle from the bubble de-

creases, the flow conditions relative to the bubble-particle pair are typically idealized and simplified to correspond to unperturbed flow (17). This allows for considerable simplifications that are universally employed in modeling the flotation microprocesses. The following discussion reviews the four microprocesses: particle capture, attachment by sliding, three-phase contact, and stability.

Particle capture

A particle must travel close enough to a given bubble if it is to be captured. Some authors refer to this process as collision or interception (e.g., (17)). When capture (or collision) occurs, it does not imply that a bubble-particle aggregate has formed; rather, it only means that short-range forces and thin-film dynamics become significant (to be discussed in the next section). A critical parameter governing the particle approach to the bubble and subsequent capture is R_c , the radius of the streaming tube within which the particle must move so as to be intercepted by the bubble. Capture radius R_c is illustrated in Fig. 1.

The corresponding probability of capture (or collision) P_c depends on the particle and bubble radii, R_p and R_b respectively, as well as on the assumptions one makes about the flow field in which the particle moves. Particles the size of typical ink particles in a flotation cell experience negligible inertial forces and tend to follow the streamlines in the flow field around the bubble. The probability of capture is then defined by Eq. 1.

$$P_c = \left(\frac{R_c}{R_b} \right)^2 \quad (1)$$

This represents the ratio of (a) the number of particles (assuming $R_p \ll R_b$) within the streaming tube of cross-sectional area πR_c^2 that encounter a bubble per unit time to (b)

the number of all particles that approach a bubble in a streaming tube with cross-sectional area πR_b^2 . The Stokes number (St) and the bubble Reynolds number (Re_b) are critical in defining the flow parameters that characterize P_c . The Stokes number represents a ratio of inertial to drag forces and is defined as

$$\begin{aligned} St &\equiv \frac{Re_b \rho_p d_p^2}{9 \rho_l d_b^2} \\ &= \frac{\rho_p d_p^2 v_b}{9 \mu_l d_b} \end{aligned} \quad (2)$$

where d_b is the bubble diameter, d_p is the particle diameter, ρ_p is the particle density, ρ_l is the liquid density, μ_l is the fluid dynamic viscosity, and v_b is the bubble rise velocity. For typical ink particles, the Stokes number is very small ($St \ll 1$) and the inertial forces have a negligible influence on particle motion. Hence, there are basically no impact collisions (i.e., such as between two billiard balls) between the particles and bubbles, and the particles follow the streamlines in the flow field around the bubble (Fig. 1). The bubble Reynolds number in Eq. 2 is defined by Eq. 3.

$$Re_b = \frac{v_b d_b \rho_l}{\mu_l} \quad (3)$$

The bubble surface mobility can also influence P_c (10), but it can be assumed that in flotation deinking, the bubble is coated with surfactants and/or other surface contaminants, and the rising bubble can be approximated as a rigid sphere. By considering only the long-range hydrodynamic forces that act on a particle as it approaches a bubble (i.e., the drag, gravitational, and buoyancy forces), a system of equations (2, 28, 29) with associated initial conditions can be written for the velocity components of the particle, whose structure de-

pends in a crucial manner on the nature of the velocity field of the liquid in a neighborhood of the bubble. For flow conditions in which $Re_b \ll 1$, the fluid flow is described by Stokes flow, and P_c can be determined by Eq. 4 (17).

$$P_c = \frac{3}{2} \left(\frac{R_p}{R_b} \right)^2 \quad (4)$$

When the bubble Reynolds number is very large, potential flow conditions exist around the bubble, and the probability of capture can be expressed by Eq. 5 (17, 30).

$$P_c = 3 \left(\frac{R_p}{R_b} \right) \quad (5)$$

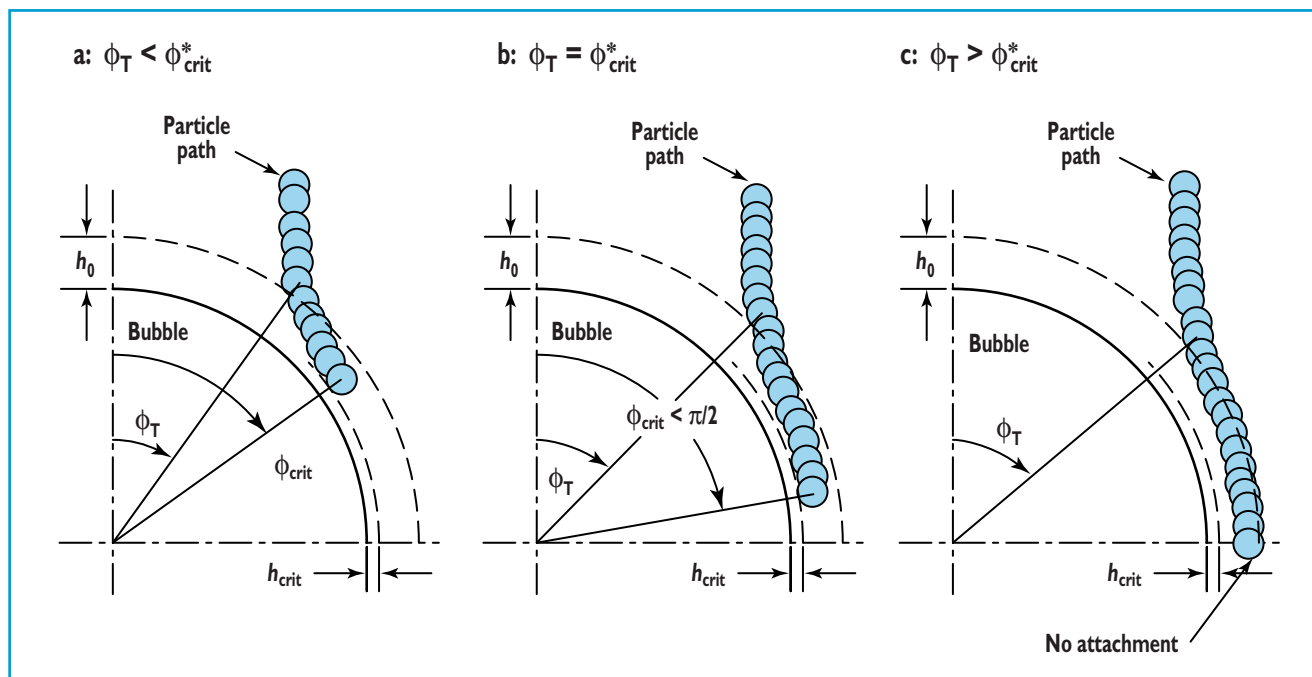
In typical flotation cells, Re_b is thought to be in the range of $1 < Re_b < 100$. Applying this bubble Reynolds number range (termed intermediate flow), a Stokes number $St \ll 0.1$, and the assumption of a rigid bubble surface, Yoon and Luttrell (17) obtained Eq. 6 by interpolating between Stokes flow and potential flow, and then correlating the resulting streamlines with experimental data obtained at various Reynolds number.

$$P_c = \left(\frac{3}{2} + \frac{4 Re_b^{0.72}}{15} \right) \left(\frac{R_p}{R_b} \right)^2 \quad (6)$$

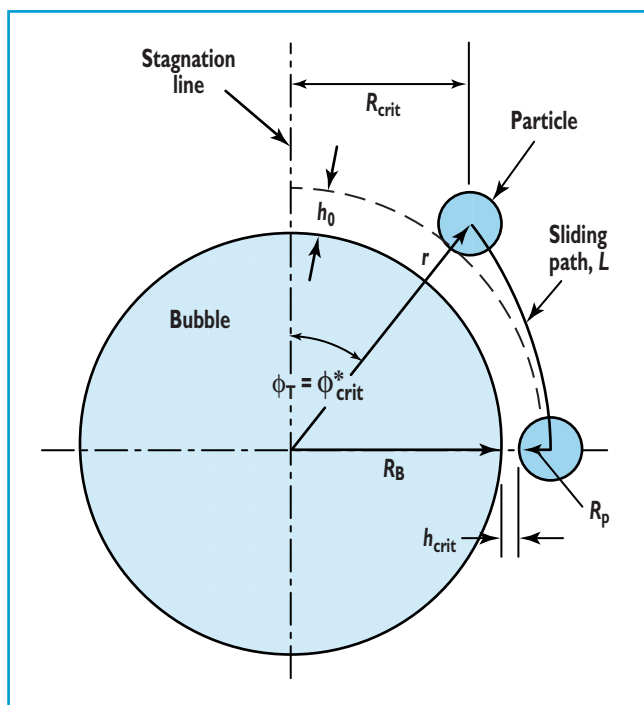
More complex correlations for P_c are available in the literature (31-33).

Attachment by sliding

Not all particles that are captured by a bubble become attached. In general, only particles that are sufficiently hydrophobic are able to attach themselves to the bubble through the formation of a three-phase contact with a finite contact angle (17). Before this three-phase contact can take place, the liquid



2. Schematic representation of the relationship among ϕ_T , h_{crit} , and ϕ_{crit}^* . (Note: diagram is not to scale.)



3. All particles within a streaming tube of radius R_{crit} will attach to a bubble by sliding. (Note: diagram is not to scale.)

layer between the bubble and particle—which forms as soon as the particle is captured by a bubble—must thin sufficiently to rupture. This liquid layer is typically referred to as a liquid film because thin-film dynamics have been used to describe the rupture process. Upon liquid film formation, the particle begins to slide over the sur-

face of the bubble and resides on it for a finite period of time, which is generally referred to as the sliding time. This sliding process subjects the film to a weak surface deformation, which tends to thin the film out and can lead to film rupture. For attachment by sliding to occur, the contact time of the particle with the liquid film must be greater than the induction (drainage) time of the film up to the point of rupture.

To study this problem, one must model the motion of a particle as it slides past the bubble with a thin liquid layer of thickness h separating the bubble and particle. The modeling of the process predicts the film thickness h as a function of the position angle ϕ of the particle. Note that both h and ϕ vary with time t and that $\phi(t=0) = \phi_T$, which is referred to as the touching angle. The following assumptions are commonly made when modeling the particle motion during the sliding process (10, 11): (a) the particles move in a quasi-stationary manner on an almost circular path along the bubble surface; (b) the sliding path $L \gg h$ and $dL/dt > dh/dt$; (c) for $0 < \phi < \pi/2$, the influence of the fluid boundary layer is negligible; and (d) as in the computation of P_c , the tangential component of the fluid velocity field can be modeled by the intermediate flow of Yoon and Luttrell (17).

Particle attachment by sliding over the bubble surface is probably the single most important microprocess in the overall flotation macroprocess, and it is also possibly the most complicated. The sliding motion of a particle is governed by a force balance as the particle slides over the bubble. Depending on the assumptions made and the magnitude of the forces involved, the following forces may be included in this force balance (2, 10, 34–36):

- The resistive force generated during the drainage of the liquid film surrounding the bubble surface
- The (apparent) weight of the particle
- The centrifugal force acting on the sliding particle
- The flow force acting on the sliding particle close to the bubble wall
- The lift force acting on the particle
- The drag force acting on the particle (which is strongly dependent on the flow field about the bubble).

From the force balance, a system of ordinary differential equations can be written in terms of (ϕ, h) . These equations govern the variation of the disjoining film thickness with respect to the position of the (sliding) particle and the associated initial condition $h(\phi_T) = h_0$. If h_{crit} denotes the critical thickness that the film must reach in order for rupture to occur, then the critical position angle ϕ_{crit}^* is defined as the largest touching angle ϕ_T ($< 90^\circ$) for which $h = h_{crit}$ will be achieved at a position angle ϕ_{crit} such that $\phi_T < \phi_{crit} < \pi/2$. **Figure 2** depicts the relationship that exists among ϕ_T , h_{crit} , and ϕ_{crit}^* . Note that the process of the particle sliding past a bubble surface does not guarantee formation of a bubble-particle aggregate.

Knowledge of film drainage and rupture is important in this micro-process to estimate the particle induction time, τ_i , or the critical film thickness at which rupture occurs, h_{crit} . Thin-film dynamic models have been used to identify the critical film thickness (19, 21, 22, 37–39), and there is continued interest in this subject because questions still remain regarding which forces are important to film rupture (37).

The probability of attachment by sliding, P_{asl} , is then defined from

knowledge of the touching angle and the sliding time. If the touching angle, ϕ_T , is less than the critical position angle, ϕ_{crit}^* , the sliding time will be sufficient for particle attachment. Therefore, all particles that satisfy $\phi_T \leq \phi_{crit}^*$ will attach to a bubble, and those particles that “touch” the bubble with $\phi_T > \phi_{crit}^*$ will not. Referring to **Fig. 3**, P_{asl} is typically described to be the ratio of the area inscribed by the limiting radius R_{crit} (the radius from the stagnation line to the line corresponding to the touching angle associated with ϕ_{crit}^*) to the area inscribed by the sum of the bubble, particle, and critical film thickness ($R_B + R_p + h_{crit}$). Since we can expect $h_{crit} \ll R_p$, it is common to write P_{asl} as given in Eq. 7 (10, 11, 17).

$$P_{asl} = \frac{R_{crit}^2}{(R_B + R_p)^2} \quad (7)$$

Relating R_{crit} to ϕ_{crit}^* , Eq. 7 can be written as

$$P_{asl} = \sin^2 \phi_{crit}^* \quad (8)$$

The critical position angle ϕ_{crit}^* is thought to be a complex function of the system parameters (i.e., bubble and particle radius, turbulent energy density, etc.) that can be determined only by numerically solving the system of ordinary differential equations that govern the adhesion-by-sliding process (10). Some authors have proposed expressions for ϕ_{crit}^* and/or expressions for the particle induction time, τ_i (17, 33, 40–42). For example, Yoon and Luttrell (17) propose

$$P_{asl} = \sin^2 \left\{ 2 \arctan \exp \left[\frac{-(45 + 8 \text{Re}_B^{0.72}) v_B \tau_i}{30 R_B (R_B/R_p + 1)} \right] \right\} \quad (9)$$

for intermediate flow conditions. However, the bubble rise velocity, v_B , and the particle induction time, τ_i ,

must be known before P_{asl} can be determined.

Three-phase contact

Once the thin film surrounding a bubble has ruptured, a relatively large three-phase contact (TPC) between the liquid, particle, and bubble must form within a sufficiently short time span (τ_{TPC}) to provide a strong enough force of attachment to prevent the bubble-particle aggregate from immediately separating. This aggregate formation is schematically represented in **Fig. 4**, where θ represents the contact angle measured in the liquid.

If τ_v denotes the average lifetime for turbulent vortices within the flotation cell, then the time required to form a three-phase contact and create a bubble-particle aggregate must satisfy $\tau_{TPC} < \tau_v$. Schulze (10) has used an exponential distribution of the approximate form

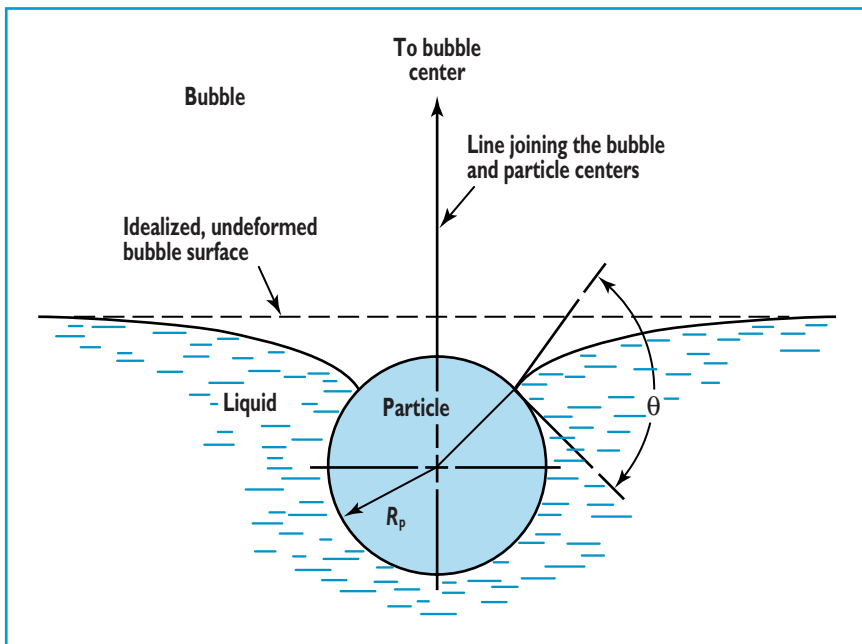
$$P_{TPC} \equiv 1 - \exp \left(\frac{-\tau_v}{\tau_{TPC}} \right) \quad (10)$$

for the probability of extension of the three-phase contact P_{TPC} . However, it has been shown that to within about 1%, $P_{TPC} = 1$ over a wide range of particle sizes (10). Indeed, many authors imply that $P_{TPC} = 1$ and omit this term from the overall probability of flotation (7, 14, 17, 25, 34, 35, 38), which seems justified.

Stability

Once a bubble-particle aggregate forms, it must remain stable on its journey to the froth layer if it is to be removed from the system. It is generally accepted (2, 10, 24, 43) that bubble-particle stability can be determined by performing a force balance on the particle attached to the bubble. **Figure 5** summarizes these forces.

Assuming spherical particles, the gravitational force (**Fig. 5a**) is specified by



4. Three-phase contact between the bubble, particle, and liquid regions.

$$F_g = \frac{4}{3} \pi R_p^3 \rho_p g \quad (11)$$

where ρ_p is the particle density, and g is the acceleration due to gravity.

The static buoyancy force that acts on the particle (Fig 5b)—assuming that the entire particle is immersed in the liquid (24)—is defined by Eq. 12.

$$F_b = \frac{4}{3} \pi R_p^3 \rho_l g \quad (12)$$

where ρ_l is the liquid density.

The buoyant and gravitational force can be combined to obtain an expression for the apparent particle weight

$$F_{wt} = \frac{4}{3} \pi R_p^3 (\rho_p - \rho_l) g \quad (13)$$

For the detaching force due to fluid drag (Fig. 5c),

$$F_d = \frac{4}{3} \pi R_p^3 \rho_p a_c \quad (14)$$

where expressions for the fluid acceleration a_c depend on both the structure and intensity of the turbulence within the flotation cell. For aggregates where the particle size is smaller than the bubble size, it has

been determined that the fluid acceleration can be related to the energy dissipation in the tank by Eq. 15 (10).

$$a_c \approx \frac{1.9 \varepsilon^{2/3}}{(R_B + R_p)^{1/3}} \quad (15)$$

where ε is the turbulent energy density.

The force generated by the capillary pressure in the gas bubble that acts on the contact area of the attached particle (Fig. 5d) is given by

$$F_{\sigma} \approx \pi R_p^2 \left(\frac{2\sigma}{R_B} - 2R_B \rho_l g \right) \sin^2 \omega \quad (16)$$

where σ is the surface tension and ω is the angle specified in Fig. 5.

The capillary force exerted on the three-phase contact line (Fig. 5e) is

$$F_{ca} = -2 \pi R_p \sigma \sin \omega \sin(\omega + \theta) \quad (17)$$

where θ is the contact angle.

Finally, the hydrostatic pressure force (Fig. 5f) of the liquid of height z_0 above the contact area radius r_p ($= R_p \sin \omega$ in Fig. 5f) is

$$F_{hyd} = \pi R_p^2 \rho_l g z_0 \sin^2 \omega \quad (18)$$

Therefore, the net detachment force that acts on a bubble-particle aggregate is given by

$$F_{detach} = F_{wt} + F_d + F_{\sigma} \quad (19)$$

while the net attachment force is given by

$$F_{attach} = F_{ca} + F_{hyd} \quad (20)$$

The stability of bubble-particle aggregates is then characterized by comparing the net detachment force with the net attachment force by the following dimensionless parameter, with $F_{hyd} \approx 0$ and F_{ca} replaced by the maximum capillary force F_{cam} (10).

$$Bo' = \frac{F_{detach}}{F_{attach}} \quad (21)$$

where

$$F_{detach} = 4R_p^2 \left(\Delta \rho_p g + \frac{1.9 \rho_p \varepsilon^{2/3}}{(R_p + R_B)^{1/3}} \right) + 3R_p \left(\frac{2\sigma}{R_B} - 2R_B \rho_l g \right) \sin^2 \left(\pi - \frac{\theta}{2} \right) \quad (22)$$

and

$$F_{attach} = \left| 6\sigma \sin \left(\pi - \frac{\theta}{2} \right) \sin \left(\pi + \frac{\theta}{2} \right) \right| \quad (23)$$

In Eq. 22, $\Delta \rho_p = \rho_p - \rho_l$. As cited in Schulze (10), taking into account the experimental results of Plate (44), a reasonable form for the probability of stability P_{stab} is

$$P_{stab} = 1 - \exp \left(1 - \frac{1}{Bo'} \right) \quad (24)$$

Recent studies suggest that once a bubble-particle aggregate forms through attachment by sliding, the bubble has a strong grip on the particle, which tends to resist breakup (45). However, these results contradict observations made by equipment manufacturers concerning tur-

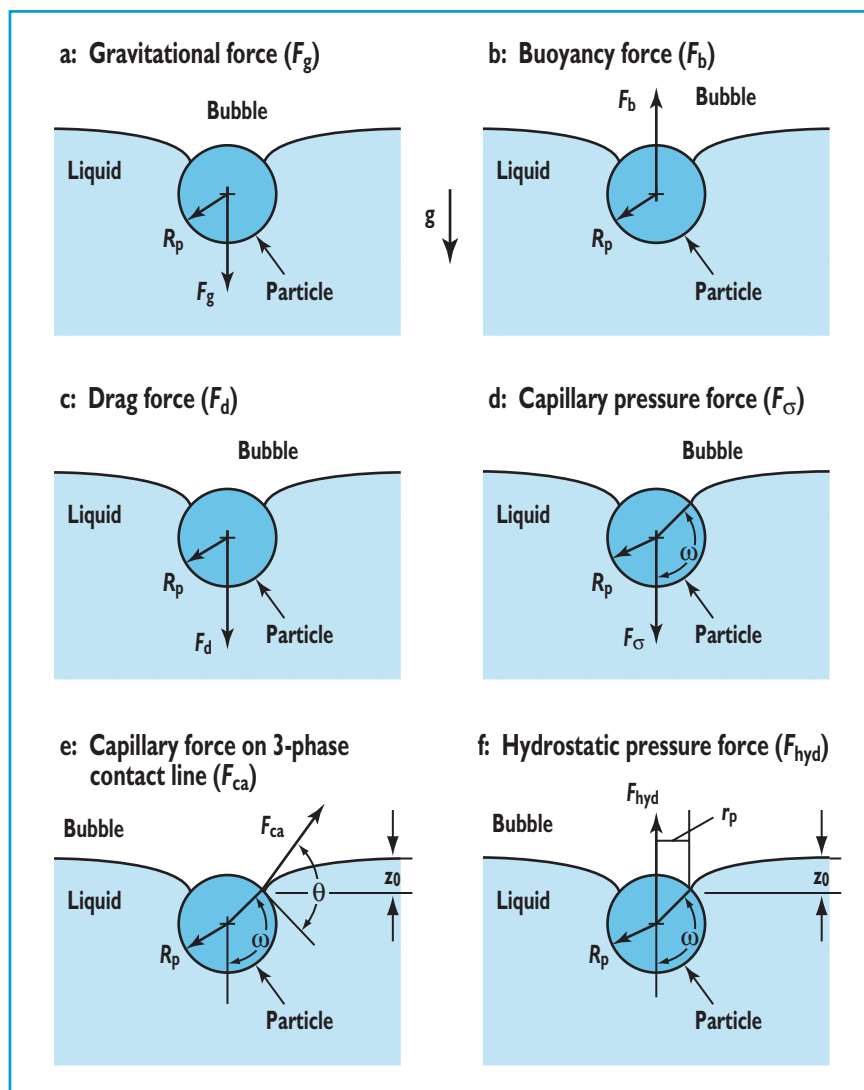
bulence (46). Additionally, flotation-cell product literature describes regions in most flotation cells where turbulence is reduced to promote the formation of stable bubble-particle aggregates (47-52). If these aggregates also collide with other aggregates on their way to the froth layer, the bubble-particle aggregate can be destroyed, with the particles falling back into the suspension. The influence of turbulence on destruction of bubble-particle aggregates has recently been studied by Julien Saint Amand (53).

FLOTATION MODELING

The microprocess probabilities are used to model the overall flotation macroprocess. The mineral flotation literature states that the simplest studies addressing the floatability of a particular ore involve at least 25 clearly identifiable variables, and a full-scale study could require consideration of at least 100 variables (3). However, there is little information about the interactions among these variables. Considering that flotation deinking involves a more complex fibrous suspension, additional variables should be considered in a full-scale study of flotation deinking.

Because experimental studies and system trials can be very expensive, there has been considerable effort to mathematically model the flotation process to predict separation efficiencies and general trends before full-scale experiments are performed. These models typically incorporate the various microprocess models within an overall flotation model.

The complexity of the flotation microprocesses has prevented the development of a flotation model based on first principles. However, in what has become a common assumption, Schuhmann (54) assumed that the individual microprocess probabilities for a given particle are not correlated. Therefore, the overall

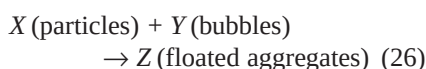


5. Stabilizing and destabilizing forces that act on a bubble-particle aggregate.

probability that a stable bubble-particle aggregate will form and be lifted to the froth layer can be written as

$$P_{\text{overall}} = P_c P_{\text{asl}} P_{\text{TPC}} P_{\text{stab}} \approx P_c P_{\text{asl}} P_{\text{stab}} \quad (25)$$

Additionally, on the basis of mineral flotation experimental evidence, the flotation process is thought to be analogous to a chemical reactor, where the overall rate of particle removal is expressed as a first-order reaction in the form (55):



If the bubble concentration is constant—and assuming that the re-

moved particles represent a small volume—then the rate of change in solids is proportional to the concentration. Therefore, flotation is assumed to be governed by Eq. 27 (2, 10, 12, 13, 30, 53, 55-59)

$$\frac{dc}{dt} = -kc \quad (27)$$

where c is the particle concentration, and k is the rate constant involving the physical, chemical, and surface properties of the system. The form of this equation suggests that given a long enough flotation time, all particles will eventually be removed, which is typically not observed in practice. Many expressions for the rate constant are available in

the literature (e.g., (17, 25, 35, 53, 55, 60)), and these may depend on the specific system being studied.

Employing the overall probability of particle removal, P_{overall} , Schulze (2, 9, 10) has written Eq. 27 in the form

$$\frac{dn_p^f}{dt} = -Z P_{\text{overall}} n_B^f n_p^f \quad (28)$$

where n_p^f corresponds to the number of free particles per unit volume available to attach to bubbles, n_B^f is the number of bubbles per unit volume available for particle attachment, and Z is related to the bubble-particle collision frequency. In the absence of bubble-particle collision frequencies specific to flotation deinking, a reasonable first estimate for Z can be represented by Eq. 29 (23)

$$Z = 2^{7/9} \frac{5}{3} \left(\frac{\varepsilon^{4/9}}{v_t^{1/3} \rho_t^{2/3}} \right) (R_p + R_B)^2 \times (R_p^{14/9} \Delta \rho_p^{4/3} + R_B^{14/9} \Delta \rho_B^{4/3})^{1/2} \quad (29)$$

where v_t is the (fluid) kinematic viscosity, $\Delta \rho_p = \rho_p - \rho_t$, and $\Delta \rho_B = \rho_B - \rho_t$. Other forms of the collision frequency are also available in the literature (e.g., (13, 61-63)). Note that Eq. 29 differs from the expression presented in a previous work by Heindel and Bloom (64) by a factor of $n_B n_p$, where n_B and n_p are the total number of bubbles and particles per unit volume, respectively. The original expression from Heindel and Bloom (64) was adopted following the work of Schulze (9, 16). However, as recently pointed out by Julien Saint Amand (53), the Z term introduced in Eq. 28 should not have the $n_B n_p$ term because it accounts for the collisions between free bubbles (n_B^f) and free particles (n_p^f), which are already incorporated into Eq. 28. Therefore, $Z n_B^f n_p^f$ is a true collision frequency, with units of numbers per unit time per unit volume.

Equations 27 and 28 represent a kinetic- or population balance-type model where, in this case, the population of free particles are modeled. The idea of a kinetic or population balance equation was extended by Heindel and Bloom (43, 64) to include a forward and reverse reaction. This model has the form

$$\frac{dn_p^f}{dt} = -k_1 n_p^f n_B^f + k_2 n_B^a \quad (30)$$

where n_B^a represents the number of bubbles with attached particles in the unit volume. The first term on the right-hand side represents the overall probability that a free particle will successfully attach to a bubble that is initially free of particles. The second term is a measure of the probability that a bubble-particle aggregate will become unstable and split to yield a "new" free particle. This term has not been explicitly included in previous flotation models (i.e., (2, 10, 12, 30, 53, 56, 60)). The kinetic constants k_1 and k_2 are positive numbers described by the various microprocess probabilities and the collision frequency:

$$k_1 = Z P_c P_{\text{asl}} P_{\text{TPC}} P_{\text{stab}} \quad (31)$$

$$k_2 = Z' P_{\text{destab}} = Z' (1 - P_{\text{stab}}) \quad (32)$$

where Z is given by Eq. 29. Based on dimensional considerations, k_2 should have units of 1/time, giving the second term on the right-hand side of Eq. 30 the units of number per time per unit volume, which is consistent with the other terms. This kinetic constant has a slightly different form than that previously presented by Heindel and Bloom (64). Since P_{destab} (or P_{stab}) is a true probability and is unitless, an additional term has to be included in k_2 , which we call Z' and has units of 1/time.

This term could be thought of as a collision rate between bubble-particle aggregates and the "thing" that makes the aggregate unstable, like a turbulent eddy or another bubble-particle aggregate. Current research is directed toward quantifying this term.

A more detailed model of the flotation process has been proposed by Schulze (2, 12), whose model includes a system of partial differential equations that govern free and attached particles in the system and accounts for particle advection and diffusion, as well as particle source and sink terms within the system. However, no attempt has been made at solving this system of partial differential equations.

Models of the flotation separation process can be very simplistic or very complicated. However, all models have the goal of predicting the trends observed in practice. If this is accomplished, the models can be used to predict changes in system performance when one or more system parameters are changed. However, current operating conditions—and any changes to these conditions resulting from a change in the desired parameter—must be known. Values of selected parameters (i.e., turbulent energy density, surface tension, etc.) can be estimated, but the utility of the resultant predictions will depend on the accuracy of these estimates.

SUMMARY

Flotation deinking is such a complicated process that it may seem fortuitous that it actually works in a recycle mill. The initial research into flotation fundamentals was carried out by the mineral processing industry, and existing flotation deinking models are based on this early research. A review of the similarities and differences between mineral flotation and flotation deinking demonstrates that the flotation fun-

damentals apply to both processes.

The overall macroprocess of flotation deinking can be explained in terms of four sequential microprocesses: particle capture, attachment by sliding, three-phase contact, and stability. This paper reviews all four microprocesses and then outlines various models of the overall

flotation process. Such models typically follow a kinetic-type formulation. **TJ**

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NOMENCLATURE

a_c = fluid acceleration	P_{overall} = overall probability of particle removal
Bo' = Eq. 21	P_{stab} = probability of stability
c = particle concentration	P_{TPC} = probability of three-phase contact
d_p = particle diameter	R_B = bubble radius
d_B = bubble diameter	R_c = capture radius
F_{attach} = net attachment force	R_{crit} = critical radius corresponding to ϕ_{crit}^*
F_{ca} = capillary force on three-phase contact line	Re_B = bubble Reynolds number
F_{cam} = maximum capillary force	R_p = particle radius
F_b = buoyancy force	r_p = radius specified in Fig. 5
F_d = drag force	St = Stokes number
F_{detach} = net detachment force	t = time
F_g = gravitational force	Z = Eq. 29
F_{hyd} = hydrostatic pressure force	Z' = collision rate between a bubble-particle aggregate and the "thing" that makes it unstable
F_{wt} = apparent particle weight	z_0 = liquid height specified in Fig. 5
F_{σ} = capillary pressure force	ε = turbulent energy density
g = acceleration due to gravity	θ = contact angle
h = film thickness (distance separating the particle from the bubble)	μ_{ℓ} = liquid dynamic viscosity
h_0 = initial film thickness	ν_{ℓ} = liquid kinematic viscosity
h_{crit} = critical film thickness when rupture occurs	ρ_B = bubble density
k = rate constant	ρ_p = particle density
k_1 = Eq. 31	ρ_{ℓ} = liquid density
k_2 = Eq. 32	$\Delta\rho_B = \rho_B - \rho_{\ell}$
L = sliding path length	$\Delta\rho_p = \rho_p - \rho_{\ell}$
n_B = total number of bubbles per unit volume	σ = surface tension
n_B^a = the number of bubbles per unit volume with particles attached	τ_i = particle induction time
n_B^f = the number of bubbles per unit volume available for particle attachment	τ_{TPC} = time required to form a three-phase contact
n_p = total number of particles per unit volume	τ_v = average lifetime of a turbulent vortex
n_p^f = the number of free particles per unit volume available to attach to bubbles	v_B = bubble rise velocity
P_{asl} = probability of attachment by sliding	ϕ = position angle
P_c = probability of capture (or collision)	ϕ_{crit} = critical touching angle when rupture occurs
P_{destab} = probability of destabilization	ϕ_{crit}^* = largest touching angle such that attachment by sliding will still take place
	ϕ_T = touching angle
	ω = angle specified in Fig. 5

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